Density-dependent mobility of a two-dimensional electron fluid

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Measurements of the low-frequency electron mobility on a helium surface are extended to higher densities and to lower temperatures than those previously reported in the literature. For electron areal densities greater than 5×10^7 cm^{-2}, the electron scattering is greater than predicted by one-electron theories by an amount which is weakly temperature dependent and increases with increasing density.

At low areal densities, electrons in surface states above a liquid-helium surface form a classical two-dimensional Coulomb gas. Both the interelectron and the electron-helium[1,2] interactions are of a very simple form. One of the simplest tests of the electron-helium interaction is a measurement of the ripplon-limited mobility, and two of the three published measurements of the low-temperature mobility[3-5] are in good agreement with theory.[2,6-8] Measurements are presented here which are more precise and are taken at higher densities and lower temperatures than previous measurements. We report deviations in the mobility from the predictions of the one-electron theories which are nearly temperature independent but which increase substantially with increasing density. Our data suggest that the one-electron theories of the mobility must be extended to include Coulomb interactions.

A cross section of our experimental cell is shown in Fig. 1. The cell consists of a plane parallel capacitor with a plate separation of $s=2$ mm, semi-immersed in liquid helium. Each plate has dimensions 1.78×2.54 cm and is divided into three sections of equal area. A dc voltage is applied to the bottom plate to hold electrons against the helium surface as the electrons are deposited onto the surface from a glow discharge. The top plate is surrounded by guard electrodes used to shape the electron density profile. Capacitance measurements between combinations of electrodes allow an adjustment of the angle of the plates relative to the helium surface to within two milliradians.

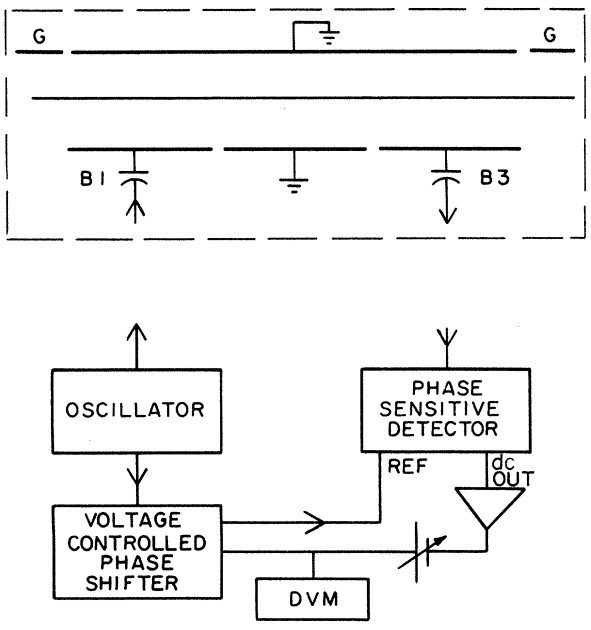

FIG. 1. Cross section of experimental cell and circuit diagram used to measure the phase of the signal relative to the phase of the excitation voltage. The electrodes marked G represent guard electrodes and the line between the plates represents the liquid level.

The mobility measurement technique of Sommer and Tanner[9] is used, and all electrodes except B_1 and B_3 are kept at ac ground. An ac voltage of angular frequency ω applied to electrode B_1 causes a redistribution of the electrons, which changes the density above electrode B_3 after a time delay determined by the amount of scattering and the effective mass of the electrons. The mobility is deduced by comparing the phase of the signal induced on electrode B_3 with the phase of the driving signal.

For low frequencies, i.e., small wave vectors $k<s^{-1}$, an electrical analog of the sample is a transmission line with per unit length values of resistance $R=(ne\mu w)^{-1}$, inductance $L=m^*/ne^2w$, and capacitance $C=w(\epsilon+1)/2\pi s$. Here n is the electron density, μ the dc mobility, w the width of the cell, m^* the effective mass of the electron, and ϵ the dielectric constant of the helium. The equation for L is obtained by equating $\frac{1}{2}LI^2$ to the total kinetic energy of the electrons and writing the current as $I=nevw$. The value of m^* may be enhanced over the electronic mass if a small dimple forms on the helium surface under each charge.

