Amorphization of a crystalline diluted Ising ferromagnet

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The amorphization of a diluted Ising ferromagnet on a square lattice is investigated with the use of a new type of effective-field theory with correlation. The exchange parameters are assumed to be distributed according to a δ function. The transition temperature, phase diagram, magnetization, and susceptibility are evaluated. We find some characteristic behavior for the amorphization.

I. INTRODUCTION

Recently, magnetism of structurally disordered alloys has become the subject of both experimental and theoretical interests as a topic of solid-state physics. A number of experimental and theoretical investigations lead to the results that magnetic long-range order may exist in amorphous systems. On the other hand, because of the disordered structure, many interesting physical properties not observed in the corresponding crystalline magnets are now becoming apparent.[1]

In what concerns the theoretical procedures, there exist a great amount of sophisticated techniques. With regard to the difficulties of the theoretical description of such complicated magnetic systems, it is sometimes necessary to do some simplifications. Therefore, for studying such systems, the lattice model of amorphous magnets has often been applied, in which the structural disorder is replaced by the random distribution of the exchange integral. In fact, experiments of the Mössbauer effect and the magnetization of amorphous ferromagnets indicate that, at least in some materials, there are large fluctuations in the exchange interaction. It is also found that the magnetization of an amorphous ferromagnet is in general lower than that of its crystalline counterpart. That fluctuation may be the underlying cause for amorphous magnets has been proposed by Gubanov,[2] Kaneyoshi,[3] and Handrich.[4] However, there are no sufficient answers to the questions: What are the conditions for the existence of definite magnetic phases as well as what kind of magnetic phase transitions can occur induced by changing the structural fluctuation? In connection with the questions the magnetic properties of the diluted amorphous systems containing nonmagnetic impurities are of great interest, since the dilution may cause an additional structural fluctuation in the systems. It is, perhaps, possible to find new phenomena in such systems which are unknown in the crystalline case.

Some of the present authors[5] have recently introduced, for the spin-$\frac{1}{2}$ pure Ising model, a new type of effective-field approximation (based on the use of a convenient differential operator in the Callen's spin-correlation identity[6]) which, within a mathematically simple framework, substantially improves the standard molecular-field approximation (MFA) results. This approach shares with the MFA a great versatility and has already been applied to a variety of interesting situations such as pure systems,[7] site-random,[8] and bond-random magnets[9,10] including spin-glasses,[11] amorphous systems,[12] transverse Ising model,[13] and surface problems.[14,15]

In this paper, the physical properties of a diluted crystalline Ising spin-$\frac{1}{2}$ system with randomly distributed exchange parameters are investigated by using the effective-field theory, in order to clarify some of the questions mentioned above. We calculate the most relevant thermodynamical quantities (transition temperature, phase diagram, magnetization, and initial susceptibility).

The outline of our paper is as follows. In Sec. II, we briefly review the basic points of the simple effective-field theory with correlation, when it is applied to the problem for the amorphization of a dilute Ising crystalline ferromagnet. In Sec. III the framework is applied to the amorphization of the dilute ferromagnet in a square lattice. The analytical forms of the relevant thermodynamical quantities are obtained. For comparison, the transition temperature in the dilute ferromagnetic square lattice is at first evaluated within the present formalism as a function of concentration of magnetic atoms. The averaged magnetization at $T=0$ is studied there; the averaged magnetization, at $T=0$, varies with the structural fluctuation δ discontinuously, and as a consequence, the system exhibits a first-order phase transition with respect to δ variation. In Sec. IV the numerical results of the relevant thermodynamial quantities are studied and discussed. We find some interesting behavior characteristic to the amorphization of the diluted Ising ferromagnet in a square lattice.

II. THEORY

The system consists of N_i identical spins, $\mu_i=\pm 1$, arranged randomly on a lattice with N sites (or $N_i \leq N$). The Hamiltonian is given by

$$H=-\tfrac{1}{2}\sum_{i,j} J_{ij}\mu_i\mu_j\xi_i\xi_j - H\sum_i \mu_i\xi_i \ , \tag{1}$$

where J_{ij} is the exchange interaction with $J_{ii}=0$. ξ_i is the random variable which takes the value of unity or zero, depending on whether the site i is occupied by a magnetic atom or not. H is the applied external field. Moreover, to describe the structural disorder in a simple way, the stochastic lattice model is used; the nearest-neighbor exchange interaction is given by independent random variables as follows:

$$P(J_{ij})=\tfrac{1}{2}[\delta(J_{ij}-J-\Delta J)+\delta(J_{ij}-J+\Delta J)] . \tag{2}$$

