PHYSICAL REVIEW B VOLUME 29, NUMBER 9 1 MAY 1984

Ordering under random fields: Renormalization-group arguments

A. Nihat Berker
Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 19 January 1984)

The possibility of long-range order under quenched random magnetic fields can be deduced from strong-coupling rescaling behavior. The lower critical dimensions of $d_l = 2$ and 4 are indicated, respectively, for Ising and n-component spin models with $n > 1$.

Magnetic systems under quenched random fields, besides being of fundamental theoretical interest,[1–4] are experimentally realizable.[5] These systems have indeed been the subject of many recent experimental[6] and theoretical[7–9] studies. The lower critical dimension d_l, below which conventional long-range order cannot occur, has been an important point of controversy in these studies.

Since the domain energy argument of Imry and Ma,[1] possible long-range order under random fields has been addressed with a variety of theoretical methods, including perturbation expansions about the upper critical dimension,[2,3] supersymmetry identifications,[4] renormalization-group treatments of domain interfaces,[7] and, more recently, Monte Carlo simulation with fractally varied dimensionalities.[9] The present paper will construct bulk renormalization-group arguments. The analysis will be carried out for the Ising model within the context of position-space renormalization group.[10] For n-vector models, the momentum-space renormalization group of fixed-length spins[11] will be used. Both cases will rely on global renormalization-group flows and on the stability of the sink of the ordered phase. An alternate line of argument was recently given in the similar low-temperature scaling analysis of Aharony and Pytte,[8] who showed that $d_l = 2$ and 4 are obtained for Ising and n-vector models if one hypothesizes a discontinuous Edwards-Anderson order parameter upon introduction of random fields.

Consider a spin system with Hamiltonian

$$-\beta\mathscr{H} = J\sum_{\langle ij\rangle}\vec{s}_i\cdot\vec{s}_j + \sum_i \vec{H}_i\cdot\vec{s}_i \ , \tag{1}$$

where the first term couples nearest-neighbor spins at sites $\langle ij\rangle$, and the second term contains the random fields. First, consider subjecting an initial condition with no random fields ($\vec{H}_i = 0$) and inside the order phase ($J > J_c$) to a rescaling transformation: (i) of course, no random fields will be generated in the rescaled (primed) system, $\vec{H}_i' = 0$; (ii) the rescaled system will be more strongly coupled, $J' > J$. Upon repeated rescalings, a renormalization-group trajectory is obtained that flows to a fixed point, the sink of the ordered phase of the pure system (point F^* in Figs. 1). It should be added that only in the lowest-level approximations or on special lattices[12] does the Hamiltonian (1) conserve its form under rescaling. In general, other types of coupling are generated. Thus the horizontal axis in the flow diagrams of Figs. 1 can be viewed as representing the interactions $\{J\}$.

Now consider the initial system with a small random field, represented by the dark circle in Figs. 1. By continuity of the renormalization-group flows (i.e., analyticity of the recursion relations),[13] the trajectory initiated here will flow alongside the horizontal axis and reach the vicinity of the order sink F^* of the pure system. It can eventually either collapse into this sink [Fig. 1(a)], or veer away from this sink [Fig. 1(b)]. In the former case, the random-field perturbation is asymptotically negligible and conventional long-range order is maintained. On the other hand, in the latter case, where the random-field perturbation is eventually amplified under rescalings, a breakdown is expected of the type of order encountered in the pure system. Either paramagnetism or a new type of order (e.g., spin-glass) characteristic of dominant quenched randomness are then the possibilities. Such a system will have thermal fluctuations at the smallest length scales (when the trajectory is near the initial point), intermediate-range conventional order (trajectory near F^*), and random-field-induced phenomena (domains) at the largest length scales (trajectory along vertical axis).