The admittance of the cell is given by[10]

$$Y=\frac{i\omega Cl\sinh(\gamma l)}{4\gamma l[3+4\sinh^2(\gamma l)]}\ , \tag{1}$$

where $\gamma=(i\omega RC-\omega^2LC)^{1/2}$ and l is one-third of the length of the cell. For small values of γl, the phase of the signal at B_3 relative to the excitation signal is given by $\phi=\tan^{-1}(\mathrm{Re}\,Y/\mathrm{Im}\,Y)$ which can be written as

$$\phi\simeq\tan^{-1}[\tfrac{7}{6}\chi(1+\tfrac{79}{105}\lambda^2+\tfrac{2063}{8820}\chi^2)]\ , \tag{2}$$

where $\chi=\omega RCl^2$ and $\lambda^2=\omega^2LCl^2$.

There are corrections to the values of C and l which depend on the degree to which electrons extend beyond the edges of the cell and the capacitive coupling of these electrons to the guard electrodes. We solved Laplace's equation subject to appropriate boundary conditions in order to determine the electron density profile and effective capacitance for a given density, holding voltage $V_\perp$, and guard voltage V_g. At the saturated density these corrections are independent of density for the fixed ratio of $V_g/V_\perp$ which we used.

A circuit diagram of our electronics is also shown in Fig. 1. The signal from electrode B_3 is fed to a phase sensitive detector and the output of the detector is used in a feedback loop with a battery to control the reference phase. The output of the detector is maintained near zero and the phase is determined by the known voltage versus phase characteristics of the voltage controlled phase shifter. A computer sets the temperature of the sample and reads the voltage controlling the phase shifter via the digital voltmeter (DVM).

The zero of the phase shift [see Eq. (2)] is determined before loading electrons on the surface by electrically floating a metallic strip located just beneath the bottom plate. Along with this primary reference phase we measured a secondary reference phase of a "dummy sample" circuit. The secondary reference is used to correct for small time-dependent phase shifts in the amplifiers. Under our operating conditions, a driving voltage of 3.5 mV rms at 1 MHz, we measure phase shifts with a precision of 1 mrad. This corresponds to a time delay of 100 psec. The phase is reproducible from day to day within 10 mrad.

In extracting the mobility from the phase we set $\lambda^2=0$ in Eq. (2). Even a relatively large effective mass for the fluid phase of ten electronic masses yields a negligible correction to the mobility.

In Fig. 2 the inverse mobility of the electron fluid at six densities is presented as a function of temperature and compared with theory. The absolute error is due to an uncertainty in the phase reference and is shown by an error bar in each graph. The relative error in the data is about 10% of the absolute error as seen from the scatter in the data. There is in addition a relative error $\Delta\mu/\mu \simeq 4\%$ due to uncertainties in the effective values of C and l. The discrepancy between the two sets of data in Fig. 2(a) may be due to a loss of electrons in one of the runs. The sharp rise at the left side of the figures occurs at the melting transition.

The data shown were taken at a frequency of 1 MHz and at the saturated density, $n_s = E_\perp/2\pi e$, where $E_\perp$ is the holding field. The mobility was also measured at 0.5 and 0.7 MHz for a density of 6.3×10^8 cm^{-2} and was found to be frequency independent to within experimental error. This frequency independence verifies that any effective mass (λ^2) contributions to Eq. (2) are small. The mobility was independent of the excitation voltage for $V_e \leq 30$ mV at $T\sim0.5$ K in agreement with the observation of Iye.[5] The density was determined by measuring the coupled mode spectra[11,12] at $n_s = 5.4\times10^8$ cm^{-2} and scaling the density with $E_\perp$.

There are three published measurements[3-5] of the electron mobility in the ripplon-dominated regime with which we can compare our data. These comparisons are difficult because of the difference in the high-frequency ($\omega\tau > 1$, see Grimes and Adams[4]) and low-frequency measurements, and because of the uncertainty in the holding field in the case of Iye.[5] However, taking these effects into account we estimate that our low density data are in agreement with the results of these workers to within the absolute errors of the experiments. Our low density mobility values are approximately an order of magnitude larger than those measured by Rybalko *et al.*[3] at 3 and 15 MHz.

