# Hysteretic (bistable) cyclotron resonance in semiconductors

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The cyclotron resonance in semiconductors under the action of quasiresonant radiation is considered in the classical single-electron approximation. It is shown that under some critical conditions, bistability and large hysteretic jumps in momentum of an electron (and related jumps in the electrical potential between the faces of the semiconductor layer) could be observed. This is due to the nonparabolicity of the semiconductor conduction band, which results in a nonlinear relation between the applied electromagnetic field intensity and the cyclotron frequency. For a magnetic field of $\sim 140$ kG, the 10.6-$\mu$m radiation of a $CO_2$ laser of 240 W/cm$^2$ is sufficient to excite a bistable resonance in InSb.

## I. INTRODUCTION

In previous work[1] by one of the authors it was shown that the relativistic mass effect of an electron can result in a bistable behavior (and in large hysteretic jumps) of the cyclotron resonance of the electron in free space. This effect is based on the dependence of the cyclotron frequency of forced oscillations on the relativistic mass of the electron, and hence on its momentum (or kinetic energy). The critical condition for bistability to occur is that the nonlinear (relativistic) shift of the cyclotron frequency must be sufficiently greater than the linewidth of the resonance. To some extent, this effect resembles hysteresis in a classic nonlinear oscillator[2] or in nonlinear parametric systems.[3] In Ref. 1 it was also pointed out that an analogous effect could be expected in semiconductors whose conduction electrons have a particularly strong dependence of their effective masses on their excitation energy.

In this paper, we consider, for the first time to the best of our knowledge, the feasibility of a bistable (hysteretic) cyclotron resonance in semiconductors. The effect is feasible due to the nonparabolicity of the semiconductor conduction band which causes a pseudorelativistic dependence[4,5] of the effective mass of conduction electrons on their momentum or energy.

The main novel features of the proposed effect as compared with the free-electron case are as follows.

(i) The nonlinearity of conduction electrons in semiconductors is many orders of magnitude larger than the relativistic nonlinearity of the free electron. This allows one to attain a fairly low critical pumping intensity for observation of the effect even taking into consideration the faster relaxation in semiconductors.

(ii) The effective mass of the electron in some semiconductors (e.g., InSb) is very small, which results in a considerable increase of the cyclotron frequency (up to 70–80 times) as compared to the free electron for the same magnetic field (or allows one to correspondingly decrease the required magnetic field). In the case of InSb, this allows one to use the 10.6-$\mu$m radiation of a $CO_2$ laser with a magnetic field $H_0 \sim 140$ kG.

(iii) In semiconductors, the nonlinear cyclotron resonance in the strong optic resonant field is accompanied by some small electrostatic potential between the faces of the semiconductor layer. This effect is due to the radiation force. This potential exhibits a hysteretic behavior as well. Therefore, the proposed effect could be the first proposed all-optical nonlinear phenomenon which results simultaneously in an optic as well as an optoelectronic bistability. This property provides for an immediate electronic indication of the effect.

The optical bistability resulting from nonlinear cyclotron resonance both in free space[1] and in semiconductors is based (in the ideal case) on an interaction of an electromagnetic (EM) wave with a single microscopic object (i.e., electron). This differs fundamentally from all kinds of optical bistability[6] presently known which so far have always been based on macroscopic nonlinear properties of the medium. One may note also that the proposed effect exhibits the so-called cavityless bistability (i.e., no resonator is required to get a bistability[7]).

We shall demonstrate the feasibility of this effect using a classical model for the interaction of an optical wave with a single electron in the conduction band of a semiconductor. The thin semiconductor layer is immersed in the homogeneous magnetic field $H_0$ which is perpendicular to the layer. The semiconductor is also subject to the action of an EM wave $\vec{E}_{in}$ of amplitude $E$ which propagates along the same direction as the magnetic field (axis $z$). In our calulations we will basically follow the approach of Ref. 1, with two exceptions. Firstly, we will consider the mass effect in its complete form (rather than take into consideration only the first-order velocity-dependent term[1] $\beta^2$) to reflect the fact that the nonlinear change of the effective mass could be quite large, in contrast to the free-electron case ($\beta^2 \ll 1$). Secondly, in semiconductors, the small static electric field $\vec{g}$ which is directed along the $z$ axis and compensates the radiation force, is not provided anymore by the external potential, but is caused by the background charge. This is due to a redistribution of the electron density between the faces of the semiconductor layer, i.e., the problem becomes self-consistent as far as the potential is concerned.

