Quantum corrections to the resistance in two-dimensional disordered superconductors above T_c: Al, Sn, and amorphous $Bi_{0.9}Tl_{0.1}$ films

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Disordered films of superconductors above their transition temperature T_c show several quantum corrections to the conductance, weak localization (WL), two contributions from the retarded Coulomb interaction, and two contributions from superconducting fluctuations, the Aslamazov-Larkin (AL) and the Maki-Thompson (MT) terms. In a magnetic field, most of them show a magnetoresistance and all are temperature dependent. In this paper the temperature and field dependence of the resistance is measured for Al, Sn, and amorphous $Bi_{0.9}Tl_{0.1}$ films. This covers the weak-coupling and the extremely-strong-coupling superconductors as well. Of particular interest are the AL and the MT terms. The latter can be reduced by a pair-breaking mechanism. This pair breaking is caused by the inelastic-scattering time of the conduction electrons and can be measured by weak localization. Al has only a small spin-orbit scattering but by covering it with $\frac{1}{4}$ atomic layer of Au it can be transformed into a strong spin-orbit scatterer. This allows an independent determination of the inelastic lifetime τ_i and spin-orbit scattering time $\tau_{s.o.}$ and therefore a complete examination of the theory and its consistency. The agreement between experiment and theory is very good for the magnetoresistance. However, the theory for the temperature dependence of the resistance fails (with the exception of the contribution of WL). For the intermediate- and strong-coupling superconductors the applicability of the existing theories is restricted. The magnetoresistance of amorphous $Bi_{0.9}Tl_{0.1}$ is essentially determined by the AL contribution.

I. INTRODUCTION

During the last few years two-dimensional anomalies of the resistance in normal conducting disordered electron systems have been discovered and intensively studied, theoretically as well as experimentally (see, for example, Refs. 1–3). One can distinguish two different effects which are generally called weak localization and the Coulomb anomaly. The first is a quantum-interference effect of the single conduction electron scattered by the impurities, while the second is caused by the retarded exchange interaction in disordered metals. At low temperature both show (for weak localization only in the absence of magnetic impurities) corrections to the residual resistance which are proportional to the logarithm of the temperature $\ln T$ or—for finite frequencies ω—proportional to $\ln \omega$.

A similar resistance anomaly proportional to $\ln \omega$ had been discovered more than 15 years ago by Maki[4] in two-dimensional disordered superconductors. It is obtained from the so-called Maki graph. This Maki anomaly was originally divergent for all temperatures at zero frequency. Thompson[5] introduced phenomenologically a pair-breaking parameter δ into the fluctuation propagator which removed the divergence at zero frequency. In the years that followed the origin of the pair-breaking parameter was discussed by several authors.[6–8]

The interplay between weak localization, electron-electron interaction, and superconductivity in disordered two-dimensional systems and its influence on the superconducting properties (T_c, H_{c2}, etc.) and the "paraconductivity," i.e., the conductivity above the superconducting transition temperature, has been investigated in several theoretical papers.[9–16] Larkin[9] has shown that in thin disordered films of superconductors the inelastic lifetime can be also obtained from magnetoresistance measurements, although the evaluation of the experiment is more complicated than in normal-conducting thin films. The difficulty in disordered films of superconductors is that the magnetoresistance (above the superconducting transition temperature T_c) is composed of several different terms: (a) the Aslamazov-Larkin fluctuations (AL), (b) the Maki-Thompson fluctuations (MT), (c) weak localization (WL), (d) the Coulomb contribution in the particle-hole channel (CPH), and (e) the Coulomb contribution in the particle-particle channel (CPP). These different contributions are briefly reviewed in Sec. III. Unfortunately, the theory of the superconducting films above T_c is very complex and approximations had to be made in order to obtain useful results. This restricts the temperature and field region of validity and makes the evaluation of the experimental results in some cases problematic.

Experimentally the resistance anomalies of superconductors have been recently studied, mainly for thin Al films by Bruynseraede *et al.*,[17] Gershenson *et al.*,[18] Shinozaki *et al.*,[19] Santhanan and Prober,[20] and Gordon *et al.*[21] Bhatnagor *et al.*[22] investigated Sn films, Kobayashi *et al.*[23] studied Zn films, and Raffy *et al.*[24] investigated W-Re films. In most experimental investigations the data are evaluated with the theory of WL and MT. This is in many cases sufficient, but a systematic study of all contributions is desirable.

Among the experimentally investigated films of superconductors, we will discuss three materials: Al, Sn, and amorphous $Bi_{0.9}Tl_{0.1}$. Al is a weak-coupling superconductor. Quench-condensed Sn is an intermediate-coupling superconductor whose gap ratio $2\Delta/T_c$ is 3.9 (Ref. 25) and therefore considerably larger than the BCS value of 3.5. Amorphous $Bi_{0.9}Tl_{0.1}$ is an extremely-strong-coupling superconductor with a ratio $2\Delta/T_c=4.6$[26] Amorphous $Bi_{0.9}Tl_{0.1}$ is an interesting material, because it was the superconductor whose superconducting fluctuations were first investigated by Glover[27] and showed nice agreement with the Al theory.

Our present knowledge obtained from the study of weak localization now allows a much better understanding of the origin of the pair-breaking parameter in the Maki-Thompson term and even an independent measurement. It is caused by inelastic processes of the conduction electrons, which destroy the phase coherence between the two partners of the Cooper pair. This means that in light of two-dimensional resistance anomalies several old questions again become interesting, and the magnetoresistance measurements give new answers because they yield the dynamics of the fluctuation. (Magnetoresistance measurements correspond to a time-of-flight experiment with conduction electrons.[28])

II. EXPERIMENTAL RESULTS

For the experimental investigation of the resistance corrections in films of superconductors above T_c, we have condensed thin films of Al, Sn, and amorphous $Bi_{0.9}Tl_{0.1}$. They cover the whole range from the weak-coupling superconductor Al to the extremely-strong-coupling superconductor $Bi_{0.9}Tl_{0.1}$, while Sn represents an intermediate case. The coupling strength indicates the (different) strength of the electron-phonon interaction and its effect on the inelastic lifetime is of great interest.

We need thin films with high resistance. Since the method of quench condensation is very favorable for preparing rather homogeneous and continuous films with high resistances, it is very suitable for the present task. In particular the classical magnetoresistance is negligible. The apparatus in which the experiments are performed has been described.[29,30] In an ultrahigh vacuum of at least 10^{-11} Torr the evaporation rate of Al (and the other superconductors) is first adjusted to about 10 to 20 atomic layers per minute. After that the film is quench condensed at a temperature which lies a few degrees above the superconducting transition temperature onto a substrate of crystalline quartz. The conductance is registered during the evaporation and the evaporation is stopped when the required resistance per square is observed. The Sn and the $Bi_{0.9}Tl_{0.1}$ are evaporated from a Ta foil—the $Bi_{0.9}Tl_{0.1}$ as an alloy. The Al is evaporated from a coil of tungsten wire. Films with resistances of the order of 100 Ω per square are condensed. The thickness of the films has been determined with a quartz oscillator which has a sensitivity better than $\frac{1}{10}$ atomic layers. After the evaporation, the film is annealed at 40 K for several minutes. The data for the films are collected in Table I.

We discuss first the experimental results for an Al film with a resistance per square of about 116 Ω. The thickness of the film is 90 Å. Figure 1(a) shows the resistance of the film as a function of $\ln T$ in zero magnetic field and in a magnetic field of 7 T perpendicular to the film. Since in a two-dimensional system the conductance is the more appropriate quantity (see Sec. III), we have added on the right-hand side the conductance scale in units of $L_{00}=e^2/(2\pi^2\hbar)\approx(80\ \mathrm{k}\Omega)^{-1}$. This is the physically essential scale which is drawn in all figures. This plot has the advantage that it combines the physically relevant scale with the resistance plot which is more familiar. Of course the conductance scale increases in downward direction.

The magnetoresistance is measured at five temperatures in the field range between -7 and $+7$ T. These results are shown below.

In a second evaporation step, the Al film is covered with a small fraction of an atomic layer of Au. Figure 1(b) shows the temperature dependence of the same Al film after a coverage with 0.25 atomic layers of Au in zero field and 7 T. The temperature dependence of the resistance changed markedly. Again, the magnetoresistance is measured at the same temperatures as before.

The magnetoresistance is strongly temperature dependent. For a comparison with the theory it is not useful to draw the magnetoresistance as function of the applied field in tesla. Therefore, we have chosen the magnetic field scale for each temperature in such a way that the magnetoresistance curves are optimally represented. The units of the field are shown on the right-hand side of each magnetoresistance curve. In Fig. 2(a) the magnetoresistance curves are plotted for the pure Al film. The points represent the experimental results. Since the magnitude of the superconducting fluctuations increases strongly if one approaches the transition temperature, we must change the units of the ordinate for different temperatures. The arrows on the right-hand side of each curve give the conductance in units of L_{00} (together with the units of the magnetic field). For the upper and lower curves the units of the resistance are given by the arrows on the left-hand side of the curves.

TABLE I. Superconducting material, film thickness, residual resistance per square, resistivity, transition temperature, derivative of the upper critical field, the thickness field, and the adjusted spin-orbit scattering field.