It is now clear that, to find out whether conventional long-range order can exist under random fields, one must study the stability of the sink of the pure-system ordered

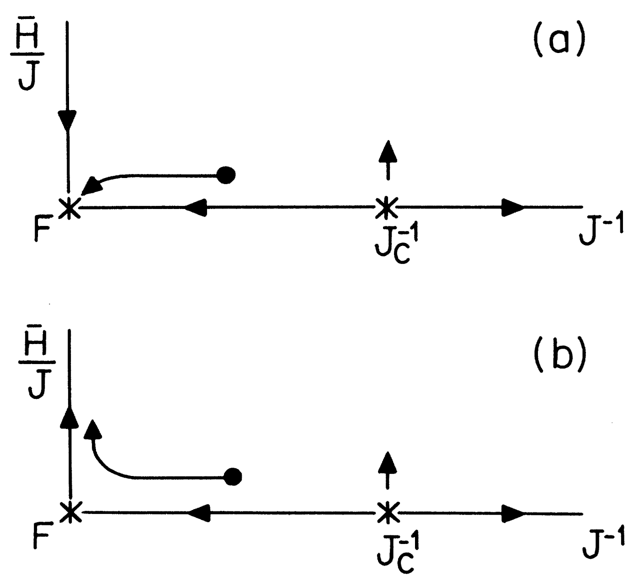

FIG. 1. Renormalization-group flows (a) above and (b) below the lower critical dimension d_l. The stability of the strong-coupling fixed point F^* to $\bar{H}/J$ determines whether conventional ordering persists under random-field perturbations.

phase. However, before doing so, one must specify more carefully the vertical axis in Figs. 1. Consider a system resulting from many renormalizations: A given spin, representing a large region of the original system, is under the influence of its neighbors via renormalized couplings and of its renormalized random field. The central question is which of these two influences is followed by this renormalized spin. Thus the ratio $\overline{H}/J$ must be monitored, where $\overline{H}$ is the average random-field strength. When this ratio, hereby used as a flow parameter, renormalizes to zero, long-range order is maintained.

The argument is carried out for the Ising case, with one-component spins $s_i = \pm 1$, most readily. For spatial dimensionality $d > 1$, the low-temperature phase of the pure system is a region of coexistence of two ordered phases, up- and down magnetized. A necessary condition[14] for the renormalization-group rendition of this coexistence is that, in the vicinity of the sink F^*, a small uniform $(H_i = H)$ field scales with eigenvalue exponent y_H equal to d, $H' = b^{y_H} H$ with $y_H = d$, where b is the length rescaling factor. A renormalization-group transformation is effected by partitioning the original system into cells containing b sites along each spatial direction.[13,10] The site spins inside each cell, numbering b^d, are coupled to a cell spin via a projection operator,[15] such that the cell spin $s'_{i'} = \pm 1$ reflects the overall alignment of the site spins s_i inside the cell. The condition $H' = b^d H$ means that applied fields felt by each one of the b^d site spins are all coherently transferred to their cell spin. Thus, in the vicinity F^*, fluctuations within the cell are statistically negligible. Only the configurations in which all site spins inside a given cell are aligned with the cell spin contribute to the statistical mechanics. [Of course, fluctuations could occur with length scales larger than b, as is the case in Fig. 1(b).] From this fact, we can deduce the scaling behavior of two other quantities.

First, by direct substitution into Eq. (1) of $s'_{i'} = s_1 = s_2 = \ldots = s_{b^d}$ for each cell,

$$J' = b^{d-1} J \tag{2}$$

is obtained for the renormalization of the spin-spin coupling constant. Secondly, an infinitesimal random field H_i is applied in the vicinity of F^*. Again, since the site spins inside a cell are aligned, from a random-field sum,

$$\overline{H}' = b^{d/2} \overline{H} \tag{3}$$

is obtained for the average random-field magnitude. Finally, combining these two equations,

$$(\overline{H}/J)' = b^{1-d/2} (\overline{H}/J) \ , \tag{4}$$

so that the simple eigenvalue exponent

$$y_R = 1 - d/2 \tag{5}$$

controls the stability of the pure-system ordered sink to random-field perturbation. This exponent changes sign and reverses stability at the lower critical dimension $d_l = 2$ for the Ising model, in agreement with other recent theories.[7(b),7(c),9] Furthermore, interest has been recently raised on Potts models in random fields.[16] Our argument above identically applies to Potts models as well, indicating the lower critical dimension $d_l = 2$.