Formal identities for the correlation functions of the Ising model have appeared in the literature for some time.[16]

The starting point for the statistics of our spin system is the exact relation due to Callen[6]

$$\langle\mu_i\rangle=\left\langle\tanh\left[h+\beta\sum_j J_{ij}\mu_j\xi_j\right]\right\rangle , \tag{3}$$

where $h=\beta H$ and the angular brackets indicate the usual ensemble average

$$\langle\cdots\rangle=\mathrm{Tr}[\exp(-\beta H)\cdots]/\mathrm{Tr}\exp(-\beta H) ,$$

and $\beta=(k_BT)^{-1}$. Here, in order to write the identity (3) in a form which is particularly amenable to approximation, let us introduce the differential-operator technique proposed by Honmura and Kaneyoshi[5] as follows:

$$\begin{aligned}\sigma_i\equiv\langle\mu_i\rangle&=\left\langle\exp\left[D\sum_j t_{ij}\mu_j\xi_j\right]\right\rangle[\tanh(h+x)]_{x=0}\\&=\left\langle\prod_j\{(1-\xi_j)+\xi_j[\cosh(Dt_{ij})+\mu_j\sinh(Dt_{ij})]\}\right\rangle[\tanh(h+x)]_{x=0} ,\end{aligned} \tag{4}$$

where $D=\partial/\partial x$ is a differential operator and $t_{ij}=\beta J_{ij}$. For deriving Eq. (4) from Eq. (3), we used both an identity

$$e^{\alpha\mu_i}=\cosh\alpha+\mu_i\sinh\alpha$$

and the relation $\xi_i^n=\xi_i$ ($n=$integer).

By assuming the statistical independence of lattice sites, namely

$$\langle\mu_i\mu_j\cdots\mu_l\rangle\cong\langle\mu_i\rangle\langle\mu_j\rangle\cdots\langle\mu_l\rangle , \tag{5}$$

Eq. (4) may be rewritten as

$$\sigma_i=\prod_\delta\{(1-\xi_{i+\delta})+\xi_{i+\delta}[\cosh(Dt_{i,i+\delta})+\sigma_{i+\delta}\sinh(Dt_{i,i+\delta})]\}[\tanh(h+x)]_{x=0} , \tag{6}$$

where δ only takes nearest neighbors of a central site i. That is to say, in a disordered system, spin-spin correlation should be more reduced than that of its corresponding nonrandom system. The approximation led, in spite of its simplicity, to quite satisfactory results. In fact, the approximation essentially corresponds to the Zernike approximation[17] in the nonrandom problem, as shown in Ref. 9. The formalism has been applied to a number of disordered magnetic systems.[8–12]

For a disordered system with random bonds and random occupation of magnetic atoms, we must perform the random configurational average to Eq. (6). In the case that the exchange interactions and random occupation of magnetic atoms are given by independent random variables, Eq. (6) reduces to, upon performing the random average,

$$\begin{aligned}m=\langle\sigma_i\rangle_r=\{&(1-p)+p[\langle\cosh(Dt_{i,i+\delta})\rangle_J\\&+m\langle\sinh(Dt_{i,i+\delta})\rangle_J]\}^Z\\&\times[\tanh(h+X)]_{x=0} ,\end{aligned} \tag{7}$$

where $\langle\ \rangle_r$ and $\langle\ \rangle_J$ express the random averages. p is the concentration of magnetic atoms defined by $p=\langle\xi_i\rangle_r=N_i/N$. Z is the number of nearest neighbors. By means of (2) the random-bond averages are then given by

$$\begin{aligned}\langle\cosh(Dt_{i,i+\delta})\rangle_J&=\cosh(2Dt\delta)\cosh(Dt) ,\\\langle\sinh(Dt_{i,i+\delta})\rangle_J&=\sinh(2Dt\delta)\sinh(Dt) ,\end{aligned} \tag{8}$$

where $t=\beta J$, and $\delta=\Delta J/2J$ is a dimensionless parameter which measures the amount of fluctuation of exchange interactions. The results (8) can be also obtained by using the so-called "lattice model" of amorphous magnets;[4] in the case of the "lattice model" of amorphous magnets discussed by Handrich, in which the structural disorders are replaced by the fluctuation ΔJ from the mean-exchange integral J in a crystalline lattice, namely $J_{ij}=J+\Delta J$, the average values of exchange interaction are approximated by

$$\langle(\Delta J)^{2n}\rangle_J=[(\Delta J)^2\rangle_J]^n \quad\text{and}\quad \langle(\Delta J)^{2n+1}\rangle_J=0 ,$$

where n is the integer. Thus, by solving Eq. (7), we can obtain the averaged magnetization m. The initial susceptibility is then defined by

$$\chi=\lim_{H\to0}\frac{\partial m}{\partial H}=\frac{t}{J}\frac{\partial m}{\partial h}\bigg|_{h=0} . \tag{9}$$

