# Excitations in the one-dimensional anisotropic classical Heisenberg chain

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The classical equations of motion for the Heisenberg chain with single-site anisotropy are shown to be equivalent to a set of nonlinear wave equations. For a zero-field $XY$ chain the excitations are linear. This agrees with magnetic specific heat measurements on $(CH_3)_4NMnCl_3$ (TMMC).

Excitations in one-dimensional magnetic systems have attracted a considerable amount of theoretical and experimental study. In particular, Mikeska[1] has shown that the classical equations of motion for the magnetic $XY$ chain in a transverse magnetic field can be mapped into the sine-Gordon equation. Also, the antiferromagnetic chain $(CH_3)_4NMnCl_4$ (TMMC) is an ideal $XY$ magnet if the temperature is below 10 K.[2] Therefore TMMC should be a physical system where nonlinear excitations can be experimentally detected. Recently[3] the magnetic specific heat of TMMC was analyzed in the temperature range between $T_N$ and 10 K with the following two conclusions: First, the experimental specific heat of TMMC in a transverse field can only be theoretically explained by nonlinear excitations. Second, the experimental specific heat with no external field can be satisfactorily explained by considering linear spin-wave contributions. In this Brief Report we will present a simple explanation of the zero-field specific heat results. The excitations can be obtained by solving the classical equations of motion for the magnetic chain with single-site anisotropy. The equations of motion are equivalent to a system of two wave equations, one of which is linear and the other is nonlinear. For an $XY$ chain the solutions to the system of equations will be linear which agrees with the zero-field specific heat results for TMMC.

The continuum approximate Hamiltonian is

$$H=\int\left(\frac{\hat{J}}{2}\left|\frac{d\vec{S}}{dz}\right|^2+\hat{A}\,(S^z)^2\right)dz, \tag{1}$$

where $\hat{A}=A/a$, $\hat{J}=Ja$, $A$ is the anisotropy constant, $J$ is the exchange constant, $a$ is the lattice constant, and the $\hat{z}$ direction is along the chain. The Hamiltonian is transformed to spherical polar coordinates,

$$\vec{S}=(\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta),$$

and using $u=\cos\theta$ and $\phi$ as canonically conjugate variables we obtain two coupled partial differential equations from the functional derivatives of the Hamiltonian density in Eq. (1):

$$\phi_t=\hat{J}\frac{u_{zz}}{1-u^2}-\hat{J}\frac{uu_z^2}{(1-u^2)^2}-\hat{J}u\phi_z^2+2\hat{A}u, \tag{2a}$$

$$u_t=\hat{J}(1-u^2)\phi_{zz}-2\hat{J}uu_z\phi_z. \tag{2b}$$

$\phi$ may be eliminated from Eq. (2b) by differentiation of this equation with respect to time and differentiation of Eq. (2a) with respect to $z$. The equations for the second-order time derivatives of $\phi$ and $u$ have terms the order of $\hat{J}^2$ and $\hat{J}\hat{A}$. It will be assumed that the wavelength of excitations are very large compared with the lattice constant. Then $a$ will be small, $\hat{J}$ is small, and $\hat{A}$ is large; therefore the terms the order of $\hat{J}^2$ will become very small as $a$ becomes small and these terms will be neglected in the equations of motion. The terms the order of $\hat{J}\hat{A}$ remain constant for any $a$.

The second-order partial differential equations for $u$ and $\phi$ are

$$u_{tt}-(c^2(u)u_z)_z=0, \tag{3a}$$

$$\phi_{tt}-(c^2(u)\phi_z)_z=0, \tag{3b}$$

where $c^2(u)=2AJ(1-u^2)$. It must be emphasized that Eqs. (3) are only valid for excitations much larger than the lattice constant. Equation (3a) is particularly simple since it only contains the dependent variable $u$. This equation is a nonlinear wave equation and the general solution is[4]

$$u=f(z-c(u)t)+g(z+c(u)t), \tag{4}$$

where $f$ and $g$ are arbitrary functions. Assume that the functions $f$ and $g$ are chosen such that Eq. (4) can be solved for $u$ as a function of $z$ and $t$. Then, since $u$ does not depend on $\phi$, Eq. (3b) is linear. Now consider Eqs. (3a) and (3b) for TMMC in the temperature range between $T_N$ and 10 K. Since TMMC has $XY$ character we will choose as a particular solution of Eq. (3a) the trivial solution $u=0$. Then $c^2(u)=2AJ$, which is independent of $u$, and Eq. (3b) becomes a linear wave equation with constant $c$ and the corresponding excitations are plane waves. Nonlinear excitations are seen to come from deviations from $XY$ character corresponding to nontrivial solutions of Eq. (3a).

In conclusion, if a magnetic chain has $XY$ character we have shown that the continuum approximate zero-field excitations are linear and magnetic measurements such as specific heat can be adequately explained with linear theory. This is in agreement with experimental specific heat results for TMMC.

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