New instability toward inhomogeneous states of nonequilibrium superconductors

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The new instability model of nonequilibrium superconductors recently proposed by Iguchi and Konno is explored to determine whether inhomogeneous superconducting states can be induced by high-energy quasiparticle injection. No superconducting states, whether homogeneous or not, can be found when the quasiparticle injection rate exceeds some critical value. For injection rates below the critical value, in addition to a stable homogeneous solution, a spatially periodic solution does exist but is, however, unstable against small fluctuations. Another model, which uses a more realistic approach to calculate the density of the relaxation phonons, is also presented and examined. This modified model does not predict a stable inhomogeneous solution either. Instead, it predicts a first-order-like transition between two homogeneous nonequilibrium states if the phonon escape time is sufficiently long.

The observation[1] of multiple-gap states in superconducting thin films under strong quasiparticle tunnel injections has imposed an intriguing theoretical problem in the field of nonequilibrium superconductivity.[2] Recently, Iguchi and Konno[3] proposed a model to explain this effect. The model is essentially based upon the well-known Rothwarf-Taylor equations.[4] The new feature of their model is the introduction of an extra phonon generation term to account for the created phonons of $\hbar\Omega \geqslant 2\Delta$ due to the *scattering* of quasiparticles from states with energy $E \geqslant 3\Delta$. Therefore the modified coupled equations become (assuming the ambient temperature $T_a = 0$)

$$\frac{\partial N}{\partial t} = I_{\rm qp} - 2RN^2 - \frac{2}{\tau_B} N_{\rm ph} + D\frac{\partial^2 N}{\partial x^2} \ , \tag{1a}$$

$$\frac{\partial N_{\rm ph}}{\partial t} = RN^2 - N_{\rm ph}(\tau_B^{-1} + \tau_{\rm es}^{-1}) + G_e \ , \tag{1b}$$

Here we have used the notations of Ref. 3 and restricted ourself to a one-dimensional problem. Using a T^* model to describe the quasiparticle energy distribution in the steady state, the added phonon generation rate is found to be proportional to the third power of N, the quasiparticle density,

$$G_e = KN^3 \ . \tag{2}$$

We have studied this model and models similar to this one, both numerically and analytically and have obtained different conclusions. First, we found that the model described by Eqs. (1) and (2) gives a spatially inhomogeneous state only when the quasiparticel injection is *below* (instead of above) a critical value. Further increase of the injection rate above this value drives the whole film normal. To show this, we first *assume* that a time-independent state exists so that $\partial N/\partial t = \partial N_{\rm ph}/\partial t = 0$. Eliminating $N_{\rm ph}$, we obtain from Eqs. (1) and (2)

$$\frac{d^2 n}{d\xi^2} + (I - n^2 + n^3) = 0 \ . \tag{3}$$

Here n is N measured in units of $N_0 \equiv R\tau_B/K\tau_{\rm es}$, I is $I_{\rm qp}$ (quasiparticle injection rate) measured in units of $2RN_0^2/(1+\tau_{\rm es}/\tau_B)$, and ξ^2 is x^2 measured in units of $2RN_0^2/(1+\tau_{\rm es}/\tau_B)DN_0$. With ξ playing the role of time, Eq. (3) represents a classical particle moving in a potential

$$V(n) = In - \tfrac{1}{3}n^3 + \tfrac{1}{4}n^4 \ , \tag{4}$$

which is plotted in Fig. 1 for three different values of the quasiparticle injection rate. Keeping in mind that only values of $n > 0$ are physical, we see that only for $I < I_c = \frac{4}{27}$ do oscillatory solutions of $n(\xi)$ exist. These solutions correspond to spatially inhomogeneous states and occur when $V_B < V < V_A$ is satisfied. Furthermore, for a fixed injection rate $I < I_c$, there are *two* uniform solutions

