Lower bounds for the width of domain walls in the random-field Ising model

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Using the replica trick we construct a "Mermin-Wagner-Hohenberg" style inequality which places a lower bound on the width of domain walls in the random-field Ising model (RFIM). We apply our inequality to two competing interface models of the RFIM, that of Pytte, Imry, and Mukamel and that of Grinstein and Ma which yield different values of d_c, the lower critical dimensionality, and the width of domain walls. If we assume replica symmetry, our lower bound is consistent with the work of Pytte *et al.* but inconsistent with that of Grinstein and Ma. Our result suggests that d_c for the RFIM is 3. However, this result is not conclusive given our assumption of replica symmetry, and the validity of the inequality as the number of replicas tends to zero. Indeed, our result suggests at least indirectly that consideration of these questions may be essential to understanding the conflicting results obtained from the two interface models.

I. INTRODUCTION

The value of d_c, the "lower critical dimension" for the random-field Ising model (RFIM) remains an open question despite eight years of both theoretical[1-10] and experimental[11,12] efforts. The first theoretical analysis was that of Imry and Ma[1] who predicted $d_c=2$ on the basis of a very simple and physical domain argument. Subsequent work,[13] however, raised doubts about this result. It was shown order by order in perturbation theory that the critical behavior of the Landau-Ginzburg representation of the RFIM in d dimensions corresponds exactly to that of the pure system in $d-2$ dimensions. Since $d_c=1$ for the pure Ising model, this correspondence suggests that $d_c=3$ for the RFIM. Binder, Imry, and Pytte[2] then suggested that the assumption of smooth, i.e., nonrough, domain walls in the Imry-Ma argument was incorrect. Specifically, if $w/L\rightarrow\infty$, where w is the width of the wall and L is the length, the wall should be considered rough and the accounting of the exchange energy in the domain argument should be modified. This observation spurred the construction of several continuum interface models[3-6] to analyze the low-temperature properties of the RFIM and in particular to calculate w/L and d_c.

In replica space, the effective Hamiltonian of the interface models constructed by Pytte *et al.*[3] and Grinstein and Ma[5] takes the form

$$H_{\text{eff}}=H_{\text{ex}}+H_{\text{field}}\ , \tag{1a}$$

where

$$H_{\text{ex}}=\sigma_0\int d^{d-1}x\sum_{\alpha=1}^{n}\{1+[\vec{\nabla}f_\alpha(x)]^2\}^{1/2}\ , \tag{1b}$$

and

$$H_{\text{field}}=\frac{\Delta}{T}\int d^{d-1}x\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}g[f_\alpha(x)-f_\beta(x)]\ , \tag{1c}$$

where $f_\alpha(x)$, a single-valued function of x, describes the interfacial profile assuming that we neglect droplets and overhangs. The coordinate x characterizes the $(d-1)$-dimensional hyperplane parallel to the flat interface with $f_\alpha(x)=0$. The energy H_{ex} corresponds to the nonrandom exchange energy in the rough phase and is simply the sum of the total surface area for each replica interface.[14,15] The factor σ_0 is the bare surface tension. The energy H_{field} describes the interaction of the spins with the random field and Δ is proportional to the variance of the field distribution (assumed to be uncorrelated between sites). The form of the function $g(y)$, $y=f_\alpha(x)-f_\beta(x)$, appearing in (1c) has been disputed. Pytte *et al.* arrive at the form (1) by analyzing the interfacial properties of the Landau-Ginzburg representation of the RFIM. Their $g(y)$ behaves as y^2 for small y and as $|y|$ for large y. The exact form of their $g(y)$ is not crucial, and we will assume that in their work $g\sim\ln(\cosh y)$ (this form reproduces the limiting behaviors mentioned above). With the use of a low-temperature renormalization-group analysis of (1), Pytte *et al.* obtained $d_c=3$ and found that $w/L\sim L^{(3-d)/2}$, thus invalidating the original domain argument. Their analysis focused on the small-y behavior of $g(y)$, and these authors argued that higher-order terms are irrelevant. Similar results were obtained by Kogon and Wallace[4] without replicas, using a supersymmetric formulation of the Landau-Ginzburg form.

