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Sliding charge-density waves. II. ac properties

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A classical model of a sliding charge-density wave (CDW) in a random pinning potential is used to determine the frequency-dependent conductivity $\sigma_E(\omega)$ in the presence of a dc electric field E well above threshold. Specific interference features are predicted for both the in-phase and out-of-phase components of $\sigma_E(\omega)$. The contribution to the real part, $\epsilon_E(\omega)$, of the dielectric constant due to the pinning potential is shown to change sign, becoming negative as ω decreases. In the low-frequency limit the field dependence is const$-\epsilon_E(\omega\to 0)\propto E^{-3/2}$. Voltage fluctuations observed with dc currents in the incommensurate CDW system $NbSe_3$ are shown to be absent, to all orders of perturbation theory, in the infinite-volume limit.

I. INTRODUCTION

In a companion paper,[1] referred to here as paper 1, a classical model of a sliding charge-density wave (CDW) was solved perturbatively for the form of the dc I-V curve, with and without a superposed ac modulation, for voltages well above threshold.

For the analogous system of a sliding magnetic vortex lattice[2,3] in type-II superconducting films, an ac linear response measurement has proven to be a useful probe, allowing greater accuracy than the nonlinear dc measurements. The corresponding CDW measurement is of the frequency-dependent conductivity $\sigma_E(\omega)$ in the presence of a dc field E well above threshold. In Sec. II this quantity will be calculated for the model presented in paper 1 and its form in various limiting cases will be given.

If the voltage reponse to a dc CDW current is spectrally analyzed it is seen[4-6] to contain broad band noise as well as one or more quite well-defined harmonic sequences. The CDW in $NbSe_3$ is incommensurate.[7] In the perturbative solution presented in paper 1, therefore, the first-order voltage is zero. The second-order correction at large fields is given by Eq. (9) in paper 1. For a purely dc current one sees that $E^{(2)}$ is purely dc and the observed voltage fluctuations remain unexplained. A natural question is whether these fluctuations are simply a higher-order effect. In Sec. III it will be shown that the answer is no. Unless the pinning potential is commensurate with the CDW and coherent over the sample volume, the voltage response to a dc current is dc, in the infinite-volume limit, to any finite order in perturbation theory. The results will be summarized in Sec. IV. This paper will use the notation and some preliminary results of paper 1.

II. FREQUENCY-DEPENDENT CONDUCTIVITY

The calculation in paper 1 determines the electric field due to a specified bulk current density, $j(t)=\rho_c\dot{u}_0(t)$, where

$$u_0(t)=vt+(a/\omega)\sin(\omega t)\ . \tag{1}$$

For the incommensurate case (e.g., both[7] CDW's in $NbSe_3$), the first-order field $E^{(1)}$ vanishes so that $\sigma_E(\omega)\equiv\sigma'_E(\omega)+i\sigma''_E(\omega)$ is given by

$$\sigma_E(\omega)=\sigma_E^{(0)}(\omega)+\sigma_E^{(2)}(\omega)\ , \tag{2}$$

where

$$\sigma_E^{(0)}(\omega)=(1+i\omega\tau)\sigma_\infty \tag{3}$$

and

$$\sigma_E^{(2)}(\omega)=\frac{-\sigma_\infty^2 E_\omega^{(2)}}{j_\omega}\ , \tag{4}$$

with $\tau=m/\lambda$, where m is the CDW effective mass density,[8] and inertial effects are very small,[9] $\omega\tau\ll 1$, for frequencies of interest here.

From (1) and (2), $j_\omega=\rho_c a/2$; the field $E_\omega^{(2)}$ is given by Eq. (9) of paper 1 by, to $O(a)$,

$$\rho_c E_\omega^{(2)}=-i\sum_g|\rho_g|^2\int\frac{d\vec{k}}{(2\pi)^3}\Gamma_E(\vec{k})k_z^2 a\omega^{-1}[\mathrm{Re}G(\vec{k}-\vec{g},\omega-\vec{k}\cdot\vec{v})-\mathrm{Re}G(\vec{k}-\vec{g},-\vec{k}\cdot\vec{v})+i\,\mathrm{Im}G(\vec{k}-\vec{g},\omega-\vec{k}\cdot\vec{v})]\ . \tag{5}$$