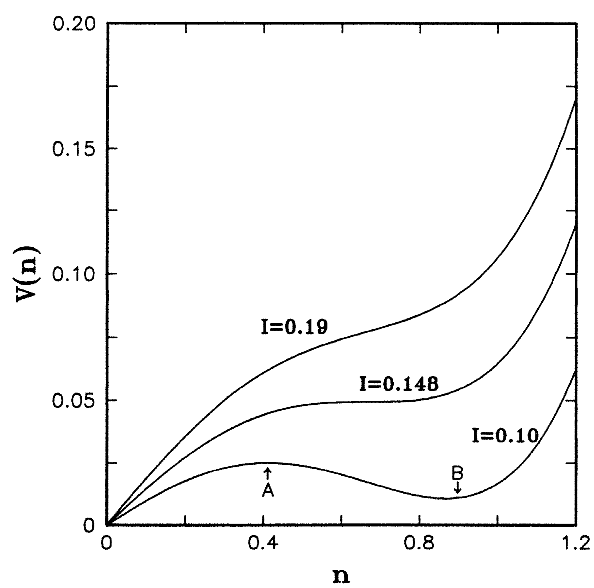

FIG. 1. Effective potential $V(n)$ vs the normalized quasiparticle density n for three different values for the normalized quasiparticle injection rate $I = 0.19$, 0.148, and 0.1. $I = \frac{4}{27} = 0.148$ is the critical value below which a spatially inhomogeneous solution of n exists.

n_A and n_B for the quasiparticle density. To investigate the stability of these homogeneous and inhomogeneous steady-state solutions, we have to consider the original Eqs. (1) and (2).

To simplify the discussion, we observe that in many cases the intrinsic phonon lifetime is much shorter than that of quasiparticles[5] and make the assumption that the phonons are in equilibrium with the quasiparticles all the time. This allows us to set $\partial N_{ph}/\partial t=0$ (Ref. 6) and we obtain an equation for the evolution of the quasiparticle density which is the same as Eq. (3) except to replace the right-hand side by $\partial n/\partial \tau$. Here τ is time measured in units of $(1+\tau_{es}/\tau_B)2RN_0$, the quasiparticle recombination time enhanced by the phonon trapping factor. A linear stability analysis similar to that of Ref. 3 for a uniform state gives

$$s=\left(\frac{\partial^2 V}{\partial n^2}\right)-q^2 \ , \tag{5}$$

where s dictates the time evolution of a small perturbation against the uniform solution. Since $s>0$ corresponds to an unstable state, we see immediately that the uniform state of quasiparticle density, n_B, is unstable with respect to long-wavelength fluctuations. The only stable uniform state is that of n_A.[7]

To study whether the nonlinear effect can stabilize an inhomogeneous state, we have studied Eq. (3) with the right-hand side replaced by $\partial n/\partial \tau$ numerically. We found that for $I>I_c$, the quasiparticle density increases as a function of time, apparently without bound. For $I<I_c$, the state of n_A is always obtained as an asymptotic solution ($t\rightarrow\infty$). Therefore the phonon generation term given by Eq. (2), evaluated from the T^* model, does not lead to a stable inhomogeneous state. It is natural, then, to ask whether some other realistic phonon generation term for the quasiparticle tunneling injection could lead to such an inhomogeneous state as observed experimentally.

To do so, we note that the intensity of the $\hbar\Omega \geqslant 2\Delta$ phonons emitted from a superconductor-insulator-superconductor tunnel junction has been measured[8,9] as a function of the junction voltage V. It was found to be linear in V for $4\Delta/e<V<6\Delta/e$ with a slope larger than that for $V\lesssim 4\Delta/e$. This suggests that the additional phonon generation term due to the inelastic scattering of quasiparticles with $E\geqslant 3\Delta$ should be linear in I_{qp}.[8] Taking into account the gap suppression due to the excess quasiparticles we found that

$$\begin{aligned} G_e&=0 \quad \text{for } V<4\Delta/e \\ &=6.4I_u\left(\frac{eV}{4\Delta_0}\right)-6.4I_u\left(1-\frac{2N}{4N(0)\Delta_0}\right) \quad \text{for } V>4\Delta/e \ . \end{aligned} \tag{6}$$

Here

$$I_u\equiv\frac{G_{nn}\Delta_0}{e^2\Omega} \ , \tag{7}$$

and G_{nn}, $N(0)$, and Ω are, respectively, the junction conductance, the normal-state single-spin electron density of states, and the volume of the junction film.

