# Role of reversed spins in the correlated ground state for the fractional quantum Hall effect

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The ground-state energy for the correlated liquid state of Laughlin with one-half of the electrons having spins antiparallel to the field is obtained at $\frac{2}{5}$ filling of the lowest Landau level. The energy is found to be lower compared with the potential energy of the spin-polarized state. The energy difference is small, however, compared with the difference of magnetic energies between the two states under the present experimental conditions. Some of the results obtained for the one-spin state are in good agreement with recent experimental observations and very accurate Monte Carlo results.

Recent discovery of the fractional quantum Hall effect with the Hall conductance $\sigma_{xy}$ quantized at certain fractional values of $e^2/h$ in a AlGaAs-GaAs heterojunction has generated much interest.[1] The correlated ground-state wave function proposed by Laughlin,[2]

$$\psi(\vec{r}_1,\ldots,\vec{r}_N)=\prod_{j<k}(z_j-z_k)^m\prod_j e^{-|z_j|^2/4l^2}, \tag{1}$$

where $z_j=x_j-iy_j$ is the coordinate of the $j$th electron and $l$ the magnetic length, has been quite successful in explaining those observations. The wave function (1) has been constructed by considering the symmetric gauge with vector potential[3] $\vec{A}=\frac{1}{2}B(x\hat{y}-y\hat{x})$, ignoring all but the lowest Landau level (so that the kinetic energy is an absolute minimum) and requiring $m$ to be an odd integer, so that the Pauli principle is obeyed. The square of this wave function is just the probability distribution of a one-component plasma and the charge neutrality of the plasma determines the filling factor $\nu=1/m$, where $\nu\equiv 2\pi\rho l^2$, $\rho$ being the total carrier density. Halperin[4,5] and Laughlin[2] have extended this approach to other rational fractions $\nu$.

In the above calculations of the partially filled Landau level, it has been assumed that only one spin state (spins parallel to the field) is occupied. However, as Halperin pointed out,[4] in GaAs both the Landé $g$ factor and the effective mass of the electrons are much smaller than the corresponding free-electron values. Therefore, the ground state for some values of $\nu$ might have some electrons with reversed spins. The magnetic energy per particle is defined as $E_m=(1-2p)g\mu_B Bs$, where $p$ is the ratio of the number of spins parallel to the field to the total number of spins, $\mu_B=e\hbar/2mc$ is the Bohr magneton and $s=\frac{1}{2}$. For GaAs with all spins parallel to the field this is given by $E_m=-0.011e^2/\epsilon l$ for $B=10$ T and $\epsilon\simeq 13$ is the dielectric constant of GaAs. This energy should be compared with the difference between the potential energies of the two states (polarized and unpolarized) to determine the possibility of the reversed spins. Even though, in the experiments to date, partially filled Landau levels with some electrons having reversed spins has not been observed, the question whether there will be electrons with reversed spins in the ground state is still open[4] for some other material or in some future experiments. In this paper we have studied the possibility of reversed spins in the ground state.

We consider a trial wave function proposed by Halperin,[4] which describes a liquid state with one-half of the electrons having spins antiparallel to the field,

$$\psi=\prod_{i<j}(z_i-z_j)^3\prod_{k<l}(\tilde{z}_k-\tilde{z}_l)^3\prod_{i,k}(z_i-\tilde{z}_k)^2\prod_i e^{-|z_i|^2/4l^2}\prod_k e^{-|\tilde{z}_k|^2/4l^2} \tag{2}$$

corresponding to a filling factor of $\nu=\frac{2}{5}$. Here $z$ and $\tilde{z}$ are the coordinates of spin-up and spin-down electrons, respectively. This state is constructed in a manner similar to the one-spin state (1), viz., (a) the electrons are in the lowest Landau level, (b) the wave function is antisymmetric under interchange of two electrons of the same spin. The energy of this state is therefore calculated in a similar way as that of the one-spin state.

