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Test of crossover scaling in the two-dimensional random-field Ising model

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The random-field-induced rounding of the specific-heat singularity observed in transfer-matrix calculations of two-dimensional Ising models by Morgenstern, Binder, and Hornreich is interpreted in terms of the Fishman-Aharony scaling theory. Results qualitatively similar to recent experimental work on $Rb_2Co_{0.85}Mg_{0.15}F_4$ are obtained.

There has been much current interest in the properties of systems with quenched random fields, particularly because of the prediction that the lower critical dimension d_l, below which a uniformly ordered state is not stable, is raised by the random field.[1-10] While there has been a controversy whether $d_l=2$ (Refs. 2 and 6–10) or $d_l=3$ (Refs. 5–7) in an Ising model, it is accepted that uniform long-range order is destroyed by weak random fields (e.g., $\pm h$ with equal probability of both signs) in two-dimensional Ising models.[6,11] In this case the sharp phase transition which occurs for $h=0$ is rounded and the width of this region where the rounding occurs is controlled by the strength of the random field.

Fishman and Aharony[12] described the crossover from the critical behavior of the pure system to the new behavior by a scaling theory; e.g., for the specific heat they obtained

$$C=|t|^{-\alpha}f(th^{-2/\phi}) \quad , \tag{1}$$

where t is the reduced distance from the mean-field transition temperature, $t=(T-T_c-bh^2)/T_c$, with b a constant and T_c the transition temperature of the pure system, α is the specific-heat exponent for the pure system, and ϕ is the crossover exponent, for which Fishman and Aharony showed $\phi=\gamma$, with γ the susceptibility exponent of the pure system. For the Ising system in two dimensions $\alpha=0$, and thus the crossover behavior is slightly more complicated,[13,14] namely, the singular part of the specific heat behaves as

$$C/k_B=f(th^{-2/\phi})-A^*\ln h,\quad t\to 0,\quad h\to 0\ ,\qquad |t|h^{-2/\phi}\text{finite}\ . \tag{2}$$

Ferreira *et al.*[14] showed that A^* is related to the amplitude A of the specific heat in the pure system ($C\propto -A\ln|t|$) by

$$A^*=2A/\phi\ . \tag{3}$$

Ferreira *et al.*[14] also provide beautiful experimental evidence for Eqs. (2) and (3) in the case of the diluted quasi-two-dimensional anisotropic antiferromagnet $Rb_2Co_{0.85}Mg_{0.15}F_4$. Strictly speaking, however, the situation in the experiment is slightly more complicated: (i) the "random staggered fields" which are induced by the uniform field in the diluted antiferromagnet[12] are correlated with the disorder in spin-spin exchange interactions (due to a locally varying coordination number); (ii) the critical behavior of the system without the field is that of a sited-disordered Ising model rather than that of a pure Ising model,[15] and hence, asymptotically close to T_c, Eq. (2) does not apply. Of course, outside a crossover region from the critical behavior of pure Ising systems to that of diluted ones Eq. (2) should be approximately valid. Such crossover effects are probably more important for three dimensions, where also some experimental evidence for them has been found.[16] The correlation effect (i) seems also to be very important at least for the understanding of some of the three-dimensional data, where often hysteresis effects are observed,[17] and it has been controversial which data are really thermal equilibrium results.[16-19] In any case, it is felt that it would be interesting to study the crossover behavior predicted in Eqs. (2) and (3) in the original random-field Ising model itself. It is the purpose of the present Brief Report to provide some evidence for the validity of Eqs. (2) and (3) in this model.

The first evidence that random fields destroy the phase transition of two-dimensional Ising systems was provided in a previous work by the author,[11] where transfer-matrix-type calculations of $L\times L$ lattices, with $L\leqslant 16$, were performed. For each configuration ($h_l=\pm h$) of random fields one obtains the free energy exactly. The results are averaged numerically over a finite sample of M different random-field configurations generated by Monte Carlo. Varying the linear dimension L it was checked to what extent the results were independent of lattice size.[11]

Typical results for the specific heat are shown in Fig. 1. In the system without random fields, for $L\to\infty$ the well-known logarithmic singularity occurs at (Ref. 20) $k_BT_c/J\approx 2.21$. Figure 1 shows that in the presence of random fields the specific-heat peak occurs at somewhat lower temperatures and with increasing strength of the random field it becomes progressively rounded. Unfortunately, fields $h/J<0.50$ could not be studied by this method since then the finite-size rounding of the specific-heat peak[21] starts to become more important than the random-field-induced rounding. A qualitatively similar limitation occurs also in the experiment[14] due to concentration gradients in the sample.