Grinstein and Ma constructed an interface model of the RFIM beginning with discrete spins rather than a Landau-Ginzburg form. In replica space their model takes the form (1) with $g(y)=|y|$ for all y. By primarily using the nonreplica version of their model, these authors argued that $d_c=2$ and $w/L\sim L^{(2-d)/3}$.[16] [The perturbative analysis of their version of (1) is complicated by the nonanalyticity of $|y|$, although this analysis also suggests that $d_c=2$.] Grinstein and Ma argued further that Pytte *et al.* may have obtained $d_c=3$ because they erroneously assumed that the small-y behavior of g determines d_c and the scaling of w/L. Possibly, an improved treatment of the model with $g\sim\ln(\cosh y)$ would yield

$d_c=2$ if it focused on the large-y regime, where both forms of $g(y)$ agree.

In this paper we attempt to elucidate the source of the discrepancy between the results of Pytte *et al.* and Grinstein and Ma. Using the replica trick, we construct a "Mermin-Wagner-Hohenberg"[17–19] (MWH) style inequality which places a lower bound on w/L. This inequality is analogous to those constructed to place an upper bound on the magnetization in the random-field vector spin model[20] and the random-axis model.[21] While MWH inequalities are generally restricted to models with continuous symmetry and hence massless modes (seemingly excluding the Ising model), an inequality can be constructed for the interfacial version of the Ising model, in the continuum limit. In this representation, the capillary waves[14,15] are analogous to the spin waves in vector models; in particular, they are massless.

With the assumption (possibly critical) of replica symmetry (i.e., that thermodynamic averages involving the fields f_α are independent of the index α) we find that $w/L \geq L^{(3-d)/2}$ regardless of whether we use $g(y)=|y|$ or $g(y)=\ln(\cosh y)$. This result is consistent with that of Pytte *et al.* but inconsistent with the result of Grinstein and Ma. Owing to the nature of this inequality we need not make any *a priori* assumptions regarding which regime of $g(y)$ is important. Constructing the inequality we find that it is the small-y behavior of $g(y)$ which determines the behavior of w/L.

Our result rules out the presence of a ferromagnetically ordered phase below three dimensions with rough domain walls, leaving open the possibility of an ordered phase with smooth walls. However, the latter possibility also conflicts with the work of Grinstein and Ma who found only rough walls in their ordered phase between two and three dimensions. In fact, the presence of an ordered phase with smooth walls between two and three dimensions in the RFIM seems unlikely, since in the pure model the domain walls are always rough between one and two dimensions, in both lattice and continuum models.

Our derivation involves two key assumptions. First, we assume that all thermodynamic averages involving f_α are independent of the index α, i.e., we assume replica symmetry. However, in the spin-glass phase of the random-bond Ising model it was found[22] that a state with replica symmetry is unstable to symmetry-breaking fluctuations. Parisi[23] has developed a replica symmetry-breaking scheme for the spin-glass phase which restores thermodynamic stability.[24] It would seem possible then that a similar phenomenon might occur in the RFIM. However, as yet no one has established the existence of a replica symmetry-breaking instability. It is also unclear how one would generalize Parisi's scheme, which applies to a phase described by a matrix order parameter $q_{\alpha\beta}$ to a ferromagnetic phase in the RFIM described by a vector f_α. Nevertheless, these questions merit further study.

Our second key assumption concerns the validity of the MWH inequality in the context of the replica method. In deriving our key results, Eqs. (17) and (26), we divide both sides of the inequality by n and eventually let n go to 0. This step could be dangerous. However, we note that similar manipulations were employed in deriving the MWH inequalities for the random-field vector spin model[20] and the random-axis model,[21] and the results of these inequalities are in agreement with all other calculations on these models.

An additional complication arises when we construct the MWH inequality for the Grinstein-Ma model. As we discuss more fully in the final section of this paper, the nonanalytic form of $g(y)$ in their model forces us to impose a nonzero, although arbitrarily small, lattice spacing a to obtain a finite domain-wall width at high dimensions. In particular, we are forced to replace $f_\alpha(x)$ by an integer-valued field representing the discrete steps of the interface. Similarly, in the perturbation theory of Grinstein and Ma, their expansion of $g(y)$ about $y=0$ yields *bare* interactions which are divergent because of the continuous nature of $f_\alpha(x)$. However, these authors find that resummation of all of these divergent terms after the fluctuation integrals have been performed yields finite results for quantities such as the surface tension. On the other hand, in our MWH inequality, Eq. (26), the limit $a \to 0$ cannot be taken.