Metal	d (Å)	R_n (Ω)	ρ (10^{-6} Ω m)	T_c (K)	H'_{c2} (T/K)	H_d (T)	$H_{\mathrm{s.o.}}$ (T)
Al	90	117	1.05	2.28	0.43	20.0	0.014
AlAu	91	114	1.03	2.28	0.43	20.0	0.42
Sn	103	97	1.00	4.42	1.0	15.3	(3)
$Bi_{0.9}Tl_{0.1}$	171	109	1.88	5.98	1.3	5.5	

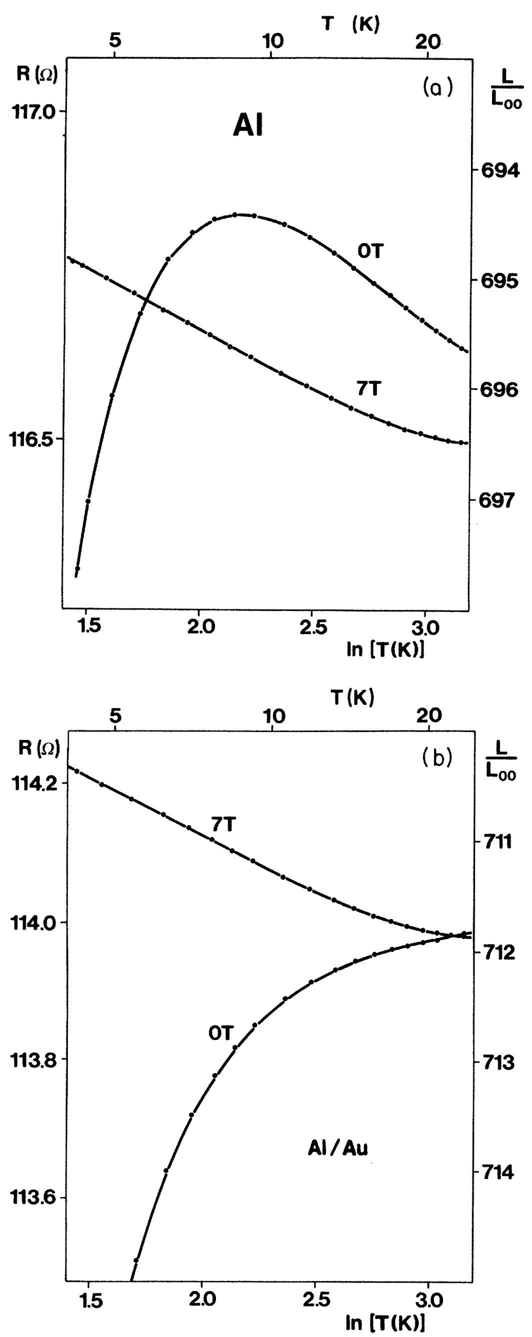

FIG. 1. (a) Resistance (i.e., the normalized conductance $L/L_{00} \approx 80$ kΩ/R using the right scale) as function of $\ln T$ for an Al film with a thickness of 90 Å. The measurements are performed in external magnetic fields of $H=0$ and 7 T perpendicular to the film. (b) Resistance (i.e., the normalized conductance) as a function of $\ln T$ for the same Al film as in (a) after 0.25 atomic layers of Au coverage. Again the measurements are performed in external magnetic fields of $H=0$ and 7 T perpendicular to the film.

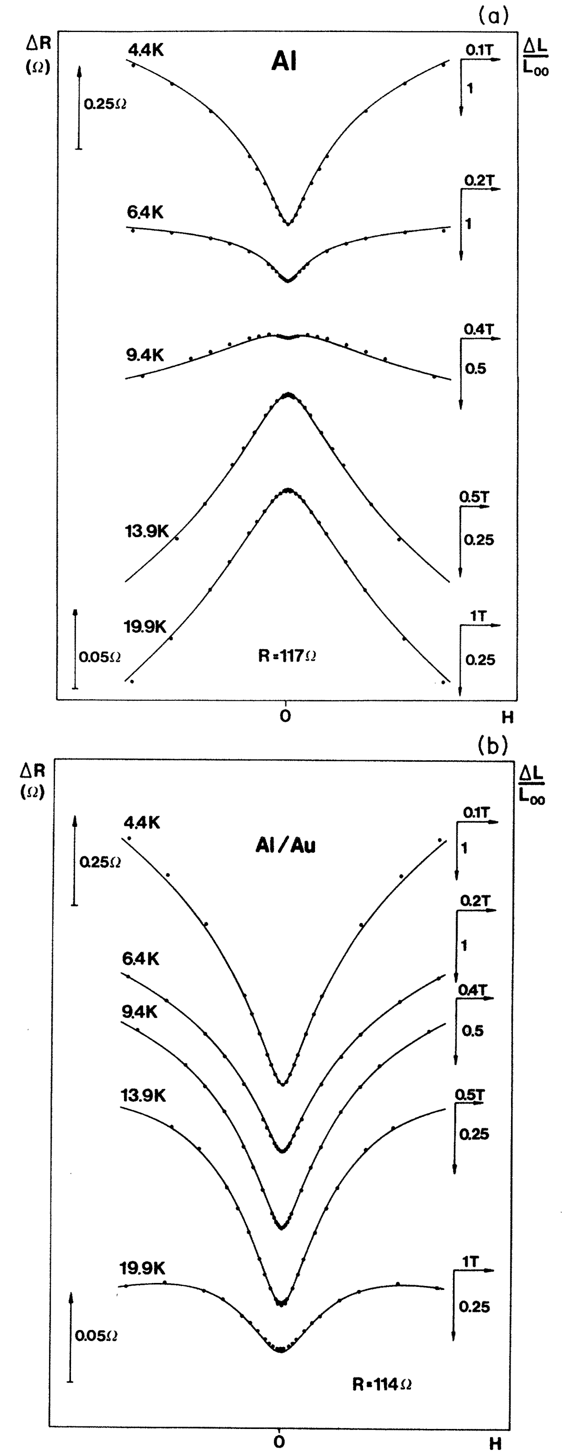

FIG. 2. (a) Magnetoresistance of a Al film ($d=90$ Å) as a function of the field H. The units of the field are shown on the right-hand side of each magnetoresistance curve. The arrows give the units of the conductance. The points represent the experimental results. The solid curves are calculated with the theory as discussed in Secs. III and IV. (b) Magnetoresistance of the same Al film after 0.25 atomic layers of Au coverage. The units of the magnetic field are the same as in (a). The points represent the experimental results. The solid curves are calculated with the same parameter set as in (a) with the exception of the value $H_{s.o.}$, the spin-orbit scattering strength which is increased by the superposition with Au.

In Fig. 2(b) the magnetoresistance is plotted for the Al/Au system. The field and resistance scale for each temperature is the same as in Fig. 2(a).

For the evaluation of the experiment it is very desirable to know the superconducting transition temperature of the Al and the derivative of the upper critical field dH_{c2}/dT.

In the apparatus which is used for the experiment, the temperature of the helium cooling the superconducting magnet determines the lowest temperature of the film. Therefore, we had to pump the helium of the superconducting magnet to reach the transition temperature of the Al. In Fig. 3 the transition curves for the Al film are plotted for different magnetic fields perpendicular to the film. The left set is for the pure Al film, and the right set is after the coverage with 0.25 atomic layers of Au. The magnitude of the field is written beside the curves. The experimental transition temperature of the quench-condensed Al is 2.28 K. This is about 1 K above the transition temperature of bulk Al without lattice defects. We do not believe that oxygen impurities which generally raise the transition temperature of Al are responsible for the T_c of 2.28 K. It is well known that quenched condensation changes the properties of the superconductor and enhances the electron-phonon interaction (see, for example, Ref. 31). Minnigerode[32] found for Al, quench condensed in an ordinary high vacuum, a superconducting transition temperature of 2.6 K. Since in the present experiment we use an ultrahigh vacuum, the lower transition temperature of 2.28 K could correspond to a quench-condensed Al without O_2. (There might be an additional effect of the small thickness on T_c.) Of course, the magnetic field shifts the transition curves to lower temperature, yielding H_{c2}. The transition curves are not noticeably changed by the coverage with 0.25 atomic layers Au. The field dependence of H_{c2} is not quite linear and allows only an approximate determination of dH_{c2}/dT for which we take the value of 0.43 T/K. Fortunately this poor accuracy is sufficient to show the minor influence of the AL fluctuations on the magnetoresistance.

As an example for an intermediate-coupling superconductor we condensed several Sn films. We present here as a typical example the experimental results for a Sn film with a square resistance of about 96.6 Ω. The thickness of the film is 103 Å. Figure 4 shows the corresponding

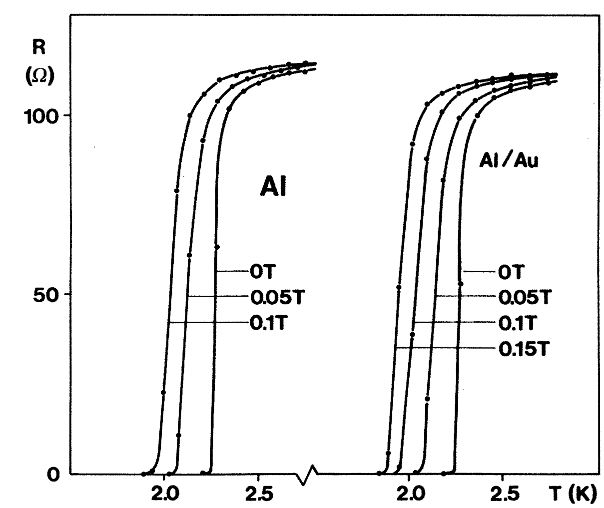

FIG. 3. Transition curves for the Al film in a magnetic field perpendicular to the film. The left set is for the pure Al film and the right set after the 0.25 atomic layers of Au coverage. The magnitude of the field is written beside the curves.