In this section, we have briefly reviewed the effective-field theory with correlation in a diluted Ising ferromagnet with random bonds. We are in a position to examine the effects of amorphization on transition temperature, phase diagram, magnetization, and susceptibility. In the following sections, we shall study the physical properties for the amorphization of the diluted ferromagnetic square lattice by using this framework.

III. AMORPHIZATION OF A DILUTED FERROMAGNETIC SQUARE LATTICE

In this section, let us study the amorphization of the diluted Ising ferromagnet on a square lattice. For the case of four nearest neighbors, Eq. (7) can be expanded, i.e.,

$$m=4Am+4Bm^3+Ch+O(h^2)\ , \tag{10}$$

with

$$A=p^4K_1+3p^3(1-p)K_3+3p^2(1-p)^2K_5+p(1-p)^3K_6\ , \tag{11}$$

$$B=p^4K_2+p^3(1-p)K_4\ , \tag{12}$$

$$C=p^4(L_1+6L_2m^2+L_3m^4)+4p^3(1-p)(L_4+3L_5m^2)+6p^2(1-p)^2(L_6+L_7m^2)+4p(1-p)^3L_8+(1-p)^4\ , \tag{13}$$

where the coefficients K_i ($i=1$ to 6) and L_i ($i=1$ to 8) are given in the Appendix. Equation (10) was then derived by expanding $\tanh(x+h)$ in Eq. (7) with h and retaining the terms linear to h.

For $h=0$, the averaged magnetization m is given by

$$m=\left[\frac{1-4A}{4B}\right]^{1/2}\ . \tag{14}$$

The critical ferromagnetic frontiers can be derived from the condition

$$4A-1=0\ , \tag{15}$$

by which the phase diagrams and transition temperatures can be determined as functions of p and δ. Then, by appling a mathematical relation $e^{aD}f(x)=f(x+a)$, all the coefficients K_i and L_i can be expressed as a sum of transcendental functions $\tanh X$ with an appropriate argument X. For instance, the coefficient K_1 is given by

$$K_1=(\tfrac{1}{2})^7\{\tanh[4t(1+2\delta)]+\tanh[4t(1-2\delta)]+4\tanh[4t(1+\delta)]+4\tanh[4t(1-\delta)]+6\tanh(4t)+2\tanh[2t(1+4\delta)]+2\tanh[2t(1-4\delta)]+8\tanh[2t(1+2\delta)]+8\tanh[2t(1-2\delta)]+12\tanh(2t)\}\ . \tag{16}$$

Here, it is worth noting that in our treatment the transition temperature $T_c=J/kt_c$ for the diluted ferromagnetic square lattice with $\delta=0$ is determined from, as a function of concentration p,

$$p^4\bar{K}_1+3p^3(1-p)\bar{K}_3+3p^2(1-p)^2\bar{K}_5+p(1-p)^3\bar{K}_6=\tfrac{1}{4}\ , \tag{17}$$

with

$$\bar{K}_1=\tfrac{1}{8}[\tanh(4t_c)+2\tanh(2t_c)]\ ,$$

$$\bar{K}_3=\tfrac{1}{4}[\tanh(3t_c)+\tanh(t_c)]\ ,$$

$$\bar{K}_5=\tfrac{1}{2}\tanh(2t_c)\ ,$$

$$\bar{K}_6=\tanh(t_c)\ ,$$

which is equivalent to the result derived by Matsudaira[18] and afterwards by Mattis.[19] For clarification, in Fig. 1, $T_c(p)$ is depicted as a function of concentration p. The critical concentration p^* is then given by $p^*=0.4284$, which compares reasonably with the exact result for critical percolation in square lattice, 0.5.[20] The reduced magnetization curves and some interesting behavior of reduced magnetization for a diluted ferromagnetic square lattice with $\delta=0$ have already been shown in Ref. 15. In the next section, some of them will be rederived, in order to complete our understanding for the amorphization of the system.