With all of these unanswered questions, we do not wish to predict values of d_c for the RFIM. Instead, we hope that our work will stimulate further investigation of these matters.

This paper is organized as follows: In the next section we construct, for the sake of illustration, a MWH inequality for the width of domain walls in the pure Ising model. In Sec. III we present our MWH inequality for the RFIM interface model (1), for both the Pytte *et al.* and Grinstein and Ma choices for $g(y)$.

II. PURE ISING MODEL

For the purpose of illustration, we construct a MWH inequality which places a lower bound on the quantity w/L in the nonrandom Ising model. We assume that the domain walls in this model can be described by the Hamiltonian (1b) with $n=1$, namely,

$$H=\sigma_0 \int d^{d-1}x \{1+[\vec{\nabla} f(x)]^2\}^{1/2} . \tag{2}$$

In terms of $f(x)$, the width w is defined as $w \equiv [\langle f^2(x)\rangle]^{1/2}$ where the angular brackets refer to a thermal average over the ensemble defined by (2).

The classical MWH inequality,[18] derived from the Schwarz inequality reads

$$\langle |A|^2 \rangle \geq \frac{T|\langle [C,A^*]\rangle|^2}{\langle [C,[C^*,H]]\rangle} \tag{3}$$

for as yet unspecified operators A and C. The classical Poisson bracket $[R,S]$ for operators R and S is defined by

$$[R,S]=\int d^{d-1}x \left[\frac{\delta R}{\delta f(x)} \frac{\delta S}{\delta P(x)} - \frac{\delta R}{\delta P(x)} \frac{\delta S}{\delta f(x)} \right], \tag{4}$$

where $f(x)$ and $P(x)$ are the canonical variables of the system. The variable $P(x)$ is the momentum conjugate to $f(x)$, i.e., the generator of translations of the interface. We choose the operators A and C appearing in (3) in the usual fashion,[21] i.e.,

$$A(\vec{k}) = \int d^{d-1}x e^{-i\vec{k}\cdot\vec{x}} f(x) , \tag{5a}$$

$$C(\vec{k}) = \int d^{d-1}x e^{-i\vec{k}\cdot\vec{x}} P(x) , \tag{5b}$$

and ultimately we integrate (3) over $\vec{k}$. Using (5) we find that

$$\int \frac{d^{d-1}k}{(2\pi)^{d-1}} \langle |A(\vec{k})|^2\rangle = \left\langle \int d^{d-1}x f^2(x) \right\rangle , \tag{6a}$$

$$|\langle [C(\vec{k}), A^*(\vec{k})]\rangle|^2 = \left[\int d^{d-1}x(1)\right]^2 , \tag{6b}$$

$$\langle [C(\vec{k}),[C^*(\vec{k}),H]]\rangle = \sigma_0 \int d^{d-1}x \left\langle \frac{k^2}{[1+(\nabla f)^2]^{1/2}} - \frac{(\vec{k}\cdot\nabla\vec{f})^2}{[1+(\vec{\nabla} f)^2]^{3/2}} \right\rangle \leq \sigma_0 \int d^{d-1}x k^2 . \tag{6c}$$

With the use of (6), the inequality (3) yields the result

$$\langle f^2(x)\rangle \geq \frac{T}{\sigma_0(2\pi)^{d-1}} \int \frac{d^{d-1}k}{k^2} . \tag{7}$$

For an interface of linear dimension L, the integral over k in (7) has a lower limit $\sim 1/L$. Thus from (7) we obtain the following bound on w/L for large L:

$$\frac{w}{L} \geq L^{(1-d)/2} \tag{8}$$

assuming that $d<1$, so that we can neglect the upper limit of the integration. This divergence of w/L is consistent with the value $d_c=1$ for the pure Ising model. Note, however, that w itself diverges below *three* dimensions as expected in a continuum interface model.[14] Consideration of the finite lattice spacing would in principle show that w diverges only below two dimensions at low temperatures.[25,26]

III. INEQUALITY FOR THE RFIM

Here, we construct an inequality for w/L in the RFIM using (1). The width of the interface in the quenched system is now defined as

$$w \equiv \lim_{n\to 0} \frac{1}{n} \sum_{\alpha=1}^{n} \langle f_\alpha^2(x)\rangle_n^2 , \tag{9}$$

where the angular brackets $\langle\ \rangle_n$ denote an average over the action generated from (1). All of the averages appearing in the inequality (3) are evaluated likewise.