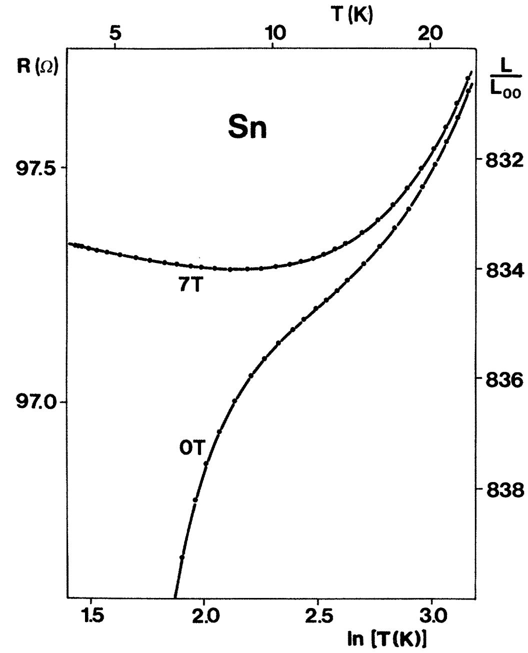

FIG. 4. Resistance (i.e., the normalized conductance $L/L_{00} \approx 80$ kΩ$/R$ using the right scale) as function of $\ln T$ for a Sn film with a thickness of 103 Å. The measurements are taken in magnetic fields $H = 0$ and $H = 7$ T.

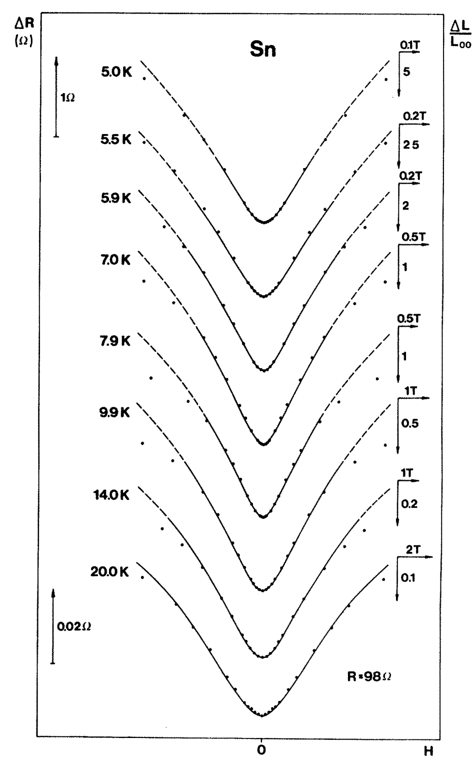

FIG. 5. Magnetoresistance (or $[L(H)-L(0)]/L_{00}$) of a Sn film as a function of the field H. The units of the field are shown on the right-hand side of each magnetoresistance curve together with the units of the conductance. The points represent the experimental results. The solid curves are calculated with the theory as discussed in Secs. III and IV.

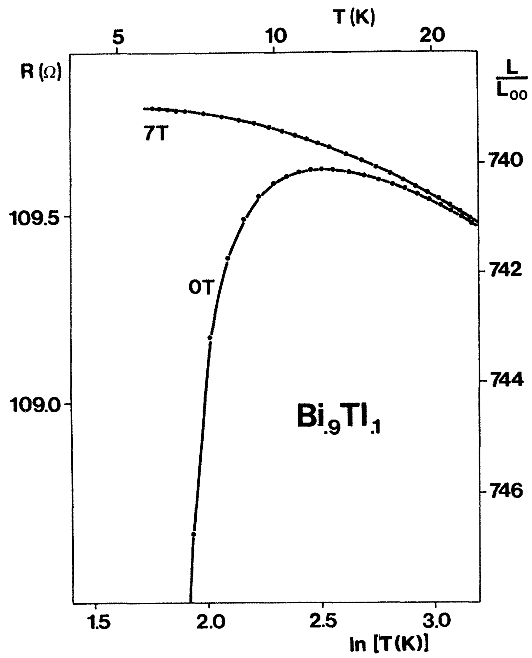

FIG. 6. Resistance (i.e., the normalized conductance $L/L_{00}\approx 80$ kΩ$/R$ using the right scale) as function of $\ln T$ for a $Bi_{0.9}Tl_{0.1}$ film with a thickness of 171 Å. The measurements are taken in magnetic fields $H=0$ and $H=7$ T.

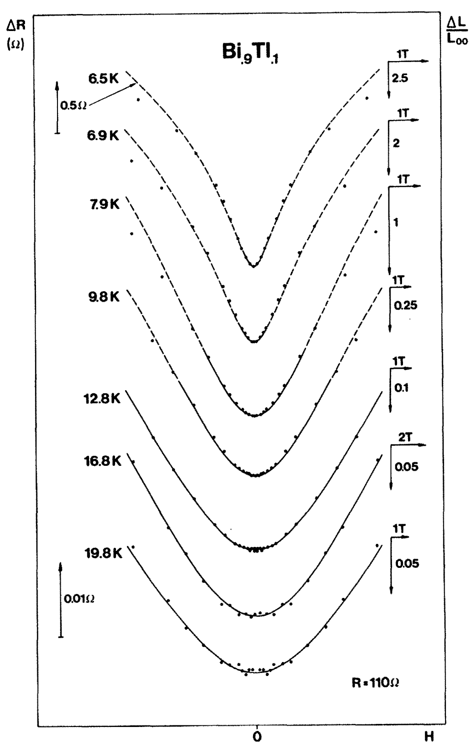

FIG. 7. Magnetoresistance (or $[(L(H)-L(0)]/L_{00})$ of a $Bi_{0.9}Tl_{0.1}$ film as a function of the field H. The units of the field are shown on the right-hand side of each magnetoresistance curve together with the units of the conductance. The points represent the experimental results. The solid curves are calculated with the theory as discussed in Secs. III and IV.

resistance as a function of $\ln T$ in a magnetic fields of 0 and 7 T. The magnetoresistance is measured at nine different temperatures. We were particularly interested in the temperature range near T_c. In Fig. 5 the magnetoresistance is plotted for these temperatures. The points represent the experimental results. The solid and dashed curves will be discussed in Sec. IV.

Again the superconducting transition curves in different magnetic fields are measured. The transition temperature of the Sn film in zero magnetic field is 4.42 K and the derivative of the upper critical field is 1.0 T/K.

$Bi_{0.9}Tl_{0.1}$ is an extremely strong coupling superconductor. We present here, as a typical example, the experimental results for a $Bi_{0.9}Tl_{0.1}$ film with a square resistance of 109 Ω. The thickness of the film is 171 Å. Figure 6 shows the corresponding resistance as a function of ln T in a magnetic field of 0 and 7 T. The magnetoresistance is measured at eight different temperatures and plotted in Fig. 7. The points represent the experimental results. The solid and dashed curves will be discussed below.

The superconducting transition curves in different magnetic fields are measured. The transition temperature of the $Bi_{0.9}Tl_{0.1}$ film in zero magnetic field is 5.98 K, and the derivative of the upper critical field is 1.3 T/K. The evaluation of the transition curves is complicated, because the superconducting films show flux flow below T_c and even above T_c the superconducting fluctuations participate in the flux flow. Therefore, the accuracy of dH_{c2}/dT is only of the order of 10%.

III. THEORETICAL BACKGROUND

A. Aslamazov-Larkin contribution

About 15 years ago Aslamazov and Larkin (AL)[33] calculated the influence of superconducting fluctuations in two-dimensional disordered superconductors on the resistance above the superconducting transition temperature T_c. They obtained a correction to the residual conductance

$$\Delta L(T)=-\frac{\Delta R(T)}{R_0^2}=\frac{e^2}{16\hbar}\frac{T_c}{T-T_c}\ . \qquad (1a)$$

At that time one was only interested in the resistance correction close to T_c. If one wants to extend the temperature range of Eq. (1a) then one must replace $(T-T_c)/T_c$ by $\ln(T/T_c)$. Then one finds

$$\Delta L(T)=(\pi^2/8)L_{00}[\ln(T/T_c)]^{-1}\ . \qquad (1b)$$

Here $L_{00}=e^2/(2\pi^2\hbar)\approx(80\ \mathrm{k}\Omega)^{-1}$ is a universal conductance which appears in all quantum corrections of the conductance. The contribution of the AL term in a magnetic field parallel to the film has been first calculated by the author[34] in an acceleration model of superconducting fluctuations. The dependence of ΔL_{AL} on a magnetic field perpendicular to the film is derived by Usadel.[35] He obtained

$$\Delta L(T,H)=\frac{\pi^2}{4}L_{00}\sum_n\frac{(n+1)p^2}{A(A+p)(A+p/2)}\ , \qquad (2a)$$

where

$$A=\ln(T/T_c)+(n+1/2)p\ ,$$

$$p=(\pi^2/4)(H/H_T)\ ,$$

and

$$H_T=(2\pi k_B T)/(4eD)=(\pi^2/8)H'_{c2}T\ .$$

The sum can be expressed by the derivative of the digamma function. The two parameters which determine the contribution of AL are the transition temperature T_c and the slope of the upper critical field dH_{c2}/dT. For finite thickness of the film one must include an additional term $(m\pi/d)^2\hbar/(2eTdH_{c2}/dT)$ in A of Eq. (2a) and sum over m from zero to infinity.