As discussed in Ref. 9, when we use the amorphization given by Eq. (2), the saturation magnetization at $T=0$ shows a characteristic behavior; the averaged magnetization, at $T=0$, varies with δ discontinuously. The result comes from the fact that the transcendental functions $\tanh[at(1-b\delta)]$ with positive a and b are included in the coefficients K_i and L_i [see Eq. (16)], and the functions, at $T=0$, can take values

$$\tanh[at(1-b\delta)]=\begin{cases}1, & \delta<\dfrac{1}{b}\\ 0, & \delta=\dfrac{1}{b}\\ -1, & \delta>\dfrac{1}{b}\end{cases}$$

depending on the value of δ. In Fig. 2(a) the saturation magnetization at $T=0$ for selected values of p are depicted as a function of δ. At a critical fluctuation δ_c depending on the selected value of p, the averaged magnetization disappears discontinuously and the system exhibits a first-order phase transition with respect to its amorphization at $T=0$. In Fig. 2(b), therefore, the critical fluctuation δ_c at $T=0$ is depicted as a function of p and at $p=p^*$ the value also disappears discontinuously.

From Eqs. (9) and (10), the inverse initial susceptibility is given by

$$(J\chi)^{-1}=\frac{D_1}{tD_2} \tag{18}$$

with

$$D_1=1-4p^4(K_1+3K_2m^2)-12p^3(1-p)(K_3+K_4m^2)-12p^2(1-p)^2K_5-4p(1-p)^3K_6\ , \tag{19}$$

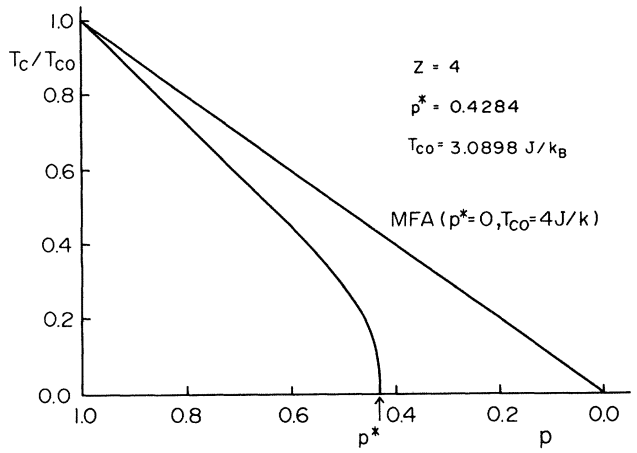

FIG. 1. Concentration dependence of the Curie temperature for the diluted square lattice. For comparison, the MFA result is also depicted.

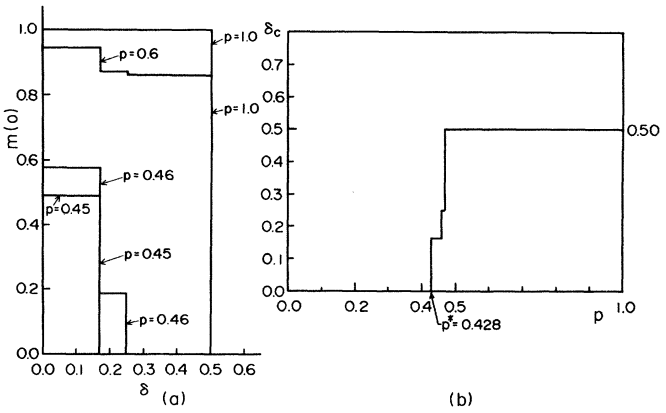

FIG. 2. (a) Averaged magnetization $m(0)$ at $T=0$ vs the structural fluctuation δ. (b) Critical fluctuation δ_c at $T=0$ vs the concentration of magnetic atoms.

$$D_2=p^4(L_1+6L_2m^2+L_3m^4)+4p^3(1-p)(L_4+3L_5m^2)+6p^2(1-p)^2(L_6+L_7m^2)+4p(1-p)^3L_8+(1-p)^4 . \quad (20)$$

We are now in a position to examine the physical properties for the amorphization of the dilute Ising ferromagnet in a square lattice numerically. The numerical results will be given in the next section.

IV. NUMERICAL RESULTS AND DISCUSSIONS

By solving Eqs. (14) and (15), the behavior of the averaged magnetization versus temperature for fixed pair of values (p,δ) are presented in Fig. 3, and the critical frontiers in the T,δ space are plotted in Fig. 4 for typical values of p.