For the operators A and C appearing in (3) we choose[20,21]

$$A_\alpha(\vec{k}) = \sum_{\beta=1}^{n} \int d^{d-1}x\, U_{\alpha\beta} e^{-i\vec{k}\cdot\vec{x}} f_\beta(x) , \tag{10a}$$

$$C_\alpha(\vec{k}) = \sum_{\beta=1}^{n} \int d^{d-1}x\, U_{\alpha\beta} e^{-i\vec{k}\cdot\vec{x}} P_\beta(x) , \tag{10b}$$

where $P_\alpha(x)$ is the momentum conjugate to $f_\alpha(x)$. The matrix $U_{\alpha\beta}$ is an orthogonal $n\times n$ matrix which diagonalizes a matrix of the form

$$\begin{bmatrix} a & b & b & \cdot & \cdot & \cdot & b \\ b & a & b & \cdot & \cdot & \cdot & b \\ b & b & a & \cdot & \cdot & \cdot & b \\ b & b & b & a & b & b & b \\ b & \cdot & \cdot & \cdot & a & b & b \\ b & \cdot & \cdot & \cdot & b & a & b \\ b & \cdot & \cdot & \cdot & b & b & a \end{bmatrix}$$

in replica space. Some useful properties of $U_{\alpha\beta}$ are

$$U_{1\alpha} = \frac{1}{\sqrt{n}} , \quad \sum_{\beta=1}^{n} U_{\alpha\beta} = \sqrt{n}\,\delta_{1\alpha} , \quad \sum_{\beta=1}^{n} U_{\alpha\beta} U_{\gamma\beta} = \delta_{\alpha\gamma} . \tag{11}$$

If we were to drop the factors $U_{\alpha\beta}$ in (10), we would obtain ultimately the useless inequality $1\geq 0$ from (3).

Using (10) and (11) we find that

$$\int \frac{d^{d-1}k}{(2\pi)^{d-1}} \sum_{\beta=1}^{n} \langle |A_\beta(\vec{k})|^2\rangle_n = n \int d^{d-1}x \langle f_\alpha^2(x)\rangle_n \tag{12a}$$

and

$$|\langle [C_\mu(\vec{k}), A_\mu(\vec{k})]\rangle_n|^2 = \left[\int d^{d-1}x(1)\right]^2 . \tag{12b}$$

For the denominator of the right-hand side of (3) we have

$$\langle [C_\mu(\vec{k}),[C_\mu^*(\vec{k}),H_{\text{eff}}]]\rangle_n = \sum_{\alpha,\beta=1}^{n} \int d^{d-1}x \int d^{d-1}x'\, U_{\mu\alpha} U_{\mu\beta} \left\langle \frac{\delta^2 H_{\text{eff}}}{\delta f_\alpha(x)\delta f_\beta(x')} \right\rangle_n e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} . \tag{13}$$

Using (1) and (11), we find from (13), with the use of replica symmetry,

$$\langle [C_\mu(\vec{k}),[C_\mu^*(\vec{k}),H_{\text{eff}}]]\rangle_n = \int d^{d-1}x \left[\sigma_0 \left\langle \frac{k^2}{\{1+[\nabla f_\mu(x)]^2\}^{1/2}} - \frac{[\vec{k}\cdot\vec{\nabla} f_\mu(x)]^2}{\{1+[\vec{\nabla} f_\mu(x)]^2\}^{3/2}} \right\rangle_n + \frac{2n\Delta}{T}(1-\delta_{\mu 1}) \left\langle \frac{d^2 g(y)}{dy^2}\bigg|_{y=f_\alpha(x)-f_\beta(x)} \right\rangle_n \right] . \tag{14}$$

The quantities in the angular brackets on the right-hand side of (14) are independent of the replica indices.