The modified Rothwarf-Taylor equations with a phonon generation term given by Eq. (6) was investigated. We found that as before, the equation that governs the quasiparticle density can be written in the form of

$$\begin{aligned} \frac{\partial n}{\partial \tau}&=I-n^2+\frac{\partial^2 n}{\partial \xi^2} \quad \text{for } V<4\Delta/e \\ &=I'-n^2+n+\frac{\partial^2 n}{\partial \xi^2} \quad \text{for } V>4\Delta/e \ . \end{aligned} \tag{8}$$

Here I is the normalized quasiparticle injection rate given before and

$$I'=I\left(1+\frac{3.2}{1+\tau_B/\tau_{es}}\right)-\frac{6.4I_u}{RN_0^2}\left(\frac{\tau_{es}}{\tau_B}\right) \ ,$$

with

$$N_0\equiv\frac{12.8I_u}{R}\left(\frac{\tau_{es}}{\tau_B}\right)\frac{1}{4N(0)\Delta} \ ,$$

$$n\equiv N/N_0 \ ,$$

$$\xi=x\bigg/\left[\frac{D}{2RN_0}\left(1+\frac{\tau_{es}}{\tau_B}\right)\right]^{1/2} \ ,$$

and τ is the normalized time as in the previous case.

It is easy to show that for a fixed bias voltage (or current) multiple steady-state solutions for the quasiparticle density exist if the junction conductance is sufficiently high or the phonon escape time is sufficiently long, so that

$$\frac{G_{nn}}{2e^2N(0)\Omega}\frac{20}{R\,4N(0)\Delta_0}\frac{\tau_{es}}{\tau_B}-1>0 \tag{9}$$

is satisfied. A possible case[10] is shown in Fig. 2. It can also be shown by an analysis similar to that applied to the previous case that, when this happens, spatially inhomogeneous states exist. To see if these states are stable or not, we calculated the time evolution of the quasiparticle density using Eq. (8). A periodic boundary condition was used along with different initial conditions. We found that the asymptotic

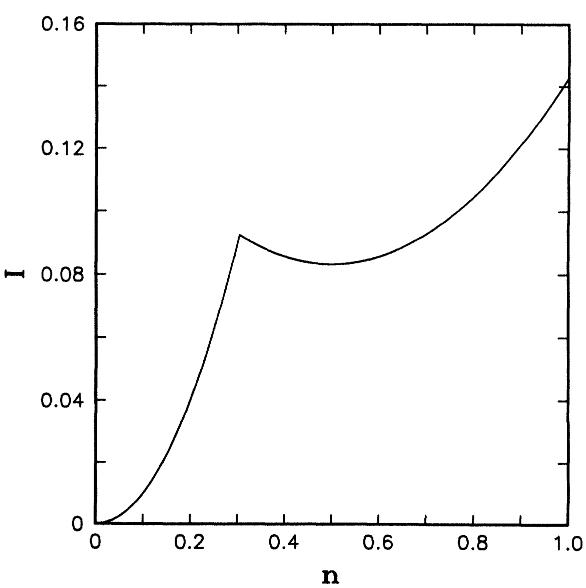

FIG. 2. Normalized quasiparticle injection rate as a function of the normalized quasiparticle density n calculated from Eq. (8) with no spatial and temporal variations. The parameters are $I_u=RN_0^2(\tau_B/\tau_{es})/10.3$ and $\tau_{es}>>\tau_B$.

state is always uniform with a density n such that $\partial I/\partial n > 0$. For small (large) I, the small (large) density state is stable. No stable inhomogeneous state was found. Finally, the criterion [Eq. (9)] requires that $\tau_{es}/\tau_B \geq 130$ for an Al junction with a resistance $G_{nn}^{-1} = 0.5\ \Omega$ and an injection volume about 10^{-8} cm^3. For Sn junctions of the same size and $G_{nn}^{-1} = 0.02\ \Omega$, the phonon trapping factor would have to be greater than 800, which seems to be too large compared with that of the actual experimental situation.[1]

In conclusion, we have studied the modified Rothwarf-Taylor-equations model proposed by Iguchi and Konno to explain the inhomogeneous multiple-gap states. Another model that uses a different phonon generation term and is believed to better describe the nonequilibrium superconducting state is also investigated. Neither of them gives *stable inhomogeneous states.* The coexistence of multiple-gap states in quasiparticle-tunnel-injected superconducting thin films remains to be an intriguing theoretical and experimental problem.

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[6] This, however, does not mean that the phonon density N_{ph} cannot change in time. N_{ph} can be a function of time when the quasiparticle density varies in time.

[7] For the case of negative diffusion constant D, one can carry out the same analysis as done here for the case of positive D. Within this simple model, however, no uniform solution can be found to be stable against short-wavelength fluctuations.

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[10] We notice that the functional form of I vs n is similar to that of pressure versus volume in the case of liquid-gas phase transition. However, the physics is quite different because we are not dealing with a thermal equilibrium state.