From Fig. 3, we can see that for selected values p,δ [see, for instance, the solid curve labeled (1.0, 0.54)] the magnetization, which does not exist until certain temperature, starts to increase, passes through a maximum value, and decreases to zero with increasing temperature. The behavior of the averaged magnetization is similar to the experimental results of amorphous ferromagnets, showing the "reentrant" phenomena. In relation to the result, it is worth noting the curves in Fig. 4. In the curve for $p=1$, for instance, there exist two possible different values of T_c for a given value of δ in the range $0.5\leq\delta\leq0.565$. If we admit the existence of a spin-glass phase below a small value of the two T_c, the result may support the reentrant phenomena that the transition from the spin-glass phase to the paramagnetic phase passing through the ferromagnetic one is possible.[27] Another interesting feature also observed in Fig. 3 [see the dotted curve labeled (0.46, 0.2)] is that the magnetization can increase from its value at $T=0$, reaches a maximum, and decreases to zero.

On the other hand a great number of experimental and theoretical works have reported that the temperature dependence of the reduced saturation magnetization of amorphous and dilute ferromagnets except some materials

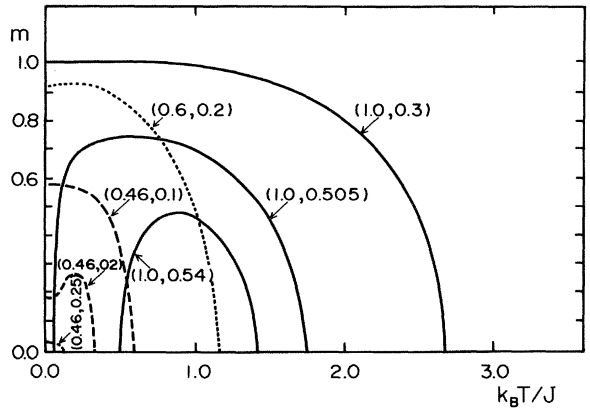

FIG. 3. Averaged magnetizations vs temperature for $Z=4$. Numerical figures associated with each curve denote the fixed pair of values p,δ.

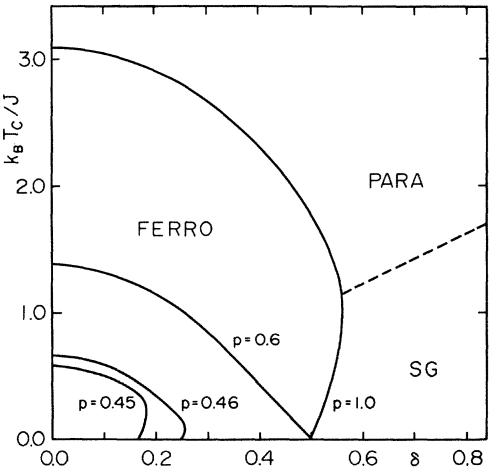

FIG. 4. Phase diagrams in the T,δ space for $Z=4$. Here the interface between the spin-glass (SG) and the paramagnetic phases (dashed line) is that predicted by the usual MFA (Ref. 19).

mentioned below has the characteristic feature; it consistently falls below that of the corresponding crystalline ferromagnets. In Fig. 5, therefore, the temperature dependences of reduced saturation magnetization are shown for some values of p and δ, except the special cases mentioned in the above paragraph. From Fig. 5(a), we can understand that the effect of increasing the disorder in the pure system with (1.0, 0.0) is generally an increase in the depression of reduced magnetization curve over the entire temperature range for $T \leq T_c$, as observed in dilute and amorphous ferromagnets.[1] Very near the critical concentration p^*, however, the behavior of the reduced magnetization curve is rather different, as depicted in Fig. 5(b). The curve labeled (0.46, 0.0) is over those of (0.6, 0.0) and (0.6, 0.2) in Fig. 5(a), and the amorphization of the curve (0.46, 0.2) exhibits the anomalous behavior, as shown in Fig. 3. The result of (0.46, 0.0) reminds us of the reduced magnetization curves observed in some amorphous and dilute ferromagnets, such as amorphous ferromagnetic $Fe_{1-x}B_x$ alloys.[1] The materials have the Invar characteristic; at first the reduced magnetization curves express the increasing depression over the entire temperature range for $T \leq T_c$, on decreasing the concentration of magnetic atoms, but from some concentration the curves approach to that of the corresponding crystalline one [or in the present case the curve labeled (1.0, 0.0), like Fig. 5(b)].