With the Pytte *et al.* choice of $g(y)=\ln(\cosh y)$, we have

$$\frac{d^2g(y)}{dy^2}=\mathrm{sech}^2 y\le 1 . \tag{15}$$

The least upper bound on (14) is then given by choosing $\vec{\nabla} f_\alpha(x)=0$ and $y=f_\alpha-f_\beta=0$ for all α and β. Thus the MWH procedure indicates that with the assumption of replica symmetry, it is the small-y regime of $g(y)$ which controls the important fluctuations. Therefore, from (14) we have

$$\langle [C_\mu(k),[C_\mu^*(\vec{k}),H_{\rm eff}]]\rangle_n \le \left[\int d^{d-1}x(1)\right]\left[\sigma_0 k^2+\frac{2n\Delta}{T}(1-\delta_{\mu 1})\right] . \tag{16}$$

Inverting (12) and (16) into (3) and performing the sum over μ, the integration over $\vec{k}$ and letting $n\to 0$ we obtain

$$w^2\ge T\int \frac{d^{d-1}k}{(2\pi)^{d-1}}\left[\frac{1}{\sigma_0 k^2}+\frac{\Delta}{T}\frac{1}{(\sigma_0 k^2)^2}\right] . \tag{17}$$

Thus, for large L we find

$$\frac{w}{L}\ge \Delta^{1/2}L^{(3-d)/2} \tag{18}$$

which is consistent with the renormalization-group result of Pytte *et al.* that $d_c=3$.

We now consider Grinstein and Ma's choice of $g(y)$. Placing a least upper bound on (14) is somewhat more difficult in this case since

$$\frac{d^2g(y)}{dy^2}=\frac{d^2}{dy^2}|y|=2\delta(y)\le\infty . \tag{19}$$

The infinite upper bound of (19) is an artifact of the continuum model which we have been using, where $f_\alpha(x)$ assumes all real values. We regularize this divergence by returning to the discrete interface model on a lattice with spacing a. The transcription of the coordinates, fields, and operators of the continuum model to their counterparts in the lattice model is as follows:

$$x\to x_i , \tag{20a}$$

$$f_\alpha(x)\to a f_\alpha(x_i) , \tag{20b}$$

$$\int d^{d-1}x\to\sum_i a^{d-1} , \tag{20c}$$

$$\frac{\delta}{\delta f_\alpha(x)}\to\frac{1}{a^{d-1}}\frac{\Delta_{f_\alpha(x_i)}}{a} , \tag{20d}$$

$$\frac{\delta}{\delta P(f_\alpha(x))}\to\frac{1}{a^{d-1}}\frac{\Delta_{P(f_\alpha(x_i))}}{a} , \tag{20e}$$

where x_i labels the lattice sites in the $(d-1)$-dimensional hyperplane parallel to the flat interface and $f_\alpha(x_i)$ is a dimensionless, integer-valued function. The forward difference operator $\Delta_{f_\alpha(x_i)}$ is defined by the relation

$$\Delta_{f_\alpha(x_i)}F(f_\beta(x_j)) =\delta_{\alpha\beta}\delta_{ij}[F(f_\beta(x_j)+1)-F(f_\beta(x_j))] , \tag{21}$$

where F is an arbitrary function of f_β. A similar equation holds for the difference operator associated with the conjugate momentum $P(f_\alpha(x_i))$. The operator $P(f_\alpha(x_i))$ generates translations and equals $\Delta_{f_\alpha(x_i)}$.

As we discussed in the Introduction, the MWH inequality can only be implemented when a continuous symmetry is present. In the continuum limit, the translational symmetry is continuous. When we consider the lattice model, we must restrict ourselves to the rough phase. This restriction arises naturally if we reconsider Mermin's derivation[18] of the MWH inequality from the Schwarz inequality. His derivation requires the use of the following relation:

$$e^{-H/T}\frac{\delta H}{\delta f_\alpha(x)}=-T\frac{\delta}{\delta f_\alpha}(e^{-H/T}) . \tag{22}$$