Now a first test of Eqs. (2) and (3) is provided in Fig. 2, noting that the specific-heat maxima should vary as $C_{\max}/k_B=\text{const}-A^*\ln h$. The straight line through the data points is of this form: $C_{\max}/k_B=0.560-0.534\ln(h/J)$. On the other hand, taking the exact result for the critical amplitude A of the pure Ising system[20] ($A\approx 0.494\,538$), we find that $A^*\approx 0.564$, which compares nicely with the numerical result $A^*\approx 0.534$: given the statistical errors of the data in Fig. 1 and considering the fact that the asymptotic behavior described by Eqs. (2) and (3) should hold only for $h\ll zJ$

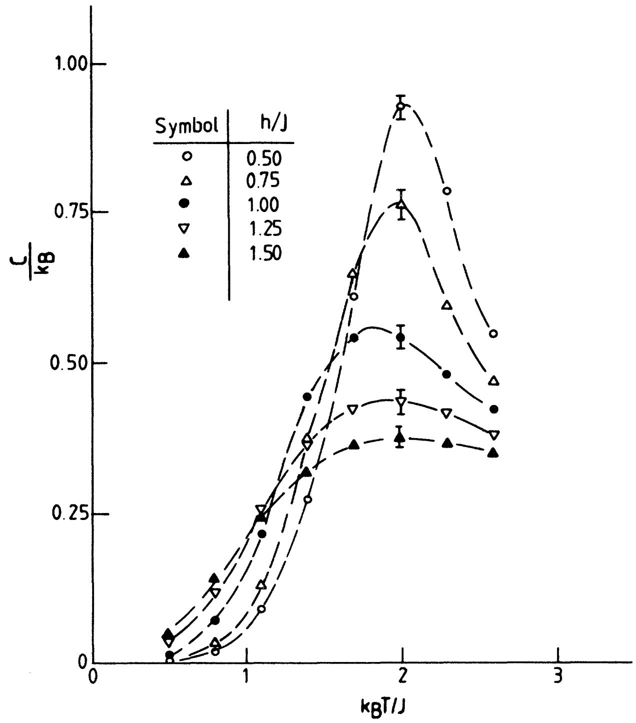

FIG. 1. Specific heat plotted vs temperature for the nearest-neighbor Ising ferromagnet with exchange constant J exposed to random fields $\pm h$, for a 12×12 square lattice, averaged over 30 configurations of the random field. Different curves refer to different values of h/J as indicated in the figure (from Ref. 11).