By solving Eq. (18) the behavior of the inverse initial susceptibility versus reduced temperature for a fixed pair of values (p,δ) are presented in Figs. 6–8. In Fig. 6 the inverse susceptibility for the amorphization of the pure (1.0, 0.0) system, in which the whole lattice sites are occupied by magnetic atoms, are depicted. Except the special case exhibiting the reentrant phenomena shown in Fig. 3, the inverse susceptibility diverges only at the transition temperature, even for the amorphization. A particular interesting behavior is the case expressing the reentrant phenomena, namely the case labeled (1.0, 0.54) in Fig. 3, for which the susceptibility diverges three times, twice at the critical points where the averaged magnetization disappears and another one at $T=0$, because of the existence of finite clusters, as shown in Fig. 6. For the amorphization of the system with the concentration of $p^* < p < 1$, on the other hand, the susceptibility diverges twice, one at the

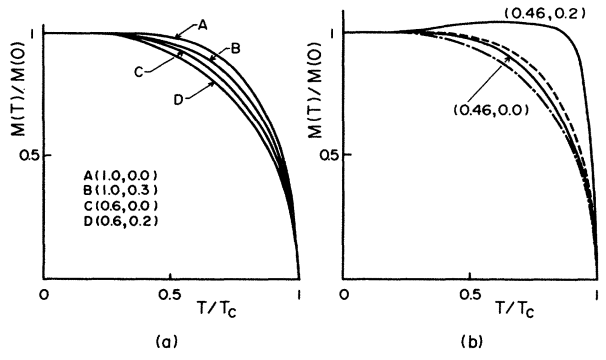

FIG. 5. (a) Reduced magnetization curves for selected values of p,δ. (b) Reduced magnetization curves for two values, (0.46, 0.0) and (0.46, 0.2). For comparison, two cases, the curve A in (a) and the curve C in (a) are depicted.

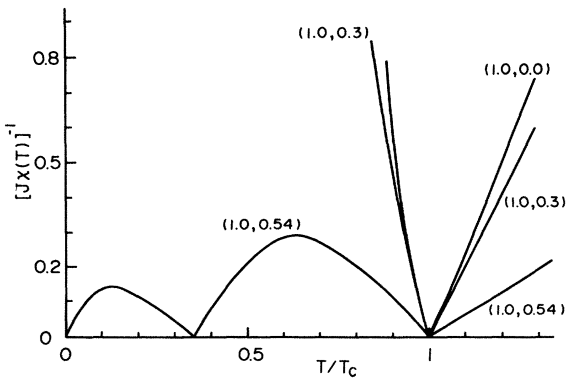

FIG. 6. Thermal dependence of the inverse initial susceptibility for selected values of δ, namely the amorphization, when p is fixed at $p=1$.

critical point (infinite-cluster contribution) and another one at $T=0$ (finite-cluster contribution), which results are depicted in Figs. 7 and 8. Thus, we observe the coexistence of a Curie-Weiss—type law with a Curie-type one within one formalism. The fact was also discussed in Refs. 10 and 22. In Fig. 7 the inverse susceptibilities for the amorphization of the system with $p=0.6$ are shown; notice that for increasing δ the low-temperature region expresses an extremely interesting behavior, although the reduced magnetization curve did not show such a characteristic, as observed in Fig. 5(a). In Fig. 8 we plot the inverse susceptibilities for the two cases of p,δ, namely (0.46, 0.0) and (0.46, 0.2), since the magnetization curve for the case (0.46, 0.2) exhibits the anomalous behavior, as indicated in Figs. 3 and 5(b). For the case labeled (0.46, 0.2), however, no anomaly for the inverse susceptibility is observed for $T < T_c$.

In order to observe the behavior of χ above the transition temperature clearly, the inverse paramagnetic susceptibility χ_{para}^{-1} is depicted in Fig. 9 for selected value of p,δ. Near the critical temperatures, the results of χ_{para}^{-1} have all downward curvatures. A characteristic behavior is that the deviation from the Curie-Weiss law is observed more remarkably than that of the pure (1.0, 0.0) system, on increasing disorder, which phenomena is observed in amor-

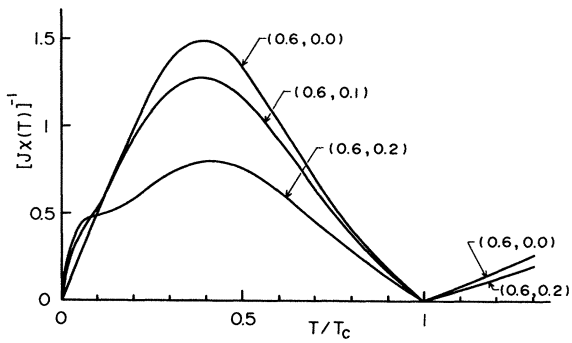

FIG. 7. Thermal dependence of the inverse initial susceptibility for the amorphization, when p is fixed at $p=0.6$.