In low magnetic fields the magnetoconductance is proportional to H^2,

$$\Delta L(H)=-\frac{\pi^2}{16}L_{00}\frac{1}{[\ln(T/T_c)]^3}\frac{1}{(T\,dH_{c2}/dT)^2}H^2\ . \tag{2b}$$

B. The Maki-Larkin contribution

Maki[4] calculated another contribution of superconducting fluctuations to the conductance of a disordered two-dimensional superconductor. Its physical interpretation is very difficult. In particular it had the disturbing property of always being divergent for a superconductor even above T_c. Thompson[5] removed this unphysical behavior by introducing an artificial pair-breaking parameter δ, which made the contribution of the Maki term finite. The origin of this pair-breaking parameter was unclear for a long time. The new phenomena of weak localization now gives an answer to these old questions and allows (in principle) a direct measurement of this pair-breaking parameter. Larkin[9] pointed out that both phenomena have the same origin: the inelastic lifetime of the conduction electrons. Each electron at the Fermi surface experiences inelastic scattering processes. After the inelastic lifetime τ_i the electron is scattered into a new energy state. If this electron was one partner in a (virtual) Cooper pair, then this Cooper pair is broken. In the former description of superconductivity, this mechanism was not included and even the Eliashberg theory includes only the pair breaking caused by electron-phonon processes. Larkin included these processes in the Maki term and obtained the following for the magnetoresistance of the Maki graph for temperatures sufficiently above T_c and not-too-large fields, i.e., for $2\pi k_B(T-T_c)\gg\hbar/\tau_i$ and $2\pi k_B(T-T_c)\gg 4eDH$:

$$\Delta L(H)=\beta(T)L_{00}[\Psi(\tfrac{1}{2}+H_i/H)-\Psi(\tfrac{1}{2}+H_0/H)-\ln(H_i/H_0)]\ , \tag{3}$$

where $\hat{\beta}(T)$ is defined and tabulated in the paper by Larkin.[9] For temperatures in the vicinity of T_c, $\beta(T)$ can be expanded in terms of $\ln(T/T_c)$: $\beta(T)\approx(\pi^2/4)/\ln(T/T_c)$, i.e., in the vicinity of T_c, the quadratic coefficient is proportional to $\ln(T/T_c)^{-1}$. H_i and H_0 express the inelastic lifetime τ_i and the elastic lifetime τ_0 in terms of magnetic fields. H_i can be directly determined in magnetoresistance measurements. The relation between τ_n and H_n is

$$H_n\tau_n=\hbar/(4eD)=\hbar eRdN/4\ , \tag{4}$$

where $D=1/(e^2NRd)$ is the diffusion constant, R the resistance per square, d the thickness of the film, and N the density of states.

In low magnetic fields the magnetoconductance is proportional to H^2,

$$\Delta L(H)\approx-[\beta(T)/24]L_{00}H^2\ . \tag{3'}$$

For the temperature dependence in zero field Altshuler and Aronov[2] give the following expression:

$$\Delta L(T)=L_{00}\beta(T)\ln(k_B T\tau_i/\hbar)\ . \tag{5a}$$

As the formula for the magnetoresistance, Eq. (5a) is restricted for temperatures not too close to T_c so that $2\pi k_B(T-T_c)\gg\hbar/\tau_i$.

Ebisawa *et al.*[14] calculated the temperature dependence of the MT contribution in the other limit close to T_c. They obtained the well-known result

$$\Delta L(T)=\frac{\pi^2}{4}L_{00}\frac{1}{\ln(T/T_c)-\delta}\ln[\ln(T/T_c)/\delta]\ , \tag{5b}$$

where the pair-breaking parameter is given by

$$\delta=\frac{\pi\hbar}{8k_B T\tau_i}\ . \tag{6}$$

Although Eq. (5b) is only valid in the vicinity of T_c, we replaced $(T-T_c)/T_c$ by $\ln(T/T_c)$. Equation (5b) does not have the puzzling property, as shown in Eq. (5a), where the temperature-dependent part of the resistance is influenced by the cutoff energy. The two formulas result from different approximations which essentially exclude each other. They have a rather different structure and it is not possible to interpolate between them. This makes an evaluation of the temperature dependence of the MT term rather difficult. To our knowledge, an explicit formula for all temperatures and fields is not available.

C. The Coulomb interaction

Altshuler *et al.*[36] and Fukuyama[37] calculated an additional contribution to the conductance at low temperature caused by the Coulomb interaction, which is modified by impurity scattering in two-dimensional and quasi-two-dimensional systems. They obtained a correction for the conduction of a thin film:

$$\Delta L_C=-\Delta R/R^2=-L_{00}(1-F)\ln(H_T/H_0)\ , \tag{7}$$

where F was originally defined as a screening factor. According to Finkelstein,[38] F had to be redefined and we take it as an adjustable parameter whose value in thin films is, of the order of 0.2–0.25.[39]

A magnetic field hardly has an orbital effect on the Coulomb anomaly and therefore causes no magnetoresistance. However, Lee and Ramakrishnan[40] showed that there is an effect of the field on the spins of the electrons which causes a positive magnetoresistance. The asymptotic behavior is given by the following equation, but in the evaluation of the experimental results the complete formula is used:

$$[L(H)-L(0)]/L_{00}\approx\begin{cases}(F/2)0.084h^2\ , & h\ll 1\\ (F/2)\ln(h/1.3)\ , & h\gg 1\end{cases} \tag{8}$$

where $h = g\mu_B H / k_B T$.

The spin-orbit coupling modifies the correction to the resistance. The Kubo graph, which arises from the Hartree particle-hole graph and which yields the part proportional to F, is reduced by a factor of 2 (Ref. 41) in the presence of dominating spin-orbit coupling. However, in a real system and for temperatures above 4.2 K the spin-orbit coupling is not dominant. Because of the complex formalism, only the limits of vanishing and dominating spin-orbit coupling have been calculated so far.

D. Coulomb interaction with particle-particle propagators

The contribution of WL and the Coulomb anomaly cannot be simply added. A mixed contribution exists which modifies the first results severely.[37,42,9] It arises from a class of Kubo graphs which one could classify as Fock and Hartree terms with particle-particle propagators including the electron-electron interaction. The latter consists of the Coulomb interactions as well as electron-phonon interaction. The effective interaction strength is given by $\Lambda(T)$, where

$$\Lambda^{-1}(T) = 1/\lambda + \ln[\gamma\eta/(\pi k_B T)] \ , \tag{9a}$$

where λ is the effective electron-electron interaction constant. $\gamma = \exp(0.577)$ and η is the cutoff parameter. For a superconductor with electron-phonon interaction, λ is given by

$$\lambda = \lambda_{e\text{-ph}} + \lambda_0 / [1 + \lambda_0 \ln(E_F/\omega_D)] \ . \tag{9b}$$

Here the cutoff parameter η is given by the Debye frequency ω_D. For a superconductor with the transition temperature T_c, $-\Lambda(T)$ is given by $g(T) = 1/\ln(T/T_c)$ [we introduce a minus sign in the definition of $g(T)$ in contrast to Larkin's definition, because we are only interested in temperatures $T > T_c$].

These Kubo graphs yield, according to Altshuler *et al.*,[42] the following magnetoresistance:

$$\Delta L(H) = 1/\ln(T/T_c) L_{00} \phi_2(H_T/H) \ . \tag{10a}$$

The function $\phi_2(x)$ is, according to Altshuler *et al.*,

$$\phi_2(x) = \int_0^\infty \frac{t\,dt}{sh^2(t)} (1 - xt/sh(xt)) \ . \tag{10b}$$

Again $H_T = \pi k_B T / 2eD$ corresponds to the time $\tau_T = \hbar/(2\pi k_B T)$. In low magnetic fields the magnetoconductance is proportional to H^2,

$$\Delta L(H) \approx 1/\ln(T/T_c)[\zeta(3)/4] L_{00} (H/H_T)^2 \ , \tag{10c}$$

where $\zeta(3)$ is the Riemann zeta function.

The temperature dependence of the particle-particle channel is given by Altshuler and Aronov[2] for $T \gg T_c$ as

$$\Delta L(T) = -L_{00} \ln\left[\frac{\ln(k_B T_c \tau_0/\hbar)}{\ln(T_c/T)}\right] \ . \tag{11}$$

The application of this equation for a superconductor is quite problematic since its range of validity is only qualitatively known.

E. Weak localization

Anderson *et al.*[43] and Gorkov *et al.*[44] predicted the following for the temperature-dependent conductance of weak localization:

$$\Delta L = -\Delta R / R^2 = -L_{00} \ln(H_i/H_0)$$
$$= p L_{00} \ln T + \text{const} \ . \tag{12a}$$

There are, however, complications because of the influence of spin-orbit scattering on weak localization which causes a decrease of the resistance with decreasing temperature and changes weak localization into weak antilocalization.[45] In addition, magnetic impurities block weak localization. Hikami *et al.*[46] included spin-orbit and magnetic scattering in their analysis of quantum interferences. Instead of using the characteristic times τ_i, $\tau_{s.o.}$, etc., we use the corresponding characteristic fields H_i, $H_{s.o.}$, etc. These fields can be directly determined in magnetoresistance measurements. The relation between τ_n and H_n is given by Eq. (4). Expressed in terms of the characteristic magnetic fields, the result by Hikami *et al.* for the temperature dependence of the conductance is

$$\Delta L_{\rm WL} = -\Delta L / R^2$$
$$= -L_{00}[\ln(H_1/H_2) - \ln(H_3/H_4)/2] \ . \tag{12b}$$

The H_n are defined in the following manner:

$$\begin{aligned} H_1 &= H_0 + H_{s.o.} + H_s \ , \\ H_2 &= \tfrac{4}{3} H_{s.o.} + \tfrac{2}{3} H_s + H_i \ , \\ H_3 &= 2H_s + H_i \ , \\ H_4 &= \tfrac{4}{3} H_{s.o.} + \tfrac{2}{3} H_s + H_i \ , \end{aligned} \tag{12c}$$

where H_0 corresponds to the elastic lifetime τ_0, H_i to the inelastic lifetime τ_i, $H_{s.o.}$ to the spin-orbit scattering time $\tau_{s.o.}$, and H_s to the magnetic scattering time τ_s. (The distinction between the x and the z component of the scattering times is neglected because it hardly affects the evaluation.)