This (admittedly heuristic) renormalization-group argument thus arrives at the conclusion of the original domain energy argument of Imry and Ma.[1] Note, however, that the domain argument involves consideration of the large length scales, whereas the renormalization-group argument here involves consideration of only the smallest length scale of b lattice constants. The domain argument assumed that domain-wall roughening is unimportant, a point which led to the recent theoretical interchanges, but which now appears valid. Our argument also contains an assumption: Eq. (2) is derived in the limit $\overline{H} \rightarrow 0$, but is assumed applicable in the limit $\overline{H}/J \rightarrow 0$, where reconsideration of Eq. (3) shows that, under repeated rescalings, $\overline{H}$ diverges although it is strongly dominated by J. Thus the assumption is probably valid, although it is clearly the weak point of the argument.

Systems of spins with $n > 1$ components are studied in the low-temperature limit by momentum-shell recursion relations of fixed-length spins. Here, the notation and pure-system results of Nelson and Pelcovits[11] will be used. In this work[11] the recursion relation

$$J' = b^{d-2} J \tag{6}$$

was obtained for the spin-spin coupling constant (which is equivalent to the inverse temperature of this previous work).

The recursion of random-field perturbations is determined similarly to Pelcovits's treatment of random-axis perturbations.[17] In the course of the renormalization-group transformation, the substitutions

$$r' = b^{-1} r, \quad k' = bk, \quad s'(k') = \zeta^{-1} s(k) \tag{7}$$

are made for the position, momentum, and momentum-space spin-field variables. The equivalent substitution for the position-space spin field is

$$s'(r') = b^d \zeta^{-1} s(r) \ . \tag{8}$$

The random-field term occurs in the Hamiltonian as

$$\begin{aligned} -\beta \mathscr{H} &= \cdots + \int (dr) \vec{\mathrm{H}}(r) \cdot \vec{\mathrm{s}}(r) \\ &= \cdots + \int (dr') \vec{\mathrm{H}}(br') \cdot \zeta \vec{\mathrm{s}}'(r') \ , \end{aligned} \tag{9}$$

where Eqs. (7) and (8) are used. Thus the identification

$$\vec{\mathrm{H}}'(r') = \zeta \vec{\mathrm{H}}(br') \tag{10}$$

is made. Then, straightforward algebra leads to

$$\begin{aligned} \overline{H}' &= \left[\int (dr') [\vec{\mathrm{H}}'(r')]^2 \right]^{1/2} = \zeta b^{-d/2} \left[\int (dr) [\vec{\mathrm{H}}(r)]^2 \right]^{1/2} \\ &= \zeta b^{-d/2} \overline{H} \ . \end{aligned} \tag{11}$$

This renormalization follows directly from algebra, but can also be understood intuitively: The factor ζ is due to spin rescaling and the factor $b^{-d/2}$ is due to performing an average over thinned-out random variables. The spin-rescaling factor ζ is determined as b^d by the pure-system calculation,[11] at zero temperature. Combining Eqs. (6) and (11),

$$(\overline{H}/J)' = b^{2-d/2} (\overline{H}/J) \ , \tag{12}$$

revealing the eigenvalue exponent

$$y_R = 2 - d/2 \ . \tag{13}$$

This exponent controls the stability of the pure-system ordered sink to random-field perturbation. It changes sign at the lower critical dimension[1,2] $d_l = 4$. There appears to be no caveat in this argument, unlike the Ising case, since the appropriate diagrammatic expansion is in orders of H/J.