## II. EQUATIONS OF MOTION

In narrow-gap semiconductors which can be described by the Kane two-band model[4] with isotropic nonparabolic bands, the conduction-band energy $W$ can be written as

$$W(p)=(m_0^{*2}v_0^4+p^2v_0^2)^{1/2}\ , \tag{1}$$

where $\vec{p}$ is the momentum of the conduction electron, $m_0^*$ is its effective mass at the bottom of the conduction band, $v_0=(W_G/2m_0^*)^{1/2}$ is some characteristic speed, and $W_G$ is the band gap [the energy $W$ in Eq. (1) is measured with respect to the middle of the gap]. The velocity $\vec{v}$ of the conduction electron is given by[8] $\vec{v}(\vec{p})=\partial W(p)/\partial\vec{p}$. By virtue of Eq. (1), this yields

$$\vec{p}=m_0^*\vec{v}/(1-v^2/v_0^2)^{1/2}\ , \tag{2}$$
$$\vec{v}=\vec{p}/m_0^*(1+p^2/p_0^2)^{1/2}\ ,$$

where $p_0\equiv m_0^*v_0=(W_Gm_0^*/2)^{1/2}$ is some characteristic momentum. One can see from Eqs. (1) and (2) that relations among $W$, $v$, and $p$ are completely relativistic, with $v_0$ playing a role as an "effective speed of light," and $W_G/2$ as an "effective rest energy" of the electron. For InSb, $W_G=0.24$ eV, $m_0^*=0.014m_0$ (where $m_0$ is the rest mass of an electron in vacuum), so that $v_0\sim1.15\times10^8$ cm/sec. For narrow-gap semiconductors[9] such as $Pb_{1-x}Sn_xSe$, $Pb_{1-x}Sn_xTe$, or $Cd_{1-x}Hg_xTe$, typically $W_G\sim0.05$ eV, $m_0^*\sim10^{-2}m_0$, and $v_0\sim6.3\times10^7$ cm/sec.

The relativistic mass effect of the free electron in vacuum is a basic phenomenon for the cyclotron maser ("gyrotron"),[10,11] and recently it was proposed to develop a solid-state cyclotron maser[12] by exploiting the pseudo-relativistic behavior of conduction electrons in semiconductors.

In the classical model, the equation of motion of an electron is[13]

$$\frac{d\vec{p}}{dt}=e\vec{E}_\Sigma+\frac{e}{c}\vec{v}\times\vec{H}_\Sigma-\Gamma\vec{p}\ . \tag{3}$$

Here $\vec{H}_\Sigma=H_0\hat{e}_z+\vec{H}_{EM}$ is the total magnetic field (including the EM wave component $\vec{H}_{EM}$), $\vec{E}_\Sigma=\vec{E}_{in}-g\hat{e}_z$ is the total electric field (comprised of the EM wave component $\vec{E}_{in}$ and static component $g$ which is due to the redistribution of charges in the semiconductor layer), and $\Gamma\vec{p}$ is a damping term which represents energy losses of the electron (in our case, it is basically due to collisions). $\Gamma$ is an inverse momentum relaxation time and in the high-energy case could depend on $p$. For our (basically, demonstration) purpose it can be considered as a constant. For relatively low momentum values one has also $\Gamma\vec{p}\simeq\Gamma m_0^*\vec{v}$.