Theoretically the total inverse mobility is determined by the sum of the individual ripplon and vapor atom scattering contributions. There are two terms in the electron-ripplon interaction: one results from the applied holding field which forces the electrons toward the troughs of the ripplons and the other is due to the polarization of the helium. In this latter interaction the electron also has a lower energy in a trough because of its proximity to more of the liquid.

Shikin and Monarkha[2] were the first to write down the correct electron-ripplon interaction, but in order to obtain a closed form of the dc mobility they used an approximation which resulted in ignoring the polarization interaction. In a better approximation, Platzman and Beni[6] and Saitoh[7] derived an integral expression for the dc mobility. Later Monarkha[8] wrote down an exact integral expression for the one-electron, ripplon-limited mobility for $E_\perp=0$. We extended Monarkha's theory to finite $E_\perp$ by including in the electron-ripplon interaction a term $Q_q \rightarrow eE_\perp$, where Q_q is the amplitude of a ripplon of wave vector $\vec{q}$. We then evaluated the mobility numerically for each value of $E_\perp$ and T. Saitoh also calculated the helium vapor scattering contribution to the one-electron mobility using a simplified contact pseudopotential for the electron-helium interaction. The theoretical predictions for the mobility shown in Fig. 2 are given by our extension of Monarkha's theory and Saitoh's vapor scattering calculation.

At low densities our values of the mobility are in striking qualitative (and, to within our absolute errors, quantitative) agreement with the theoretical predictions for both ripplon and vapor atom scattering. This confirms that the electron-ripplon and electron-helium atom interactions are well understood.

The discrepancy between theory and experiment at higher densities increases with increasing density and depends only weakly on the temperature. This weak dependence on temperature over such a large temperature range would appear to rule out an explanation in terms of coupling to excitations in a hexatic liquid crystal phase. On the other hand, the density dependence of the discrepancy suggests[13] that Coulomb interactions are important in the scattering process. The coupling between single particle motion and plasma modes with wave vector $k \sim n^{1/2}$ has been observed in computer simulations of the two-dimensional (2D) electron gas. This coupling is manifested in oscillations in the velocity autocorrelation function.[14,15] This same coupling may lead to the absorption and emission of plasmons in electron scattering processes. In this context it is important to note that the thermal wave vectors which dominate the scattering are much less than the Debye wave vector $k_D = 2\pi ne^2/k_BT$.

The observed discrepancy between experiment and theory cannot be explained by the altered dispersion relation for a charged surface.[16] The dispersion relation is appreciably altered only for saturated densities $n_s \gtrsim 2\times10^9$ cm^{-2} while we observe deviations from theory for $n\sim10^8$ cm^{-2}. This change in the dispersion relation is also negligible for the wave vectors k which are involved in the scattering processes, i.e., $k >> k_0 = E_\perp^2/\pi\sigma$. Here σ is the surface tension and in the present experiment $k_0 < 7$ cm^{-1}.

We attempted to measure the mobility as a function of density at fixed $E_\perp$ and T. These measurements were unsuccessful due to a loss of electrons in the low density, high $E_\perp$ experiments. We plan to redesign our cell in order to measure μ at $n < n_s$.

In summary, we have presented high-precision measurements of the electron mobility and confirmed the correctness of the one-electron theories at low densities. We have further observed a density dependence in the mobility of a two-dimensional electron fluid for densities $\gtrsim 5\times10^7$ cm^{-2}. This density dependence suggests that the theory of the