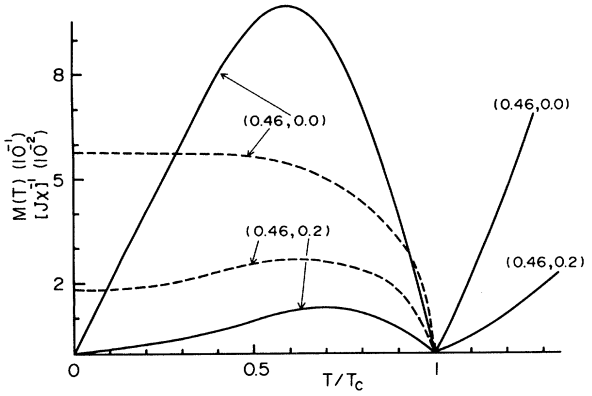

FIG. 8. Thermal dependence of the inverse initial susceptibility for the two cases of δ, namely $\delta=0.0$ and $\delta=0.2$, when p is fixed at $p=0.46$. For reference, the averaged magnetizations for the two cases are also depicted as dashed lines.

phous ferromagnets.[1] In view of the result, we evaluate the effective exponent $\gamma(T)$ of the paramagnetic susceptibility defined by

$$\gamma(T)=(T-T_c)\chi_{\text{para}}\frac{d\chi_{\text{para}}^{-1}}{dT}\ , \tag{21}$$

which was first introduced by Kouvel and Fisher.[23] By the use of Eq. (18) we can solve Eq. (21) numerically. The results for selected value of p,δ are depicted in Fig. 10. At $T=T_c$ the results all reduce to the value of unity, since our approach is essentially a molecular-field approximation. For higher temperatures, the exponent also approaches gradually to the value of unity. The effective exponent shows a maximum at a temperature $T_{\max}$. A characteristic feature of the result is that the maximum value at $T=T_{\max}$ increases for both the amorphization and the dilution of the pure system. For crystalline systems, however, it is well known that the effective exponent decreases monotonically with increasing temperature from the value at $T=T_c$. Accordingly, the result for the pure (1.0, 0.0) system cannot have any physical meaning for the

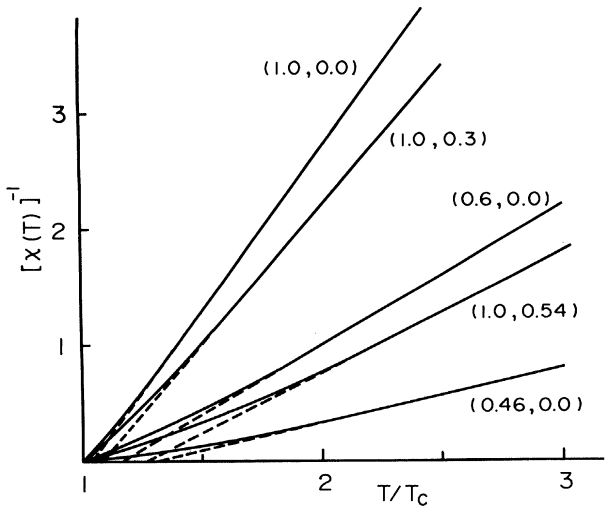

FIG. 9. Thermal dependence of the inverse paramagnetic susceptibility for selected values of p,δ.

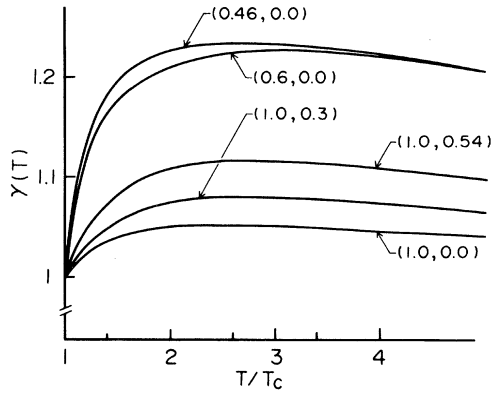

FIG. 10. Temperature dependence of $\gamma(T)$ for selected values of p,δ.

"real" critical region, since our theory is a molecular-field approximation. In amorphous and dilute ferromagnets, on the other hand, the effective exponent runs through a maximum and the real critical range is expected to become narrower than that for the pure system.[1] Thus, our result may have some relation to the experimental results.