Taking into consideration that for the plane EM wave $\vec{H}_{EM}=\vec{k}\times\vec{E}_{EM}/\omega$, Eq. (3) can be rewritten for the interaction of the "magnetized" electron with an arbitrary propagating plane wave $\vec{E}_{in}$

$$\frac{d\vec{p}}{dt}+\Gamma\vec{p}=e\left[\vec{E}_{in}+\frac{1}{c}\vec{v}\times\vec{H}_0\right]+e\left[\frac{\vec{v}}{\omega}\times(\vec{k}\times\vec{E}_{in})-\vec{g}\right]\ , \tag{4}$$

where the term $e\vec{v}\times(\vec{k}\times\vec{E}_{in})/\omega$ is a radiation force applied to the electron. Usually this term is neglected[10–12] except in Ref. 14 in which, however, losses and other possible forces such as counterpotential $\vec{g}$ were not considered. However, all of these interactions become important when considering excited steady states (and multistability) of the electron under the action of the sufficiently intense resonant EM wave.[1]

Under the assumed geometry one has $\vec{H}_0||\vec{k}||\vec{g}||\hat{e}_z$. Assuming now that $p_z\ll p$ (which is valid at least in the neighborhood of steady states), and writing Eq. (4) in terms of $\vec{p}$ using Eq. (2), one gets

$$\frac{d\vec{p}}{dt}+\Gamma\vec{p}=e\vec{E}_{in}+\frac{\omega_c}{(1+p^2/p_0^2)^{1/2}}\vec{p}\times\hat{e}_z$$
$$+e\hat{e}_z\left[\sqrt{\epsilon}\frac{\vec{p}\cdot\vec{E}_{in}}{(1+p^2/p_0^2)^{1/2}}\frac{1}{m_0^*c}-g\right]\ , \tag{5}$$

where $\omega_c=(eH_0/m_0^*c)$ is the "unperturbed" cyclotron frequency which corresponds to the effective mass at the bottom of the conduction band,[15] and $\epsilon$ is the susceptibility of semiconductor at the given frequency $\omega$. Under the conditions $v_0=c$, $p\ll p_0$, Eq. (5) coincides with Eq. (4) of Ref. 1 if written in terms of $\vec{v}$. Assuming now that the EM wave is circularly polarized[1] and rotates in the same direction as the electron:

$$\vec{E}_{in}=E(\hat{e}_x\sin(\omega t-kz)+\hat{e}_y\cos(\omega t-kz)]\ , \tag{6}$$
$$k=\omega\sqrt{\epsilon}/c$$

where $\omega$ is the frequency of the wave, the solution to (5) can be written in the form

$$\vec{p}=p[\hat{e}_x\sin(\omega t-kz+\phi)+\hat{e}_y\cos(\omega t-kz+\phi)]+p_z\hat{e}_z\ , \tag{7}$$

which after substituting into (5), yields the set of truncated equations

$$\frac{dp}{dt}+\Gamma p=eE\cos\phi\ , \tag{8a}$$

$$\frac{d\phi}{dt}=\frac{p_z\omega\sqrt{\epsilon}}{cm_0^*(1+p^2/p_0^2)^{1/2}}-\frac{eE\sin\phi}{p}-\left[\omega-\frac{\omega_c}{(1+p^2/p_0^2)^{1/2}}\right] \tag{8b}$$

$$\frac{dp_z}{dt}+\Gamma p_z=e\left[\sqrt{\epsilon}\frac{pE/m_0^*c}{(1+p^2/p_0^2)^{1/2}}\cos\phi-g\right]$$
$$\equiv eE_z\ , \tag{8c}$$