The field dependence of the conductance was first calculated by Altshuler *et al.*[47] in the absence of spin-orbit coupling. Hikami *et al.*[46] and Maekawa and Fukuyama[48] extended the calculation by including spin-orbit coupling and magnetic scattering. The formula of Hikami *et al.* is given by Eq. (13):

$$\Delta L_{\rm WL}/L_{00} = -\{\Psi(\tfrac{1}{2} + H_1/H) - \Psi(\tfrac{1}{2} + H_2/H)$$
$$+ [\Psi(\tfrac{1}{2} + H_3/H) - \Psi(\tfrac{1}{2} + H_4/H)]/2\} \ , \tag{13}$$

where Ψ is the digamma function and H the applied field.

For finite film thickness one may replace the arguments in the various digamma functions by $\frac{1}{2} + H_n/H + m^2 H_d/H$ and sum over m. Here H_d gives a field characteristic for the film thickness. The film is, in given magnetic field at a given temperature, two-dimensional as long as $H, H_i \ll H_d$. H_d is given by

$$H_d = \frac{\hbar\pi^2}{4ed^2} , \tag{14}$$

where d is the film thickness.

In low magnetic fields the magnetoconductance is proportional to H^2,

$$\Delta L(H)/L_{00} \approx -\tfrac{1}{24}[H_1^{-2} - H_2^{-2} + (H_3^{-2} - H_4^{-2})/2]H^2 . \tag{13'}$$

IV. EVALUATION OF THE EXPERIMENTAL RESULTS

There are so many theoretical contributions to the resistance (conductance) of superconductors above T_c that the evaluation of the experimental results appears to be rather difficult. In particular, at temperatures sufficiently above T_c (for example, $T > 2T_c$) weak localization (WL), the Coulomb anomaly in the particle-hole channel (CPH) and in the particle-particle channel (CPP), the Maki-Thompson (MT) terms, and the Aslamasov-Larkin (AL) terms can be of the same order of magnitude. Since most of the contributions depend on one or several parameters, it is difficult to extract the single terms from the experiment. Therefore, one must look for simple criteria to estimate the different contributions. One possibility is to consider first the temperature region not too far from T_c. For $T_c < T < 1.5T_c$, the contribution of the AL and the MT contribution are by an order of magnitude larger than the other terms discussed in Sec. III. However, one must pay for this simplification, because several complications arise: (i) The mean-field theory of superconductivity becomes problematic. Inelastic scattering processes of the two electrons belonging to a Cooper pair reduce the superconducting transition temperature. In thin disordered films such inelastic processes are enhanced and important. In particular the resulting pair-breaking parameter δ which is given by Eq. (6) is temperature dependent. As a consequence, the reduction of T_c is temperature dependent. In other words, if one raises the temperature in a disordered film, the critical temperature which enters the physical properties is lowered. This generates, for example, a nonlinear temperature dependence of the upper critical field.[13] The superconducting fluctuations cause a similar change of T_c. The total effect can be measured by a T_c dependence on the resistance or thickness of the superconducting film. In the evaluation of the superconducting fluctuation one should therefore replace T_c by $T_c(T)$ in the function $g(T) = 1/\ln(T/T_c)$. We neglect this effect in the following evaluation. (ii) Some of the theoretical formulas are less reliable in the vicinity of T_c, because the complexity of the problem requires simplifications and sometimes the q dependence of the fluctuation propagator or the verticies have been neglected. This applies in particular for the evaluation of the Maki graph by Larkin which has the two restrictions that $2\pi k_B(T - T_c) \gg \hbar/\tau_i$ and $2\pi k_B(T - T_c) \gg 4eDH$. Sometimes the upper cutoff of the frequency summation is taken at $2\pi k_B(T - T_c)$, which is, close to T_c, surely not a reasonable upper cutoff energy. (Not all of the theoretical approximations can be easily recognized from the published derivations, so this remark is a more general warning.)

On the other hand, the theoretical contributions of AL, the Coulomb anomaly CPH, and CPP are theoretically determined if T_c and dH_{c2}/dT are experimentally given. The two other contributions, WL and MT, require two adjustable parameters: H_i, which is temperature dependent, and $H_{s.o.}$, which should be temperature independent. If we rely on the theory then we have no more adjustable parameters than in the case of a normal metal. However, the main problem is that the calculation of the magnetoresistance of the MT is restricted in the temperature and magnetic field range. The error of the calculation (if one approaches the limit) is not well known.

A. Magnetoresistance

Before we present the evaluation of the magnetoresistance curves it appears useful to obtain a survey about the different contributions for the different metals as a function of the temperature. For this purpose we consider the simplest information of the magnetoresistance curves for an orientation. This is the coefficient of the quadratic field dependence which we discussed already in Sec. III. For $Bi_{0.9}Tl_{0.1}$ and Sn, this coefficient varied between $T_c + 0.5$ K and about 20 K by almost a factor of 10^4. We determine experimentally the coefficient of the quadratic field dependence of the magnetoconductance as

$$\Delta L(H)/L_{00} \approx a_2 H^2 ,$$

and plot the logarithm of the absolute value of a_2, i.e., $\ln(|a_2|)$ as a function of the logarithm of $g(T) = 1/\ln(T/T_c)$. This is done in Fig. 8 for Al/Au, in Fig. 11 for Sn, and in Fig. 13 for $Bi_{0.9}Tl_{0.1}$. In the same figures we plot the theoretically expected contributions from the AL term [Eq. (2a)] and the two contributions from the Coulomb anomalies [Eqs. (8) and (10b)], which require only the knowledge of T_c and dH_{c2}/dT. The difference between the experimental value of $\ln(|a_2|)$ and the three theoretical contributions should be due to WL and MT. This plot concentrates on small magnetic fields but gives reasonable insight into the contributions of the different terms. For vanishing spin-orbit scattering the contributions of WL and MT are opposite. At $T = 2.7T_c$ they are opposite and equal and for lower temperature the MT term exceeds the WL. For large spin-orbit scattering both terms have the same sign and yield positive magnetoresistance. Generally the determination of the spin-orbit scattering, i.e., $H_{s.o.}$, is very uncertain, but the superposition of Au on the same film allows a good analysis of the spin-orbit scattering. This shall be discussed for the different superconductors.

1. Magnetoresistance of the Al and Al/Au films

First we determine the coefficient of the quadratic field dependence a_2 for the Al/Au film (for the pure Al film a_2 changes sign and is less suited for this purpose) and plot $\ln(|a_2|)$ versus $\ln[g(T)]$, in Fig. 8, together with the theoretical dependence of $\ln(|a_2|)$ for the different corrections of the conductance. The dashed curve gives the AL values which are calculated with the experimental

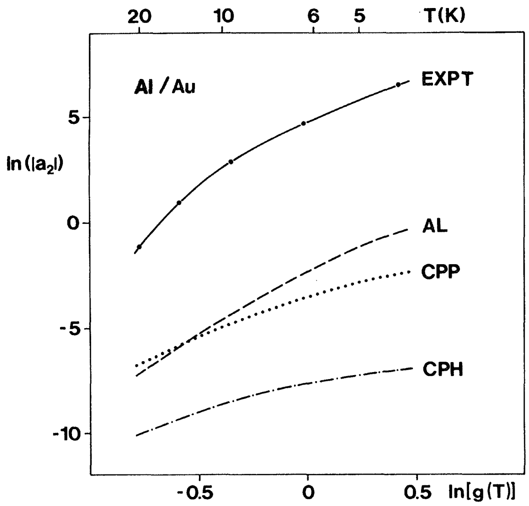

FIG. 8. Magnetoresistance curves start with a quadratic field dependence $\approx a_2 H^2$. The logarithm of the absolute value of a_2, i.e., $\ln(|a_2|)$ is plotted versus $\ln[g(T)]$, where $g(T)=1/\ln(T/T_c)$. The solid curve gives the experimental result for the Al film with 0.25 atomic layers of Au coverage. The dashed curve gives the theoretical contribution of the Aslamazov-Larkin part (AL) (using the experimental value for $dH_{c2}/dT=0.43$ T/K), the dotted curve gives the theoretical contribution of the Coulomb anomaly in the particle-particle channel (CPP), and the dashed-dotted curve gives the contribution of the Coulomb anomaly in the particle-hole channel (CPH). The curves show the different contributions to the experimental magnetoresistance curves.

value of $dH_{c2}/dT=0.43$ T/K. The dotted curve gives the particle-particle contribution of the Coulomb anomaly to $\ln(|a_2|)$ (it yields a negative magnetoresistance) and the dashed-dotted curve gives the particle-hole contribution with $F\approx 0.25$. One realizes that these three theoretical terms are a factor of several hundred smaller than the experimental value for Al/Au. Therefore, they play only a minor role in the magnetoresistance although they influence the shape of the magnetoresistance curves at larger fields as we shall see below.