V. CONCLUSION

We have discussed the amorphization of diluted spin-$\frac{1}{2}$ Ising ferromagnet in a square lattice. Within the effective-field theory with correlation, we evaluated the most relevant thermodynamical quantities, namely critical temperature, phase diagram, susceptibility, and effective exponent of paramagnetic susceptibility. Some interesting effects of amorphization come up in the thermal behavior; the susceptibility shows the effect of the eventual coexistence, in the system, of an infinite cluster with finite ones. The magnetization exhibits the reentrant phenomena versus temperature for selected values of structural fluctuation δ, and also the discontinuous changes at $T=0$ for increasing the structural fluctuation. Except the special cases showing the reentrant phenomena, the reduced magnetization curve falls below that of the corresponding crystalline ferromagnets, which phenomena is generally observed in amorphous and dilute ferromagnets. Moreover, we found some interesting facts within the present formalism that for amorphization near the percolation concentration the magnetization increases and then decreases, on increasing temperature, such as the curve with (0.46, 0.2) in Fig. 3, and the susceptibility with (0.6, 0.2) in Fig. 7 shows an anomalous behavior at low temperatures for amorphization. It will be an interesting problem to study whether the phenomena are general or not by using more elaborated theories.

In these calculations we have applied a decoupling in the effective-field framework (based on the use of a conventional differential operator) introduced by Honmura and Kaneyoshi.[5] As discussed in Ref. 9, the approximation essentially corresponds to the Zernike approximation.[17] This formalism is, from the analytical standpoint, almost as simple as the standard mean-field approximation, and because of negligence of multispin correlation, shares with it the fact that the critical exponents are all

Landau-type, and the related fact that the topology of the system is only partially taken into account, essentially through the coordination number, although the effective exponent of paramagnetic susceptibility expresses a characteristic behavior on increasing the disorder from the pure system. Nevertheless, we verify that its results are quite superior to the other effective-field theories and exhibit some characteristic behavior for amorphization of a diluted Ising ferromagnet in a square lattice.

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APPENDIX

The coefficients K_i $(i=1-6)$ and L_i $(i=1-8)$ in Eqs. (11)—(13), are given as follows:

$$K_1=\cosh^4(2Dt\delta)\sinh(Dt)\cosh^3(Dt)[\tanh(x)]_{x=0}\,,$$

$$K_2=\cosh^4(2Dt\delta)\sinh^3(Dt)\cosh(Dt)[\tanh(x)]_{x=0}\,,$$

$$K_3=\cosh^3(2Dt\delta)\cosh^2(Dt)\sinh(Dt)[\tanh(x)]_{x=0}\,,$$

$$K_4=\cosh^3(2Dt\delta)\sinh^3(Dt)[\tanh(x)]_{x=0}\,,$$

$$K_5=\cosh^2(2Dt\delta)\cosh(Dt)\sinh(Dt)[\tanh(x)]_{x=0}\,,$$

$$K_6=\cosh(2Dt\delta)\sinh(Dt)[\tanh(x)]_{x=0}\,,$$

and

$$L_1=\cosh^4(2Dt\delta)\cosh^4(Dt)[\mathrm{sech}^2(x)]_{x=0}\,,$$

$$L_2=\cosh^4(2Dt\delta)\sinh^2(Dt)\cosh^2(Dt)[\mathrm{sech}^2(x)]_{x=0}\,,$$

$$L_3=\cosh^4(2Dt\delta)\sinh^4(Dt)[\mathrm{sech}^2(x)]_{x=0}\,,$$

$$L_4=\cosh^3(2Dt\delta)\cosh^3(Dt)[\mathrm{sech}^2(x)]_{x=0}\,,$$

$$L_5=\cosh^3(2Dt\delta)\cosh(Dt)\sinh^2(Dt)[\mathrm{sech}^2(x)]_{x=0}\,,$$

$$L_6=\cosh^2(2Dt\delta)\cosh^2(Dt)[\mathrm{sech}^2(x)]_{x=0}\,,$$

$$L_7=\cosh^2(2Dt\delta)\sinh^2(Dt)[\mathrm{sech}^2(x)]_{x=0}\,,$$

$$L_8=\cosh(2Dt\delta)\cosh(Dt)[\mathrm{sech}^2(x)]_{x=0}\,.$$

The coefficients can easily be calculated by applying a mathematical relation, $e^{\alpha D}f(x)=f(x+\alpha)$.

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