where $E_z$ is total electric field acting on the electron in $z$ direction. Since the quasistatic electric field $g$ is developed by the redistribution of electrons inside the layer, these equations are to be supplemented by the equation for the field $g$. Strictly speaking, the single-electron model cannot provide the required equation. The complete theory of the dynamics of the system can be developed only in a framework of a kinetic approach which takes into consideration the distribution function of electrons in the system. However, in order to get results for the steady-state regime as well as equations for small perturbations of that regime, one can use these arguments. By the analogy with other effects in which redistribution of charges is caused by an external force (e.g., as in Hall effect, or in the case when an external electrostatic field is applied to the layer), it is easy to understand that the distribution of electrons remains almost homogeneous inside the layer such that no free bulk charges occur while the induced charges are localized at the surfaces of the layer. The thickness of the charged surface layer is of the order of the Debye's radius $r_D$, which is fairly small (e.g., for $\epsilon \sim 5$ and $N_0 \sim 0.5 \times 10^{15}$ cm$^{-3}$, where $N_0$ is the concentration of electrons, one has[8] $r_D \sim 10^{-5}$ cm at room temperature). The entire layer then can be considered as a capacitor with capacitance $C = \epsilon/4\pi d$ per unit area (where $d$ is its thickness) whose potential $U$ determines the field $g$: $g = U/d = q/Cd$. Here, the electrical charge per unit area $q$ is related with the current $J$ per unit area by the relationship $dq/dt = J = \sigma E_z$, where $\sigma$ is a conductivity of the material.

From these relationships one gets $q = Cdg = \epsilon g/4\pi$, or, after taking time derivative, $\epsilon \dot{g}/4\pi = \dot{q} = J = \sigma E_z$. Making use of Eq. (8c), one finally gets the required equation for $g$:

$$\tau_\sigma e \frac{dg}{dt} = \left\langle \frac{dp_z}{dt} + \Gamma p_z \right\rangle = e \langle E_z \rangle , \tag{9}$$

where $\tau_\sigma = \epsilon/4\pi\sigma$ is a relaxation time for redistribution of the charge, and angular brackets denote an averaging over ensemble of electrons.

## III. STEADY STATES AND BISTABILITY

Equations (8) and (9) present a complete set of equations needed to solve the problem. The averaging over an ensemble in Eq. (9) brings a cooperative interaction aspect into the problem (this is because of the fact that during a transient period, electrons interact with each other through the potential $U = gd$ developed by their collective motion manifested by the current $J$). However, as far as steady states are concerned, one can assume that all electrons are interacting coherently with the EM wave such that any quantity averaged over ensemble of electrons in Eq. (9) coincides with the respective quantity for each electron. The main condition of the steady-state regime [$d/dt = 0$ in Eqs. (8) and (9)] then yields

$$(eE)^2 = p^2 \left[ \Gamma^2 + \left[ \omega - \frac{\omega_c}{(1+p^2/p_0^2)^{1/2}} \right]^2 \right] , \tag{10}$$

$$\tan\phi = -\Gamma[\omega - \omega_c/(1+p^2/p_0^2)^{1/2}] , \tag{11}$$

$$eg = \Gamma p^2 \sqrt{\epsilon}/m_0^* c(1+p^2/p_0^2)^{1/2}) , \quad p_z = 0 . \tag{12}$$

Under condition $m_0^* = m_0$, $v_0 = c$, $p \ll p_0$, these equations coincide with Eqs. (8) and (9) in Ref. 1. Figures 1(a) and 1(b) show the behavior of the square of the momentum $p^2$ (and of the intensity of the wave $E^2$) as functions of the magnetic field $H_0$ ($\propto \omega_c$). It is seen that above some critical intensity and magnitude of magnetic field, the steady-state solution for $p^2$ [Eq. (10)] becomes three valued, which results in bistability (since at least one out of these three states is unstable) and hysteresis. Under the natural condition $\Gamma/\omega_c \ll 1$ (i.e., the bandwidth of the cyclotron resonance is much less than a central frequency of the resonance), Eq. (10) provides lower limit $|E_1|^2$ on the EM wave intensity for the hysteresis to occur:

$$|eE_1|^2 = \frac{8}{3\sqrt{3}} W_G m_0^* \Gamma^3/\omega_c , \quad \omega_c - \omega > \frac{\Gamma\sqrt{3}}{\omega_c} , \tag{13}$$