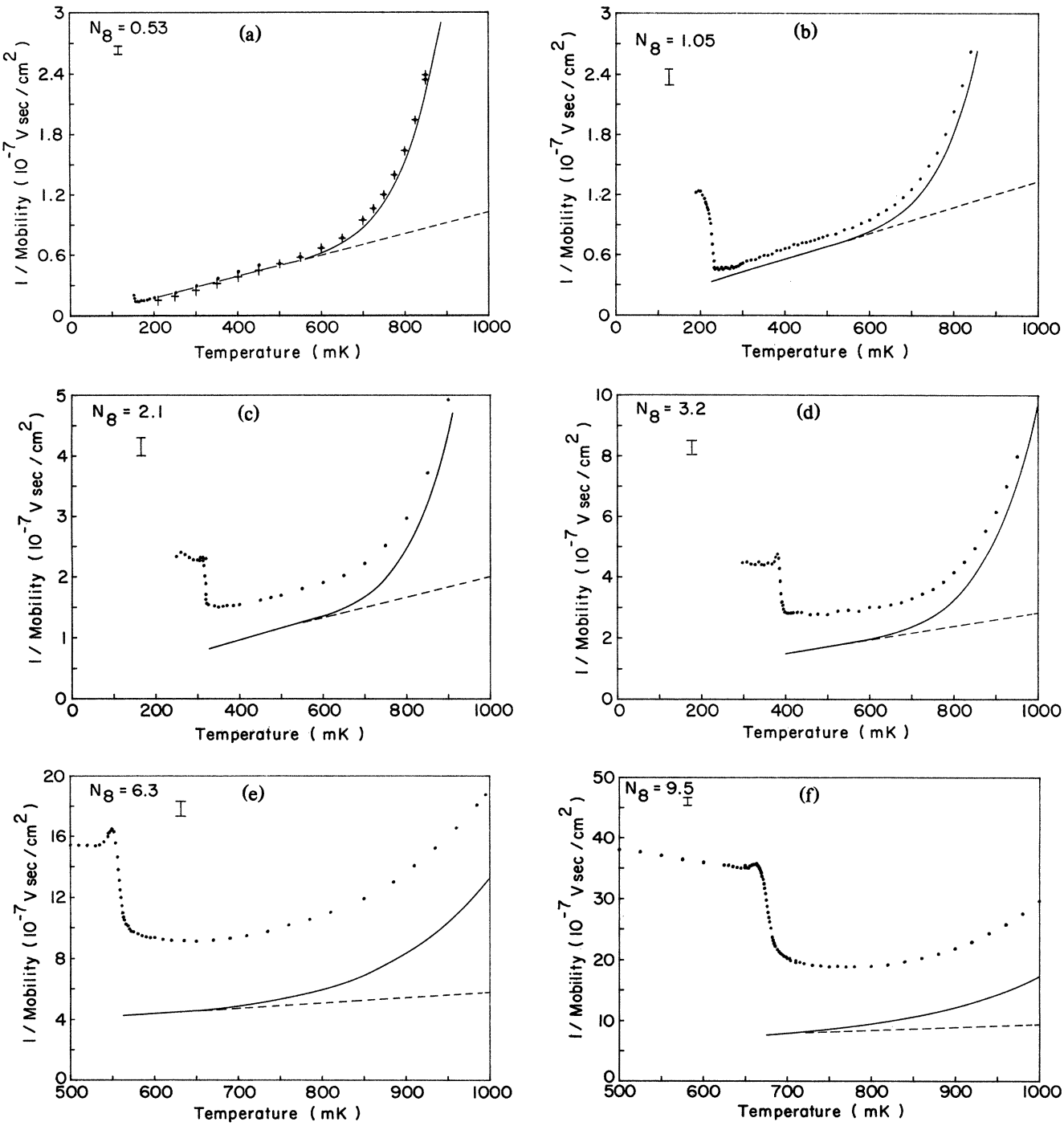

FIG. 2. Inverse mobility vs temperature for various densities. The solid curve is the theory for scattering from ripplons and helium gas atoms. The dashed curve is the contribution from ripplon scattering alone. The densities in units of $n_8 = 10^8$ cm^{-2} are, respectively, (a) $n_8 = 0.53$, (b) $n_8 = 1.05$, (c) $n_8 = 2.1$, (d) $n_8 = 3.2$, (e) $n_8 = 6.3$, (f) $n_8 = 9.5$. The + symbols in (a) are data from a second experimental run.

electron mobility should be extended to include Coulomb interactions in the scattering Hamiltonian. Such a theory may give insight into the nature of excitations at large wave vectors which cannot be detected directly.

During the writing of this paper we were informed that a density-dependent mobility has recently been observed in a higher-density 2D electron system on a neon substrate coated with a thin helium film.[17]

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