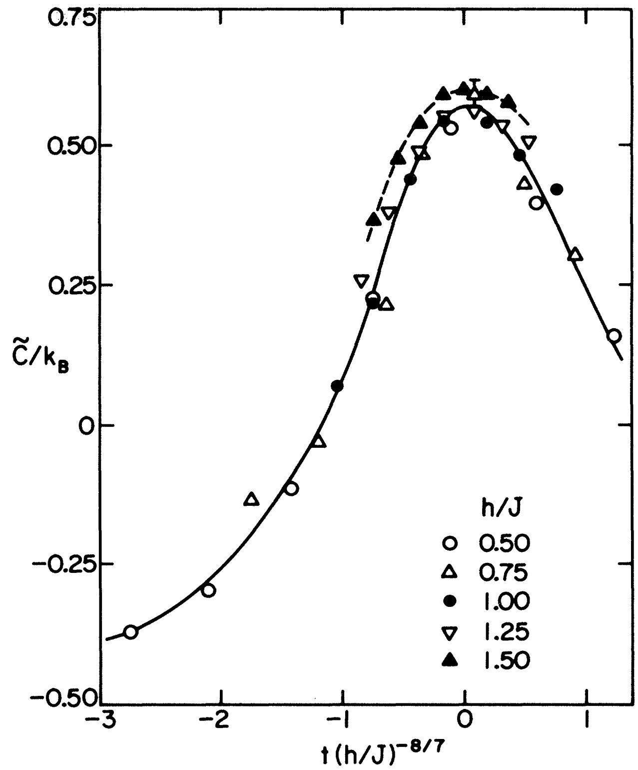

FIG. 3. Scaling function $\tilde{C}/k_B = C/k_B + A^* \ln h/J$ plotted vs the scaling variable $t(h/J)^{-8/7}$, where $t \equiv k_B(T-T_{\max})/J$, $T_{\max}$ being the temperature of the specific-heat maximum. The full curve is our estimate for the scaling function; broken curve (connecting points for $h/J = 1.5$) is a guide to the eye for illustrating deviations due to corrections to scaling. For A^* the theoretical prediction $A^* \approx 0.564$ was chosen.

in our model, where the coordination number $z=4$ for the square lattice, any better agreement anyway would be fortuitous.

A more complete test of Eqs. (2) and (3) is provided by noting[14] that the function $\tilde{C}/k_B = C/k_B + A^* \ln h$ should no longer depend on the variables t and h separately, but rather on the scaled combination $th^{-2/\phi} = th^{-2/\gamma} = th^{-8/7}$ only. Figure 3 shows that this scaling works out reasonably well. Only the data for $h/J = 1.5$ seem to fall systematically above the estimated scaling function, which is not at all surprising, since these are the data farthest away from the scaling region; for all other data the systematic errors due to corrections to scaling seem to be smaller than the statistical error. Note that in Fig. 3 there are no adjustable parameters whatsoever, also not in the coordinate scales, and hence the success of this scaling representation is nontrivial.

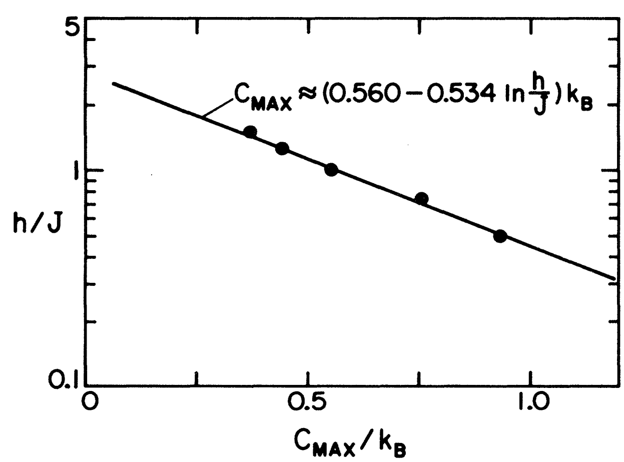

FIG. 2. Plot of $\ln h/J$ vs the maximum of the specific heat, $C_{\max}$, taken from Fig. 1.

In conclusion, it has been shown that the crossover analysis of Eqs. (2) and (3) describes the rounding of the transition due to the random field over a wide range of fields; the present results for the random-field nearest-neighbor Ising square lattice are very similar to the corresponding experimental results for the diluted quasi-two-dimensional anisotropic antiferromagnet $Rb_2Co_{0.85}Mg_{0.15}F_4$ in a field.[14] What is still lacking, however, is an explicit theoretical prediction for the scaling function $f(th^{-2/\phi})$ itself, as well as analytic results for the structure of spin correlations near this rounded transition from a disordered state (for $t>0$) to a "domain state" (for $t<0$, $h\neq 0$).[11] In Ref. 11 it was speculated that this change of correlations could still be associated with a subtle phase transition seen only in correlations of the type $[\langle s_i s_j\rangle^2]_{\rm av}$ describing spin-glass-type order. However, in the spherical limit[22] where it is possible to calculate these correlations analytically the spin-glass order gradually changes and there is no longer a phase transition of any kind for $h\neq 0$.

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[1] P. Lacour-Gayet and G. Toulouse, J. Phys. (Paris) 35, 425 (1974).

[2] Y. Imry and S.-K. Ma, Phys. Rev. Lett. 35, 1399 (1975).

[3] G. Grinstein, Phys. Rev. Lett. 37, 944 (1976); A. Aharony, Y. Imry, and S.-K. Ma, *ibid.* 37, 1367 (1976); A. P. Young, J. Phys. C 10, L257 (1977).