i.e., hysteresis occurs if the intensity $E^2$ and frequency detuning $\omega_c - \omega$ become sufficiently large ($|E|^2 > |E_1|^2$). It is interesting to note, though, that according to Eq. (10), when the intensity $E^2$ becomes orders of magnitude larger than the value $E^2$ defined by Eq. (13), the width of the hysteresis loop diminishes. Finally, under conditions

$$|eE|^2 > |eE_2|^2 = \omega_c^2 p_0^2 , \quad \frac{\omega}{\omega_c} < 4 \left[ \frac{\Gamma}{\omega_c \sqrt{3}} \right]^{3/2} , \tag{14}$$

(where $|E_2|^2$ is the upper limit of the EM wave intensity) the hysteresis disappears altogether. This corresponds to fairly large EM field, when the forced motion of the electron action of the EM field prevails over the cyclotron motion imposed by the magnetic field $H_0$. However, here we are talking about very highly excited motion ($p^2/p_0^2 > \omega_c \sqrt{3}/\Gamma \gg 1$) and it is not quite clear whether the simplified model of Eqs. (1) and (2) is still applicable in this range. On the other hand, one may note that in order to attain the lowest critical threshold of hysteresis, $|E_1|^2$, it is sufficient to have a considerably lower level of excitation. Indeed, for the momentum $p_1$ which corresponds to the condition (13), one gets

$$p_1^2 \simeq (eE_1)^2/\Gamma^2 \simeq p_0^2 \Gamma/\omega_c \ll p_0^2 . \tag{15}$$

Let us make some qualitative estimates for the threshold intensity $E_1^2$ [Eq. (13)] of the incident wave. In the case of InSb, $m_0^* = 0.014\, m_0$, $W_G = 0.24$ eV. The damping rate $\Gamma$ is usually a large quantity, $\Gamma \sim 10^{13} - 10^{14}$ sec$^{-1}$, which depends primarily on the concentration of impurities. We assume it to be $\Gamma \sim 2 \times 10^{13}$ sec$^{-1}$, such that the resonant bandwidth is $\Gamma/\omega_c \sim 10^{-2}$ for $\gamma_c \sim 10.6\ \mu$m, if a $CO_2$ laser is used as a source of the driving radiation. The magnetic field required to get a cyclotron resonance in the neighborhood of $\gamma \sim 10.6\ \mu$m is $H_0 \sim 140$ kG, and Eq. (13) provides for a critical field required to observe hysteresis: $E_1 \sim 300$ V/cm. This corresponds to the incident power of about 240 W/cm$^2$, which can be readily attained.

Let us now discuss briefly the quasistatic potential $U = dg$ ($d$ is the thickness of the layer) which accompanies the phenomenon. From Eq. (12), one can see that the field $g$ must experience hysteresis as long as momen-