At 20 K the influence of the MT term is roughly a factor of 3–6 smaller (depending on the spin-orbit scattering) than the contribution of WL; i.e., the contribution of WL is dominant. As a consequence the magnetoresistance curves are very different for Al and Al/Au. This allows a reliable and independent determination of the spin-orbit scattering field $H_{\text{s.o.}}$ and the inelastic field $H_i(20\text{ K})$. The author recently described the fitting procedure in detail[49] for films of Mg and Mg/Au. Both sets of curves for Al and Al/Au for all temperatures are iteratively used to obtain a consistent set of parameters. This yields one parameter set of $H_i(T)$ which applies for both Al and Al/Au and two different temperature-independent values for $H_{\text{s.o.}}$, one for pure Al and the second for Al/Au. The two values are 0.014 T for Al and 0.42 T for Al/Au. Before we discuss H_i and its temperature dependence the consistency of the agreement between the experimental data and the theoretical evaluation should be emphasized. The fact that both sets of curves reproduced well by the same temperature-dependent $H_i(T)$ means essentially that both WL and MT are correctly described by the theory. If one would subtract the magnetoresistance curves of Al and Al/Au at each temperature, all theoretical terms with the exception of WL cancel, and the agreement of the experimental difference with the theory (since each single curve shows this agreement) demonstrates the agreement of the theoretical contribution of WL with the experimental. From the agreement between each single experimental curve with the theoretical result, one easily concludes that the same agreement between experiment and theory is given for the MT part as well.

For the magnetoresistance curve of Al/Au at 4.4 and 19.9 K we want to show how the total theoretical contribution is composed. In Figs. 9(a) and 9(b) the contributions of WL, MT, AL, and the Coulomb anomaly in the CPP to the total theoretical magnetoresistance curves are plotted. The dashed curve *a* shows the single contribution WL, the dashed curve *b* shows the sum of WL and MT, and the dashed curve *c* shows the sum of WL, MT, and Al. Finally, the solid curve *d* gives the total contribution as a sum of WL, MT, AL, and CPP. The points represent the experimental results. (At 4.4 K, curves *c* and *d* are almost identical.)

From the inelastic field we can calculate, by means of relation (4), the inelastic lifetime of the conduction electrons as a function of temperature. This is plotted in Fig. 10 in a log-log plot. In the temperature range between 4.4

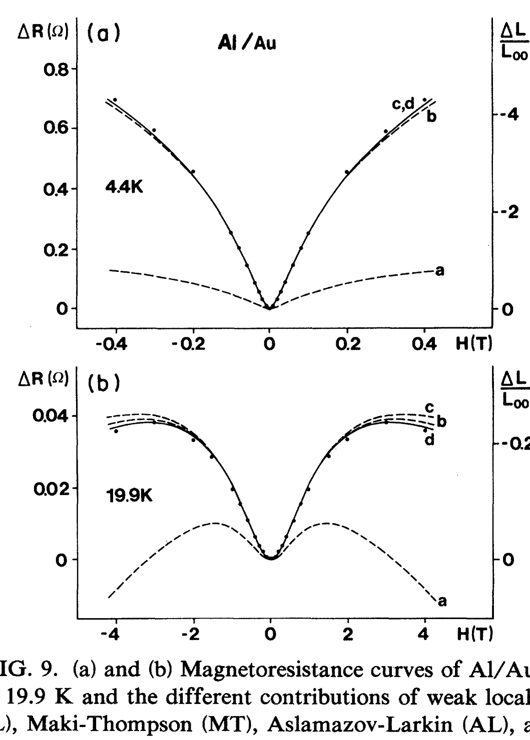

FIG. 9. (a) and (b) Magnetoresistance curves of Al/Au at 4.4 and 19.9 K and the different contributions of weak localization (WL), Maki-Thompson (MT), Aslamazov-Larkin (AL), and the Coulomb anomaly in the particle-particle channel (CPP) to the total theoretical magnetoresistance curves. The dashed curves *a* show the single contribution WL, the dashed curves *b* the sum of WL and MT the dashed curves *c* the sum of WL, MT, and AL, and finally the solid curves *d* give the total contribution as a sum of WL, MT, AL, and CPP. The points represent the experimental results.

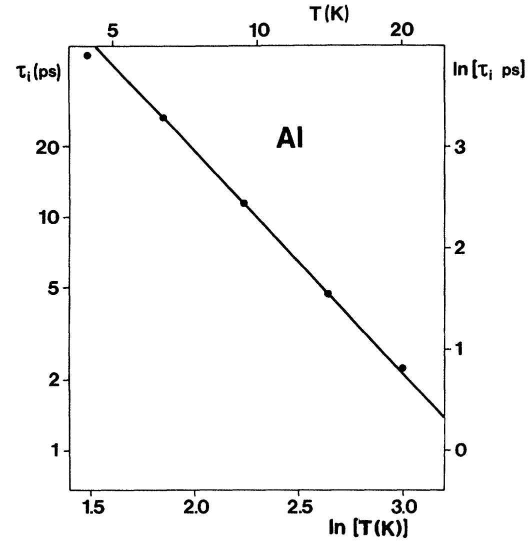

FIG. 10. Inelastic lifetime of the conduction electrons in the Al films as a function of temperature in a log-log plot.

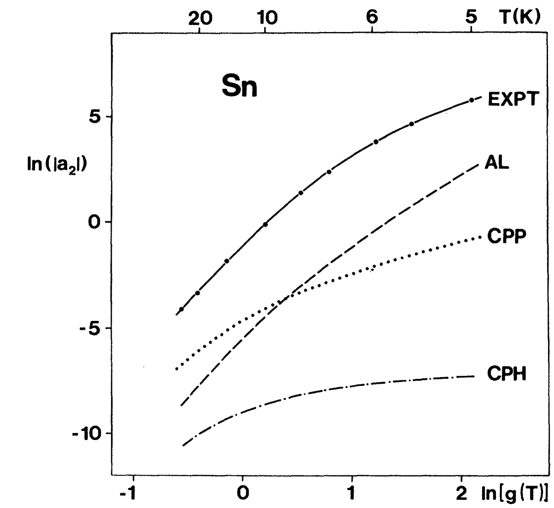

FIG. 11. Logarithm of the absolute value of a_2; the coefficient of the quadratic magnetic field dependence of the magnetoresistance is plotted versus the logarithm of $g(T)=1/\ln(T/T_c)$. The solid curve gives the experimental results from the magnetoresistance curves for the Sn film, shown in Fig. 5. The dashed curve gives the theoretical contribution of the Aslamazov-Larkin part (AL) (using the experimental value for $dH_{c2}/dT=1.0$ T/K), the dotted curve gives the theoretical contribution of the Coulomb anomaly in the particle-particle channel (CPP), and the dashed-dotted curve gives the contribution of the Coulomb anomaly in the particle-hole channel (CPH).

and 20 K the inelastic lifetime varies between 1.5 and 40 ps. It is roughly the same as for quench-condensed Mg. The inelastic lifetime depends on temperature as a power law $1/\tau_i \propto T^p$; the experimental value of p is 2.13, i.e., essentially a quadratic dependence. The spin-orbit scattering times which one obtains from the two values of $H_{\text{s.o.}}$ are 46 ps for Al and 1.5 ps for Al/Au. Meservey, Tedrow, and Bruno[50] obtained for a similar Al film from the superconducting tunneling with polarized electrons $\tau_{\text{s.o.}} \approx 20$ ps. Our spin-orbit scattering time for the Al film is quite close to that of quench-condensed Mg which possesses the value of 68 ps.

For magnetic fields above 1 T the Al film shows generally a negative magnetoresistance, even at 4.4 K [this is beyond the field range of Fig. 2(a)]. Such a behavior can not be reproduced by the present theory, because at $T \approx 2T_c$ the positive MT term is larger than the negative contribution of WL (even in the limit of vanishing spin-orbit scattering). Again for this field range the Larkin theory of the MT term requires an improvement.

2. Magnetoresistance of Sn films

In Fig. 5 the magnetoresistance of the Sn film is shown. First we determine the coefficient of the quadratic field dependence a_2 and plot $\ln(|a_2|)$ versus $\ln[g(T)]$ in Fig. 11 together with the theoretical dependence of $\ln(|a_2|)$ for the different corrections of the conductance. Again the dashed curve gives the AL values which are calculated with the experimental value of $dH_{c2}/dT=1$ T/K. The dotted curve gives the particle-particle contribution CPP of the Coulomb anomaly to $\ln(|a_2|)$ (with a negative magnetoresistance). The particle-hole contribution of the Coulomb anomaly is given by the dashed-dotted curve. As we shall discuss below, the experimental data can only be reproduced by the theory if we assume that the Sn film is, even at the highest temperature, in the limit of strong spin-orbit scattering. In this case the contribution of weak (antilocalization) depends very little on the value of $H_{\text{s.o.}}$ and we have essentially only one temperature-dependent parameter, H_i. In Fig. 5 the theoretical curves are plotted together with the experimental results. The theoretical curves are composed of the contribution of WL, the particle-particle part of the Coulomb anomaly, the AL part, and the MT part of the superconducting fluctuations. Since the calculation by Larkin for the MT part neglects the depression of the superconducting transition temperature by the magnetic field and the q dependence of the fluctuation propagator, it can only be applied for a limited magnetic field range which is, expressed in the parameters of the Sn film, $H \ll 0.47(T-T_c)$ [the coefficient is $2\pi k_B/(4eD)$]. For fields larger than this limit the theoretical curves are dashed. One realizes that the safe range of magnetic field in which the Larkin theory can be applied is rather small. (Larkin gives the $1/2\pi$ fraction of the value which limits the theory to only the quadratic dependence.) If one keeps this restriction in mind then the agreement between experiment and theory is surprisingly good.