First we consider the one-spin state. The ground-state energies are obtained by employing the hypernetted-chain (HNC) technique for the classical one component plasma in two dimensions.[6,7] Laughlin used a modified HNC scheme where the elementary diagrams are evaluated by an approximate method given in Ref. 6. We have chosen to ignore the elementary diagrams instead (HNC/0 approximations), which have usually very small contributions. This is done in order to compare the result with that of the two-spin state, which is calculated within the same approximations. We have reproduced the radial distribution functions $g(r)$ of Ref. 7 for $m=5$. The energy values for the one-spin state are given in Fig. 1, together with Laughlin's results[2] and the charge-density-wave crystal energies[8] for $\nu=\frac{1}{3}$, $\frac{1}{5}$, and $\frac{1}{7}$. While our results lie above Laughlin's results, the present work indicates that a liquid to solid transition might take place for $\frac{1}{7}<\nu<\frac{1}{5}$. Laughlin's result converges with the crystal energy near $m=10$. Our HNC/0 prediction is apparently in agreement with the experimental observations by Mendez *et al.*,[9] which seem to indicate that the fractional quantized Hall effect does not occur for $\nu\leqslant\frac{1}{7}$. At $\nu=\frac{1}{3}$ and $\frac{1}{5}$, we have also compared our liquid

 

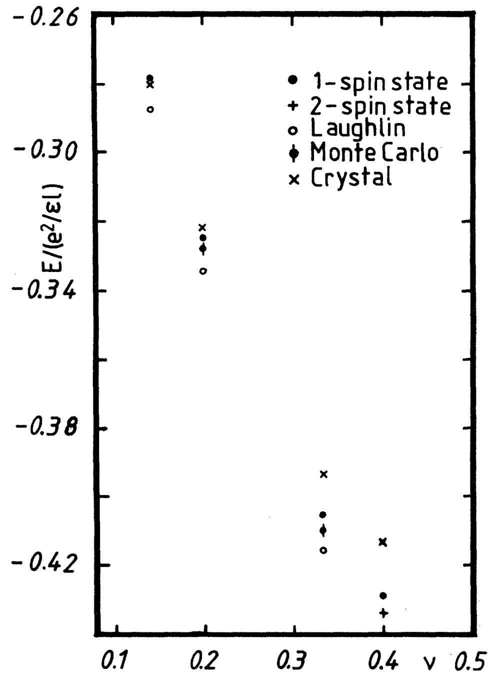


FIG. 1. Potential energy per particle vs the fractional filling of the lowest Landau level. The electron lattice results (×) are from Ref. 8. The modified HNC results (●) of Laughlin, Ref. 2, and the present one-spin (○) and two-spin (+) state HNC results. The Monte Carlo results (⧫) are from Ref. 10.

results with the recent Monte Carlo results.[10] The agreement is quite satisfactory.

As stated above, the one-spin state of Laughlin (1) corresponds to the filling factor $\nu = 1/m$ with $m$ being an odd integer. However, Laughlin's study indicates that the energy at $\nu = \frac{2}{5}$ is still well approximated[2] by the energy obtained with the wave function (1). Very recently, Halperin[5] has constructed stable states at various filling fractions iteratively by adding quasiparticles or quasiholes to lower stable states and estimated the energy correction (at $\nu = \frac{2}{5}$) to be $\simeq 0.009 e^2/\epsilon l$. We have added that energy correction to the plasma energy obtained with the wave function (1). The resulting one-spin state energy at $\nu = \frac{2}{5}$ is plotted in Fig. 1.

The ground-state energy for the two-spin state (2) is obtained by considering the spin-up and spin-down electrons as two different particles. The total interaction energy is then written as

$$E_{\text{int}} = \frac{1}{2}\pi\rho \int_0^\infty \frac{e^2}{r}[g_{11}(r) + g_{12}(r) - 2]\, r\, dr, \tag{3}$$

where $g_{\alpha\beta}(r)$ are the partial pair-correlation functions. They are obtained by solving the two-component HNC/0 equations, a generalization of the one-component HNC equations.[11] We have used the standard procedure of separating the long- and the short-range part of the Coulomb interaction employed in the one-spin state calculations.[7] The resulting energy is found to be lower than the energy obtained for the one-spin state at $\nu = \frac{2}{5}$ (Fig. 1). The difference of the potential energy between the two states $\Delta E = 0.005 e^2/\epsilon l$ is, however, not big enough to overcome the magnetic energy of the spin-polarized state, under the present experimental conditions. (The latter energy vanishes for the present two-spin state.) Therefore, the *total* energy for the spin-polarized state is lower, in accordance with the experimental observations.

As the magnetic energy depends upon the experimental conditions, by choosing suitable values for the Landé $g$ factor, effective mass, etc., it might still be possible for the potential energy difference to overrride the magnetic energy. That would result in lowering the total energy of the present two-spin state.

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