For the quantities calculated in this paper, as in paper 1, the CDW response function takes the simple form

$$G^{-1}(\vec{k},\omega)=D(\vec{k})-i\omega\lambda f(\hat{k})\ , \tag{6}$$

where

$$f(\hat{k})=1+\hat{k}_z^2\sigma_\infty/\sigma(\hat{k})\ , \tag{7}$$

for sufficiently high remnant normal electron conductivity, $\sigma(\hat{k})$. For classical hydrodynamics to be valid the combination of response functions in (5) must be sharply peaked at $\vec{k}=\vec{g}$. Considering, for incommensurate CDW's, the contribution of defects, the slowly-varying $\Gamma_d(\vec{k})$ can then be taken out of the integral. Furthermore, the frequency argument $\vec{k}\cdot\vec{v}$ can be replaced by $\vec{g}\cdot\vec{v}$ as long as $|\omega-\vec{g}\cdot\vec{v}|$ is not too small. Performing the integrals then gives

$$\sigma_E^{(2)}(\omega)=\sigma_\infty \sum_g B_g(a_g+ib_g) , \tag{8}$$

where

$$B_g \equiv (4\sqrt{2}\pi)^{-1}|\rho_g|^2\Gamma_d(\vec{g})g_z^2\left\langle\left[\frac{f(\hat{q})}{\lambda K^3(\hat{q})}\right]^{1/2}\right\rangle_{\hat{q}} , \tag{9}$$

$$\omega a_g=\mathrm{sgn}(\vec{g}\cdot\vec{v}-\omega)\,|\vec{g}\cdot\vec{v}-\omega|^{1/2} , \tag{10}$$

and

$$\omega b_g=|\vec{g}\cdot\vec{v}|^{1/2}-|\vec{g}\cdot\vec{v}-\omega|^{1/2} . \tag{11}$$

This result shows interference features,[2] centered at $\omega=\vec{g}\cdot\vec{v}$. While the approximation leading to this result breaks down if $|\omega-\vec{g}\cdot\vec{v}|$ is small, the following conclusions can be drawn. The in-phase response $\sigma_E'(\omega)$ should show steps, and the bulk dielectric constant $\epsilon_E(\omega)=1-(\epsilon_0\omega)^{-1}\sigma_E''(\omega)$ will exhibit downward peaks, centered at $\omega=\vec{g}\cdot\vec{v}$. These features have recently been observed[10] in conductivity measurements on $NbSe_3$.

Combining Eqs. (2), (3), (8), and (11) gives, for the dielectric constant,

$$\epsilon_E(\omega)=1-(\sigma_\infty/\epsilon_0)\times\left[\tau+\sum_{\vec{g}} B_g\omega^{-2}(|\vec{g}\cdot\vec{v}|^{1/2}-|\vec{g}\cdot\vec{v}-\omega|^{1/2})\right] .$$

The contribution to $\epsilon_E(\omega)$ due to the pinning potential is thus positive for large ω, but changes sign so that

$$\epsilon_E(\omega\to 0)=1-(\sigma_\infty/\epsilon_0)\left[\tau+\sum_{\vec{g}}(B_g/8)|\vec{g}\cdot\vec{v}|^{-3/2}\right] , \tag{12}$$

and ϵ_E-1 is negative at low frequencies and large dc fields. Physically, at high frequencies, the sliding CDW appears pinned by the pinning potential, and ϵ_E is positive. At low frequencies, the dc sliding motion becomes apparent so ϵ_E becomes negative. Furthermore, the field dependence at large E is seen to be $\mathrm{const}-\epsilon_E(\omega\to 0)\propto E^{-3/2}$.

III. VOLTAGE OSCILLATIONS

It was seen in paper 1 that the equation of motion of CDW with bulk current density $j=\rho_c\dot{u}_0(t)$ can be written as

$$u(\vec{r},t)-u_0(t)=\int d\vec{r}\,dt'G(\vec{r}-\vec{r}',t-t')[\rho(\vec{r}')E_p(\vec{r}'+\vec{u}(r',t'))+\rho_c\widetilde{E}(t')] \tag{13}$$

with

$$\rho_c\widetilde{E}(t)=-\langle\rho(\vec{r})E_p(\vec{r}+\vec{u}(\vec{r},t))\rangle_r , \tag{14}$$

where $\langle\ \rangle_r$ denotes a volume average.