In the evaluation we assumed a spin-orbit scattering field of $H_{\text{s.o.}}=3$ T. This corresponds almost to the limit of strong spin-orbit scattering, because the inelastic magnetic field at 20 K is $H_i(20\text{ K})=1.12\text{T} \ll H_{\text{s.o.}}$. It is not possible to achieve a reasonable agreement between experiment and theory with small spin-orbit scattering. In the limit of small spin-orbit scattering, weak localization shows a negative magnetoresistance and this contribution is considerably larger than the MT term because $T/T_c > 4$ and at a ratio of 2.7 both contributions are equal. For intermediate values of $H_{s.o.} \approx H_i$ the theory yields a magnetoresistance curve with a maximum at finite fields in contrast to the experiment. The large spin-orbit scattering for Sn is somewhat puzzling, because for Ag, whose nuclear charge is not so different from Sn, the author[51] found that at 20 K H_i was about twice $H_{s.o.}$. To check this point, a similar Sn film has been covered after the measurement of the magnetoresistance with 0.27 atomic layers of Au. Such a Au coverage increased the value of $H_{s.o.}$ in Al by about 0.4 T. However, if the Sn is already in the limit of strong spin-orbit scattering, then the Au should have little effect on the magnetoresistance curves. This is indeed found for the magnetoresistance curves for the Sn film and the Sn/Au sandwich. The effect of the Au is very small and in agreement with the assumption that the pure Sn film already has a large value of $H_{s.o.}$ of the order of 3 T. (However, if one starts with a small $H_{s.o.}$ which does not fit the experimental data, then the theory predicts a much larger change of the magnetoresistance curves by the Au.)

In Fig. 12 the adjusted inelastic field H_i^* of Sn is plotted as a function of the temperature in a log-log plot. Since the Larkin theory of the MT term is not really applicable to the intermediate-coupling superconductor such as quench-condensed Sn, we hesitate to interpret the H_i^* measurements in terms of the inelastic lifetime at least for temperatures close to T_c.

3. Magnetoresistance of the amorphous $Bi_{0.9}Tl_{0.1}$

In Fig. 13 the logarithm of the absolute value of the coefficient $\ln(|a_2|)$ is plotted against $\ln[g(T)]$. The AL contribution is almost identical with experimental points.

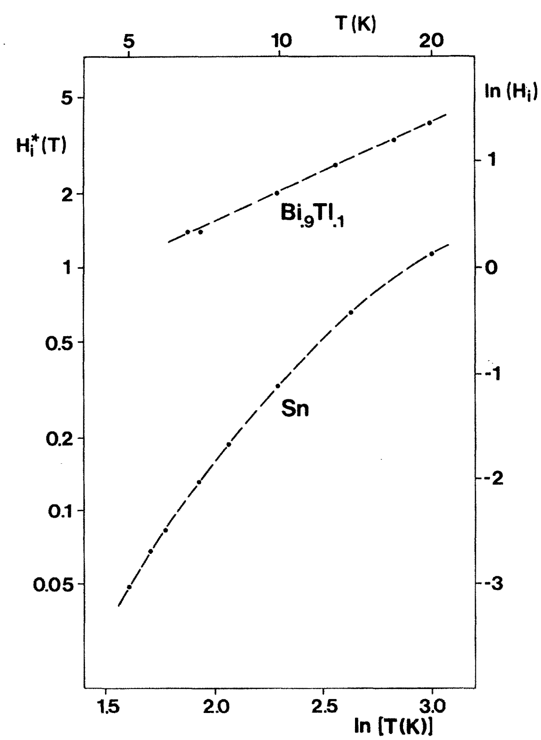

FIG. 12. Field H_i^* of the Sn and the $Bi_{0.9}Ti_{0.1}$ films as a function of temperature in a log-log plot.

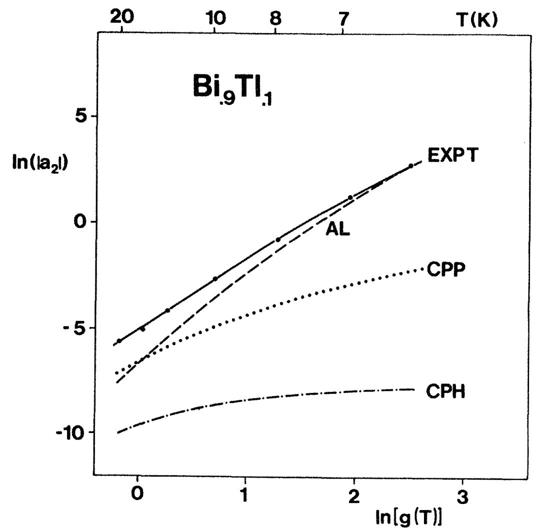

FIG. 13. Logarithm of the absolute value of a_2; the coefficient of the quadratic field dependence of the magnetoresistance is plotted versus the logarithm of $g(T) = 1/\ln(T/T_c)$. The solid curve gives the experimental results from the magnetoresistance curves for the $Bi_{0.9}Ti_{0.1}$ film, shown in Fig. 7. The dashed curve gives the theoretical contribution of the Aslamazov-Larkin part (AL) (using the experimental value for $dH_{c2}/dT = 1.3$ T/K), the dotted curve gives the theoretical contribution of the Coulomb anomaly in the particle-particle channel (CPP), and the dashed-dotted curve gives the contribution of the Coulomb anomaly in the particle-hole channel (CPH).

This means that AL is the dominant contribution in the magnetoresistance curves and the other terms yield only corrections, in particular at low temperatures. The theory by Larkin for MT is not really applicable for the amorphous $Bi_{0.9}Tl_{0.1}$. The Larkin limit requires that the magnetic field range is restricted, $H \ll 0.87\,(T - T_c)$, and that the temperature is sufficiently above T_c, i.e., $T - T_c \gg H_i/0.87$. Since it results that an adjusted H_i is of the order of a few tesla, in the whole experimental temperature range the Larkin theory is not really applicable. (One must keep in mind that Larkin had the sharper condition with an additional factor of 2π.) Since there is no other theoretical treatment for the magnetoresistance of the MT part, we formally evaluate the experimental magnetoresistance curves with the existing theories by fitting the H_i^* and take it as a fit parameter which cannot be identified with the inelastic lifetime. The theoretical curves in Fig. 7 are calculated as before for Al and Sn by adjusting H_i^* and assuming the limit of strong spin-orbit scattering (because of the great atomic charge Z of Bi and Tl and the short mean free path). The other terms are calculated as before. The adjusted H_i^* are plotted in Fig. 12 together with the Sn values. They vary between 1.4 and 4 T. Even at low temperature (6.5 K), where the AL contribution is completely dominant, the total shape of the theoretical magnetoresistance curve is influenced by WL and MT.

B. Temperature dependence of the resistance

For the Al film, the coverage with 0.25 atomic layers of Au clearly changed the temperature dependence of the resistance as Figs. 1(a) and 1(b) show. This change is caused by the influence of the spin-orbit scattering. In contrast to WL, the other contributions are essentially independent of the spin-orbit scattering. This applies to the superconducting fluctuations AL and MT and the Coulomb anomaly in the particle-hole channel CPH. The Coulomb anomaly in the particle-particle channel depends in principle on the spin-orbit scattering, however, this requires that $H_{s.o.} > H_T$. The superposition of the Al with the Au does not change AL, MT and CPH. Since for the Al/Au film $H_{s.o.} = 0.42$ (in units of T) while $H_T = 0.53$ T (units of T/K) we find that even the Al/Au film is, for CPP, still in the limit of small spin-orbit scattering (in the temperature range $4.4 < T < 20$ K). Therefore, the only change in the resistance corrections is caused by WL. We plot the difference in the resistances between the Al and the Al/Au film as a function of ln (T) and compare it with the difference that the theory yields for WL using the $H_i(T)$ and the two values of $H_{s.o.}$ which are 0.014 T for Al and 0.42 T for Al/Au. In Fig. 14 the points represent the experimental difference and the solid curve gives the theoretical difference. This is essentially an absolute plot of the resistance difference. [The film resistance was reduced by the Au evaporation and the following annealing at 40 K. However, the theoretical resistance versus temperature curves in a magnetic field of 7 T for the Al and Al/Au (which are almost parallel to each other) can be used to adjust the shift between the two experimental curves.]

The agreement between the experimental and the theoretical difference is quite satisfying. This means that the contribution of WL is correctly determined and represents another cross check of the evaluation. Now we may examine the temperature dependence of the other terms. For MT we are now in the nice position that we know H_i or τ_i and, therefore, the pair-breaking parameter directly from the experiment. Since experimentally H_i varies proportional to T^2 the pair-breaking parameter is proportional to T, and with the experimental data we find $\delta \approx 0.0033T$. In Fig. 15 we have plotted the experimental temperature dependence of the Al resistance (solid curve). Now we subtract from the experimental curve the different corrections that the theory yields. If the theoretical contributions are correct, then after the subtraction we expect a constant temperature-independent resistance curve. The dashed curves a, b, c, d, and e are obtained by the successive subtraction of WL, AL, MT, CPP, and CPH. For the MT contribution the result of Altshuler and Aronov [Eq. (5a)] is used. We find that Eq. (5a) overestimates the MT contribution strongly. The temperature dependence of the MT term is not correctly reproduced by the theory.