[4] G. Parisi and N. Sourlas, Phys. Rev. Lett. 43, 744 (1979); A. Niemi, *ibid.* 49, 1808 (1982); J. L. Cardy, Phys. Lett. 125B, 470 (1983).

[5] E. Pytte, Y. Imry, and D. Mukamel, Phys. Rev. Lett. 46, 1173 (1981); K. Binder, Y. Imry, and E. Pytte, Phys. Rev. B 24, 6736 (1981); H. S. Kogon and D. J. Wallace, J. Phys. A 14, L527 (1981); D. Mukamel and E. Pytte, Phys. Rev. B 25, 4779 (1982).

[6] G. Grinstein and S.-K. Ma, Phys. Rev. Lett. 49, 685 (1982); Phys. Rev. B 28, 2588 (1983); J. Vaillain, J. Phys. (Paris) 43, L55 (1982); K. Binder, Z. Phys. B 50, 343 (1983); T. Nattermann, J. Phys. C 16, 6407 (1983); J. F. Fernandez, G. Grinstein, Y. Imry, and S. Kirkpatrick, Phys. Rev. Lett. 51, 203 (1983).

[7] A. Aharony and E. Pytte, Phys. Rev. B 27, 5872 (1983).

[8] D. P. Landau, H. H. Lee, and W. Kao, J. Appl. Phys. 49, 1358 (1978); E. B. Rasmussen, M. A. Novotny, and D. P. Landau, *ibid.* 53, 1925 (1982); D. Andelman, H. Orland, and L. C. R. Wijewardhana, Phys. Rev. Lett. 52, 145 (1984).

[9] M. Schwartz (unpublished).

[10] M. Grant and J. D. Gunton (unpublished).

[11] I. Morgenstern, K. Binder, and R. M. Hornreich, Phys. Rev. B 23, 287 (1981).

[12] S. Fishman and A. Aharony, J. Phys. C 12, L729 (1979).

[13] T. Niemeijer and J. M. J. van Leeuwen, in *Phase Transitions and Critical Phenomena,* Vol. 6, edited by C. Domb and M. S. Green (Academic, New York, 1976), p. 446.

[14] I. B. Ferreira, A. R. King, V. Jaccarino, J. Cardy, and H. J. Guggenheim (unpublished).

[15] V. S. Dotsenko and Vl. S. Dotsenko, Pis'ma Zh. Eksp. Teor. Fiz. 33, 40 (1981) [JETP Lett. 33, 37 (1981)]; J. Phys. C 15, 495, L557 (1982); P. Hoever and J. Zittartz, Z. Phys. B 49, 39 (1982).

[16] D. P. Belanger, A. R. King, V. Jaccarino, and J. L. Cardy, Phys. Rev. B 28, 2522 (1983).

[17] R. A. Cowley, J. Magn. Magn. Matter. 31-34, 1439 (1983); P. Wong and J. W. Cable, Phys. Rev. B 28, 5361 (1983).

[18] H. Yoshizawa, R. A. Cowley, G. Shirane, R. J. Birgeneau, H. J. Guggenheim, and H. Ikeda, Phys. Rev. Lett. 48, 438 (1982); Y. Shapira, J. Appl. Phys. 53, 1931 (1982); H. Rohrer, *ibid.* 52, 1708 (1981); M. Hagen, R. A. Cowley, S. K. Satija, H. Yoshizawa, G. Shirane, R. J. Birgeneau, and H. J. Guggenheim, Phys. Rev. B 28, 2602 (1983).

[19] D. P. Belanger, A. R. King, and V. Jaccarino, Phys. Rev. Lett. 48, 1050 (1982); R. A. Cowley and W. J. L. Buyers, J. Phys. C 15, L1209 (1982).

[20] L. Onsager, Phys. Rev. 65, 117 (1944).

[21] A. E. Ferdinand and M. E. Fisher, Phys. Rev. 185, 832 (1969); M. E. Fisher and M. N. Barber, Phys. Rev. Lett. 28, 1516 (1972).

[22] R. M. Hornreich and H. G. Schuster, Phys. Rev. B 26, 3929 (1982).