A scheme for solving (13) and (14) to order n in E_p is as follows. For $m=1,2,\ldots,n$, calculate, to $O(n)$,

$$\rho_c E_m(\vec{r},t)\equiv\begin{cases}-\rho(\vec{r})E_p(\vec{r}+\vec{u}_0), & m=1\\ -\rho(\vec{r})[E_p(\vec{r}+\vec{u}_0+\vec{u}^{(m-1)})-E_p(\vec{r}+\vec{u}_0+\vec{u}^{(m-2)})], & m>1\end{cases}$$

[step (a)], where $u^{(m)}\equiv\sum_{j=1}^m u_m$,

$$\widetilde{E}_m(t)\equiv\langle E_m(\vec{r},t)\rangle_r$$

[step (b)], and

$$u_m(r,t)=\int d\vec{r}'dt'G(\vec{r}-\vec{r}',t-t')\rho_c[\widetilde{E}_m(t')-E_m(\vec{r}',t)]$$

[step (c)].

Then to $O(n)$, $u^{(n)}(\vec{r},t)$ and $E^{(n)}(t)$ $[(\equiv\sum_{m=1}^n\widetilde{E}_m(t)]$ satisfy (13) and (14) and the bulk electric field for $j=\rho_c\dot{u}_0(t)$ is $E(t)=E_0(t)+E^{(n)}(t)$, where $E_0(t)$ is the zeroth-order ($E_p=0$) field.

For a dc current ($u_0=vt$) it will now be shown by induction that E is independent of t, in the infinite-volume limit, to *any* finite order in perturbation theory.

For n any positive integer, assume that there is some positive integer N, such that, for $1\le m\le N$, $u_m(\vec{x}-\vec{v}t,t)$ satisfies the following condition. When evaluated to $O(n)$ as described above in terms of integrals of products of ρ's, E_p's and derivatives of E_p, and G's, $u_m(\vec{x}-\vec{v}t,t)$ is assumed to depend on t only in the combination $\vec{x}-\vec{v}t$ in the argument of ρ, and on $\vec{x}$, other than this, in and only in the argument of every E_p. From step (a) above, $E_{N+1}(\vec{x}-\vec{v}t,t)$ has the same t and $\vec{x}$ dependence. Performing steps (b) and (c) then shows that the same is true of $u_{N+1}(\vec{x}-\vec{v}t,t)$. It is easy to see that $u_1(\vec{x}-\vec{v}t,t)$ satisfies this condition, so that by induction $u_m(\vec{x}-\vec{v}t,t)$ and $E_m(\vec{x}-\vec{v}t,t)$ have this t and $\vec{x}$ dependence for $1\le m\le n$. Thus the bulk field $E^{(n)}(t)$ has its t dependence in terms with the form

$$\left\langle\exp[i\vec{g}\cdot(\vec{x}-\vec{v}t)]\prod_{j=1}^n\left[\left[\frac{\partial}{\partial z}\right]^{p_j}E_p^{r_j}(\vec{x}+\vec{c}_j)\right]\right\rangle_x ,$$

where the p_j and r_j are positive integers or zero, and the

$\vec{c}_j$ are independent of t and $\vec{x}$. Unless E_p is commensurate and coherent over the whole sample, however, this volume average vanishes in the infinite-volume limit if $\vec{g}\neq 0$. Thus only dc components of E can survive the sample averaging in the infinite-volume limit, to *any* finite order in perturbation theory.

This argument is independent of the detailed form of G, and thus applies, for example, even in the presence of long-range electrostatic interactions.

IV. SUMMARY

The frequency-dependent conductivity $\sigma_E(\omega)$ of an overdamped CDW sliding over defects in a dc field E well above threshold has been calculated. The in-phase response $\sigma'_E(\omega)$ is predicted to show downward steps, and the out-of-phase response $\sigma''_E(\omega)$ is predicted to show peaks, centered at $\omega=\vec{g}\cdot\vec{v}$. The real part, $\epsilon_E(\omega)$, of the dielectric constant was calculated. The CDW contribution due to the interaction with the pinning potential changes sign from positive to negative as ω decreases through $\omega=\omega_x\sim|\vec{g}\cdot\vec{v}|$. At low frequencies the field dependence has the form const $-\epsilon_E(\omega\to 0)\propto E^{-3/2}$.

The experimental observation of voltage fluctuations in response to a dc current was also considered. It was seen that for a pinning potential that is not both commensurate and phase coherent over the whole sample, the electric field due to a dc CDW current is dc, in the infinite-volume limit, to any finite order in perturbation theory.

Since the CDW's in $NbSe_3$ are incommensurate,[7] this result applies to that system. The observed voltage fluctuations have not yet been satisfactorily accounted for. Two mechanisms under consideration are finite-size effects[11] and contact effects.[12]

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