Finally in this section we want to examine the two

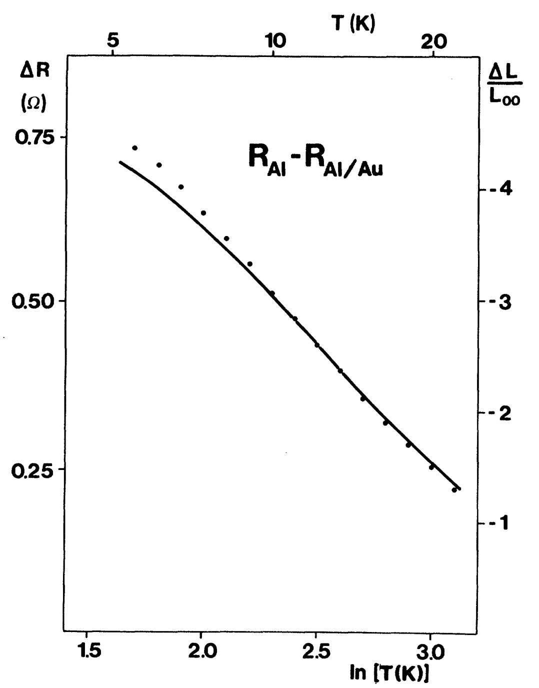

FIG. 14. Difference between the measured resistance of the Al and the Al/Au film as a function of temperature (circles). The solid curve gives the theoretical differences using the common set of $H_i(T)$ and the two different values of $H_{s.o.}$, which are evaluated from the magnetoresistance in Figs. 2(a) and 2(b).

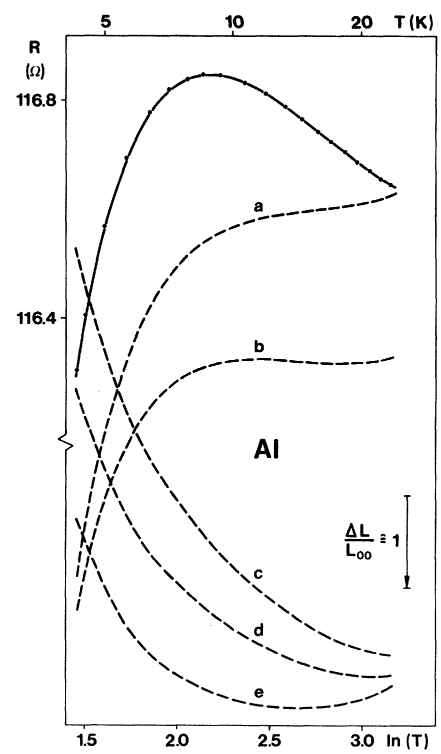

FIG. 15. Resistance of the Al after the subtraction of the different theoretical contributions. The solid curves with the points present the experimental result. The dashed curve a is obtained after the subtraction of WL. The dashed curve b is obtained after the subtraction of WL and AL. The dashed curve c is obtained after the subtraction of WL, AL, and MT. The dashed curve d is obtained after the subtraction of WL, AL, and MT, and CPP. The dashed curve e is obtained after the subtraction of WL, AL, MT, CPP, and CPH. The absolute values of the theoretical resistance corrections are irrelevant, because they depend on cutoff energies.

dimensionality of the thin films. For some of the theoretical contributions this can be done numerically, because the corresponding equation can be generalized for finite film thickness which requires an additional summation over k_z planes. This is discussed in Sec. III for weak localization. The similarity in the structure of the Larkin result for the MT term suggests the same generalization. However, since the published work does not contain the intermediate calculation we hesitate in doing so. There is an additional qualitative condition which allows the estimation of the relative role of higher k_z planes. The field range in which the two dimensionality is fulfilled is given by the thickness field H_d [see Eq. (14)]. The film behavior is two dimensional as long as the other characteristic fields such as H, H_i, and H_T are much smaller than H_d. H_d is given in Table I. We see that for Al the two dimensionality is well fulfilled. For Sn the condition is not so perfectly fulfilled, and indeed at 20 K the contribution of WL changed by about 10% when the next k_z plane was included (the other planes did not contribute additionally). Since the contributions for nonzero k_z planes of the other terms are not published, it would be inconsistent to include the higher planes only for WL, AL, and possibly MT. Therefore, we prefer to present only the two-dimensional evaluation. For $Bi_{0.9}Tl_{0.1}$ even AL yields a small contribution from the next k_z plane at 20 K. The other terms would surely require a summation over higher k_z planes. However, since the evaluation here is only qualitative, it is not reasonable at the present state of the theory to devote great effort to the evaluation of dimensionality, because the results would not be more conclusive.

V. DISCUSSION

In the preceding section we saw that the theory for the magnetoresistance of superconductors is restricted in its temperature and field range. For the weak-coupling superconductor Al the theory worked quite nicely in the experimental range ($T > 2T_c$). This allows a reliable determination of H_i and $H_{s.o.}$. Therefore, we could check whether the old question of the Maki-Thompson graph and its contribution to the temperature dependence of the resistance is consistent with the microscopic parameters of the superconductor. We conclude that this is not the case, and the problem remains unsolved.

Before we compare the present results with former results from other authors, it should be emphasized that quench-condensed Al is not the same metal as pure Al or granular Al when electron-phonon interactions and superconducting properties are concerned. The author[52,53] had pointed out 15 years ago that in a disordered superconductor the impurities participate in the motion of the phonons and change the electron-phonon interaction and T_c (violating the Anderson theorem that impurities do not change the superconducting transition temperature). At that time it was quite difficult to study this effect experimentally. (Superconducting tunneling which determined the Eliashberg function was not sufficiently sensitive at small energies.) The magnetoresistance of the quantum corrections of the resistance are now an adequate method to study these questions quantitatively as a function of the disorder. We expect that the inelastic lifetime is smaller in quench-condensed metal films than in the pure metal.

Bruynseraede *et al.*[17] investigated Al films with a resistance per square between 1.5 and 60 Ω. They evaluated the large field region in fitting the magnetoresistance curves to an $\ln H$ dependence and obtained $1/\tau_i$ proportional to the temperature. They assumed vanishing spin-orbit scattering, which is questionable. Gershenson *et al.*[18] used for the analysis the spin-orbit scattering time as determined in tunneling experiments and evaluated the magnetoresistance curves below 10 K in the weak spin-orbit scattering limit and above 10 K in the strong spin-orbit scattering limit. They found a T^2 dependence of $1/\tau_i$, and at 10 K one obtains from their plot roughly $\tau_i \approx 20$ ps. Santhanam and Prober[20] investigated very-low-resistance films whose resistance per square varied between 0.15 and 8 Ω. The spin-orbit scattering time they fitted varied between 10 and 100 ps. For the inverse inelastic lifetime they obtained a temperature dependence as $A_1T + A_3T^3$. At 10 K, τ_i was of the order of a few hundred picoseconds, depending on the sample. Gordon *et al.*[21] investigated granular Al film with resistances per square between 15 and 200 Ω. Their spin-orbit scattering fields are between 0.004 and 0.11 T. They interpret their inverse inelastic lifetime as a sum of electron-electron and electron-phonon processes.

For amorphous $Bi_{0.9}Tl_{0.1}$ the Aslamazov-Larkin contribution is dominant. The inelastic lifetime is so short that its relative contribution to the magnetoresistance is very small. In addition, the presently existing theories hardly apply to $Bi_{0.9}Tl_{0.1}$, because of the restricted temperature and field range. Here it would be desirable to improve the theory.

Quench-condensed Sn as an intermediate-coupling superconductor has also an intermediate position in the quantum corrections to the resistance. Only at higher temperatures can the presently existing theories be applied, but here the two dimensionality is not completely fulfilled.

For the determination of the inelastic lifetime from H_i one needs to know the product $H_i\tau_i$, which is given by relation (4). In a normal-conducting metal one may calculate this product within the free-electron model or from detailed Fermi-surface properties. In the superconducting case one has the advantage that the derivative of the upper critical field allows an experimental determination of the product, because

$$H_n\tau_n = \frac{\pi}{16}\left[\hbar\frac{dH_{c2}}{dt}\Big/k_B\right]. \tag{4'}$$

Only in the case of strong-coupling effects does one have to include a correction[54] which is for rather strong-coupling superconductors, of the order of 30%. In the case of Al, the two values obtained from the free-electron model and the upper critical field were in good agreement.

For normal disordered metal films the magnitude of $1/\tau_i$ as obtained from the experiment is not well understood. The existing models which use electron-electron interaction and electron-phonon interaction do not yield such large values. It was even unclear which of the mechanisms was responsible for the experimental values. The author[55] obtained from a heating experiment that the electron-phonon interaction is an important or even the essential mechanism at temperatures above 4 K. Our experimental results on superconductors for H_i confirm this result. A strong-coupling superconductor is distinguished from a weak-coupling one by the strength of the electron-phonon interaction. Therefore, one expects an increase of $1/\tau_i$, i.e., H_i going from the weak-coupling to the strong-coupling case if the electron-phonon interaction is the dominant mechanism for the inelastic processes. This we indeed found qualitatively in going from Al to $Bi_{0.9}Tl_{0.1}$, where the H_i increased strongly.

VI. CONCLUSIONS

The experimental results presented in this paper allow the following conclusions.

(1) The magnetoresistance of weak-coupling superconductors is well described by the present existing theories of weak localization, superconducting fluctuations, and the Coulomb anomaly. In the case of intermediate and strong-coupling superconductors, the theories yield a qualitative understanding; however, it would be desirable to improve the theory so that the allowed temperature range is extended towards the superconducting transition temperature and the field range towards higher fields.

(2) The temperature dependence of the Maki-Thompson term is not reproduced by the theory. The magnetoresistance measurements allow an independent determination of the pair-breaking parameter δ, and δ is no longer a fit parameter for the temperature-dependent resistance formula. All relevant parameters of the Maki-Thompson term are experimentally determined but none of the existing theories yield the correct temperature dependence. The increase of the inelastic field H_i with increasing coupling strength of the superconductor confirms the interpretation that the electron-phonon interaction is essentially responsible for the finite inelastic lifetime in the investigated temperature range.

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