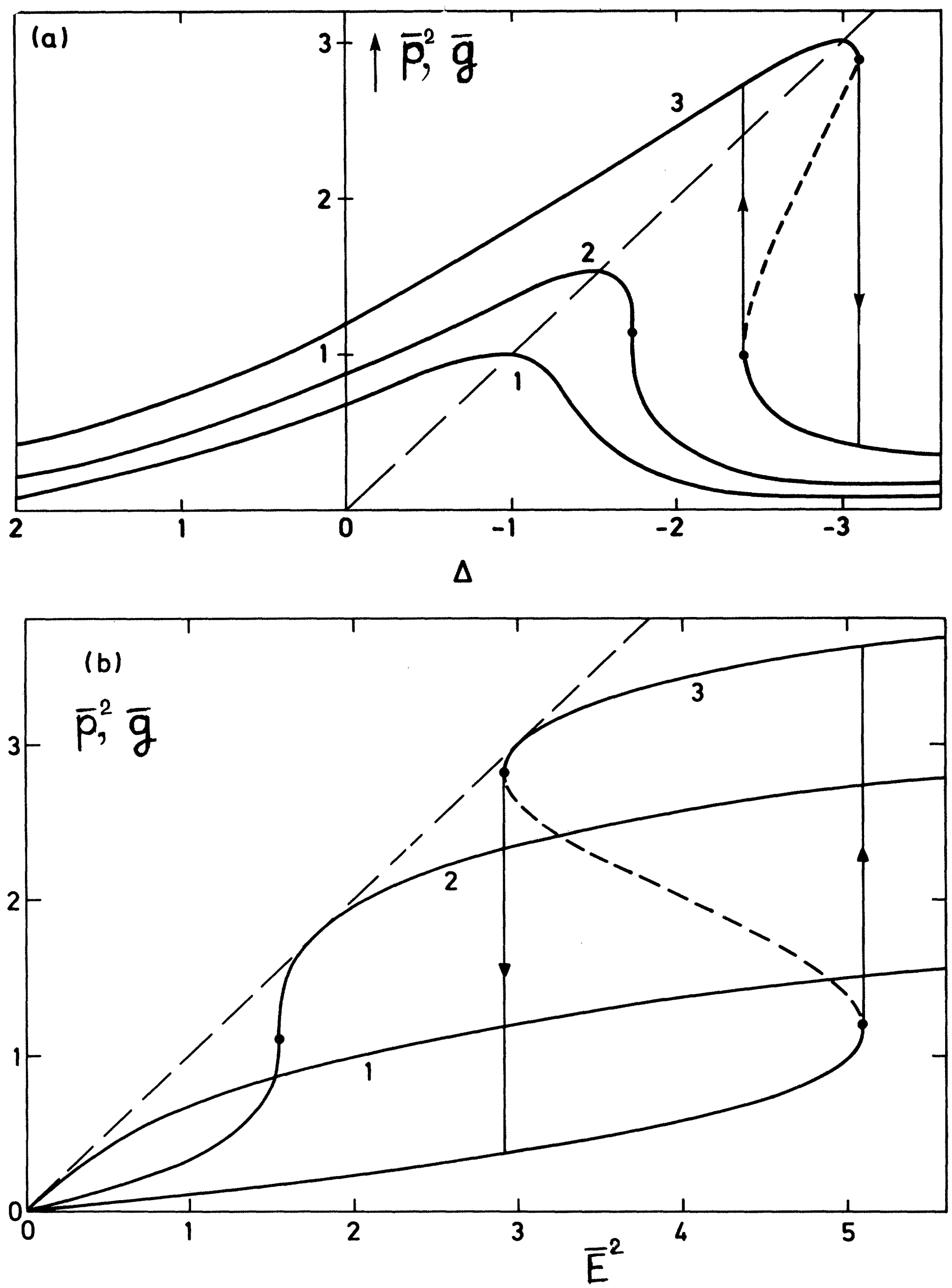


FIG. 1. Plots of the normalized square of the momentum of the electron $\bar{p}^2=p^2/2p_c^2$ and normalized electric field $\bar{g}=gce/W_G\Gamma\sqrt{\epsilon}$: (a) vs normalized resonant detuning of the constant-magnetic field $\Delta=(H-H_0)\omega_c/H_0\Gamma$ for different intensities of incident EM field, and (b) vs normalized incident intensity $\bar{E}^2=8E^2/3\sqrt{3}E_1^2$ for different detunings. Curves: (a) 1, $\bar{E}^2=1$; 2, $\bar{E}^2=(2/\sqrt{3})^3$; 3, $E^2=3$; (b) 1, $\Delta=0$; 2, $\Delta=3\sqrt{3}$; 3, $\Delta=3$. With different scales, these curves reproduce the respective curves for nonlinear cyclotron resonance of free electrons in vacuum (Ref. 1).