In closing, the stability of critical behavior (fixed point J_c^{-1} in Figs. 1) to random-field perturbation can be contrasted to the stability of conventional long-range order (fixed point F^*) discussed thus far. The critical stability is governed[3,18] by the susceptibility critical exponent γ, which is usually strongly positive, meaning that pure-system criticality is unstable to random fields. The crossover is for $d > d_l$ to a new universality class of critical phenomena [Fig. 1(a)], and for $d < d_l$ either to a spin-glass transition, or to no phase transition at all ("a rounded transition") [Fig. 1(b)].

I thank A. Aharony, D. Andelman, R. J. Birgeneau, M. Kardar, P. A. Lee, and S. R. McKay for valuable discussions. This research was supported by the National Science Foundation under Grant No. DMR83-02419. The support of the Alfred P. Sloan Foundation is also acknowledged.

[1]Y. Imry and S.-k. Ma, Phys. Rev. Lett. 35, 1399 (1975).

[2]A. Aharony, Y. Imry, and S.-k. Ma, Phys. Rev. Lett. 37, 1364 (1976).

[3]A. Aharony, Phys. Rev. B 18, 3318 (1978); 18, 3328 (1978).

[4]G. Parisi and N. Sourlas, Phys. Rev. Lett. 43, 744 (1979); M. Kardar, B. McClain, and C. Taylor, Phys. Rev. B 27, 5872 (1983).

[5]S. Fishman and A. Aharony, J. Phys. C 12, L729 (1979).

[6]H. Yoshizawa, R. A. Cowley, G. Shirane, R. J. Birgeneau, H. J. Guggenheim, and H. Ikeda, Phys. Rev. Lett. 48, 438 (1982); D. P. Belanger, A. R. King, and V. Jaccarino, *ibid.* 48, 1050 (1982); P.-z. Wong and J. W. Cable, Phys. Rev. B 28, 5361 (1983).

[7](a) E. Pytte, Y. Imry, and D. Mukamel, Phys. Rev. Lett. 46, 1173 (1981); (b) G. Grinstein and S.-k. Ma, *ibid.* 49, 685 (1982); (c) J. Villain, J. Phys. (Paris) Lett. 43, L551 (1982).

[8]A. Aharony and E. Pytte, Phys. Rev. 25, 5872 (1983).

[9]D. Andelman, H. Orland, and L. C. R. Wijewardhana, Phys. Rev. Lett. 52, 145 (1984).

[10]T. Niemeijer and J. M. J. van Leeuwen, in *Phase Transitions and Critical Phenomena,* edited by C. Domb and M. S. Green (Academic, New York, 1977), Vol. 6.

[11]D. R. Nelson and R. A. Pelcovits, Phys. Rev. B 16, 2191 (1977).

[12]A. N. Berker and S. Ostlund, J. Phys. C 12, 4961 (1979); M. Kaufman and R. B. Griffiths, Phys. Rev. B 24, 496 (1981).

[13]K. G. Wilson, Phys. Rev. B 4, 3174 (1971); 4, 3184 (1971); K. G. Wilson and J. Kogut, Phys. Rep. C 12, 75 (1974).

[14]B. Nienhuis and M. Nauenberg, Phys. Rev. Lett. 35, 477 (1975); M. E. Fisher and A. N. Berker, Phys. Rev. B 26, 2507 (1982).

[15]K. Subbarao, Phys. Rev. B 11, 1165 (1975).

[16]D. Blankschtein, Y. Shapir, and A. Aharony, Phys. Rev. B 29, 1263 (1984).

[17]R. A. Pelcovits, Phys. Rev. B 19, 465 (1979).

[18]Y. Shapir and A. Aharony, J. Phys. C 14, L905 (1981); D. Andelman and A. N. Berker, Phys. Rev. B 29, 2630 (1984).