Equation (22) is certainly true if $f_\alpha(x)$ is a continuous field. However, for an integer-valued field $f_\alpha(x_i)$, the discrete analog of (22) where we replace the functional derivative by its lattice counterpart [see (20d)] is valid only if $1/T\,|\Delta_{f_\alpha(x_i)}H|\ll 1$. In the pure Ising model, if we write the exchange energy in the solid-on-solid form[25]

$$H_{\rm ex}=\sigma_0 a^{d-1}\sum_\alpha\sum_{\langle ij\rangle}|f_\alpha(x_i)-f_\alpha(x_j)| , \tag{23}$$

we find

$$\frac{1}{T}\Delta_{f_\alpha(x_i)}H_{\rm ex}=\frac{\sigma_0}{T}a^{d-1}\sum_{\hat{n}}\{\,|f_\alpha(x_i+\hat{n})-[f_\alpha(x_i)+1]\,| - |f_\alpha(x_i+\hat{n})-f_\alpha(x_i)|\,\} , \tag{24}$$

where the sum in (24) is over the z-nearest neighbors of x_i. The factors a^{d-1} appearing in (23) and (24) arise because σ_0 is the quantity defined in (1b); i.e., the surface tension in the continuum limit, which is finite as $a\to 0$ [recall (20c)]. The right-hand side of (24) will be small compared to unity if $\sigma_0 z a^{d-1}/T\ll 1$. For $a\sim 1$, this condition restricts the validity of (22) to high temperatures, i.e., the rough phase. As $a\to 0$, (22) becomes valid at progressively lower temperatures. For $a=0$ we recover the continuum limit where the validity of (22) extends to zero temperature.

When we include the lattice analog of the field energy (1c) of the Grinstein-Ma model in the quantity $1/T\,|\Delta_{f_\alpha(x_i)}H_{\rm eff}|$, we find that this quantity is small if

$$\frac{z}{T}\sigma_0 a^{d-1}+\frac{\Delta}{T^2}a^d(n-1)\ll 1 , \tag{25}$$

where Δ is the variance of the field distribution and is finite as $a\to 0$. The condition (25) is equivalent to requiring that $T/\sigma_0\gg a^{d-1}$ and $\Delta<T\sigma_0/a$. Thus our MWH inequality for the lattice model will be valid everywhere for small but finite a except at very low temperatures and

high Δ. This restriction should not be a major limitation of our result since Grinstein and Ma have shown that their ordered phase between two and three dimensions persists for some finite-temperature range where $T \ll \sigma_0$. The latter region is still within the realm of validity of our inequality since we can make a arbitrarily small.

Finally, assuming that we avoid the temperature region $T/\sigma_0 \ll a^{d-1}$ we can implement our MWH prescription straightforwardly. The calculations are similar to those used to obtain (17), and the transcriptions shown in (20) are used. The analog of Eq. (16) is

$$\langle [C_\mu(\vec{k}),[C_\mu^*(\vec{k}),H_{\rm eff}]]\rangle_n \le \left[\sum_i a^{d-1}\right]\left[\sigma_0 k^2+\frac{n\Delta}{Ta}(1-\delta_{\mu 1})\right]. \quad (26)$$

Note that the discrete analogy of (19) is

$$\frac{\Delta_y}{a}\frac{\Delta_y}{a}(a\,|y\,|)=\frac{|y+2|-2|y+1|+|y|}{a}\le\frac{2}{a} \quad (27)$$

for an integer-valued function y.

We insert (27) and the discrete analogs of Eqs. (12) into (3). After summing over μ and $\vec{k}$, dividing by n, and letting $n \to 0$, we obtain

$$w^2 \ge \frac{T}{N}\sum_k \left[\frac{1}{\sigma_0 k^2}+\frac{2}{a}\frac{\Delta}{T}\frac{1}{(\sigma_0 k^2)^2}\right], \quad (28)$$

where N is the number of sites in the $(d-1)$-dimensional hyperplane. Aside from the factor of $2/a$ multiplying Δ, Eq. (28) is identical to (17). Thus with the Grinstein-Ma form of $g(y)$ we obtain the same lower bound for w/L as we did using the energy of Pytte *et al.* The quantity of w/L is bounded as shown in (18) and as explained in the Introduction, it is inconsistent with the result of Grinstein and Ma. If we let $a \to 0$ in (28), then w should be infinite for all dimensions, which seems highly unlikely. As we noted following Eq. (19) the continuum limit seems pathological in the presence of the nonanalytic field energy proportional to $|f_\alpha(x)-f_\beta(x)|$. Thus we believe that a should be kept very small but finite. Further investigation of this point would be of interest.

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