tum $p$ does. In the vicinity of the lowest threshold intensity of the hysteresis $E_1^2$, the pseudorelativistic change is very small, due to relationship (15), such that one can neglect the pseudorelativistic dependence in Eq. (12), i.e., $g\simeq\Gamma p^2/\sqrt{\epsilon}/m_0^*ce$. Then at the point of higher excitation where $p_{\max}\simeq eE/\Gamma$, this gives

$$g_{\max}\simeq eE^2\sqrt{\epsilon}/\Gamma m_0^*c \; . \tag{16}$$

In the above instance, if the pumping intensity is slightly above the threshold $E_1^2$ (for instance, $E=500$ V/cm), one gets $g_{\max}\sim 1$ V/cm. For a thickness of the layer $d\sim 0.1$ cm this gives $U=dg\sim 0.1$ V which can be readily measured. Under relatively weak pumping the field $g$ is simply proportional to $p^2$ [Eq. (12)] (see Fig. 1). The relaxation time for the potential $U$ is of order of $\tau_\sigma=\epsilon/4\pi\sigma$.

We would like to point out that the presence of two different relaxation times ($\tau_\sigma$ and $\Gamma^{-1}$) in the system can cause even more complicated behavior which depends on the ratio of these times, i.e., quantity $\tau_\sigma\Gamma$. In the limit $\tau_\sigma \rightarrow 0$, i.e., if $\tau_\sigma\Gamma \ll 1$, it can be shown by linearization of Eqs. (8) and (9) in the close vicinity of the steady states [Eqs. (10)—(12)] that two states out of the three possible steady states [Eqs. (10)—(12)] (in the case of a multivalued solution) are stable with regard to small perturbations. These two stable steady states must satisfy the condition $d(p^2)/d(E^2)>0$ [or $dW/d(E^2)>0$], the same as in Ref. 1, i.e., the states which belong to upper and lower branches of hysteresis curves at Fig. 1, whereas the intermediate branch [with $d(p^2)/d(E^2)<0$] is unstable. However, if $\tau_\sigma\Gamma \sim 1$, there could be some domains on the upper branch (which are close to the onset of the reverse jump from the upper branch to the lower one), where these steady states become unstable. This may result in excitation of self-oscillations and possibly chaotic motion. Using the usual formula for conductivity[8] $\sigma = eN\mu$, where $N$ is the concentration of conduction electrons and $\mu = e/m_0^*\Gamma$ is their mobility, one gets the expression for parameter $\tau_\sigma\Gamma$:

$$\tau_\sigma\Gamma = \frac{\epsilon m_0^* \Gamma^2}{4\pi e^2 N} = \left[\frac{\Gamma}{\omega_p}\right]^2, \quad \omega_p = \left[\frac{4\pi e^2 N}{\epsilon m_0^*}\right]^{1/2}, \qquad (17)$$

where $\omega_p$ is the plasma frequency.[8] In the above-mentioned instance of InSb with $\Gamma \sim 2\times 10^{13}$ sec$^{-1}$, this yields $\tau_\sigma\Gamma \sim 10^{18}/N$ where $N$ is expressed in units of cm$^{-3}$. Hence, it is feasible to expect a manifestation of instability if $N \sim 10^{18}$ cm$^{-1}$. One must note, though, that a more comprehensive theory of stability (as well as of the entire phenomenon) has to involve consideration of cooperative motion of electrons, i.e., the kinetics of distribution functions rather than the motion of a single electron.

## IV. CONCLUSION

In conclusion, we have demonstrated the feasibility of hysteretic behavior and bistability of cyclotron resonance in semiconductors having a strongly pronounced nonparabolicity of the conduction band, under the action of sufficiently strong pumping radiation. We found out the critical conditions of this effect in the framework of the classical, single-electron approximation. Future research should involve quantum as well as kinetic theory of the phenomenon. Even far from the onset of the hysteresis, the action of the strong pumping should cause a dramatic change in the location and the shape of the resonant curve of cyclotron resonance; in particular, the dependence on the frequency (or magnetic field) should become drastically asymmetrical. This effect may provide a new experimental method to measure the nonparabolicity of the conduction band, effective mass, and nonlinear relaxation of the momentum, etc.

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