Quenched-impurity influence on quantum critical behavior

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The effects of short-range-correlated impurities on the critical behavior of some quantum models with a free-energy spectrum $\sim k^\sigma$ $(0<\sigma\leq 2)$ are investigated by using the renormalization-group approach via the replica trick. At zero temperature, two nonsimultaneous expansion parameters $\epsilon_q = d_q - d$ ($d_q=\sigma$ or $\frac{3}{2}\sigma$) and $\epsilon = 2\sigma - d$ are involved in terms of which an unusual phase transition is predicted for $d<2\sigma$.

Recently, a number of investigations have appeared for quantum systems in the presence of quenched random fields which couple linearly to the order parameter.[1-4] At present, an analogous study about the influence on quantum critical behavior of quenched impurities described by a random field which couples quadratically to the order parameter is absent (see, however, Ref. 5). In this Communication we consider only the last problem on the basis of a quantum generalized model appropriate to a description of the interacting Bose gas,[3,4,6-12] the X-Y spin model in a transverse field,[11,13-15] and the structural phase transitions near the displacive limit.[16-18] Our approach is based on a renormalization-group (RG) treatment with the use of the replica trick.[3,19] Of course, an analogous investigation can be realized for other quantum models.[11,20] Here we limit ourselves to short-range-correlated quenched impurities. A detailed study also including the long-range case will be presented elsewhere.

We take for our discussion the functional representation

$$Z\{\phi\} = \int \mathscr{D}[\psi]\, e^{-\mathscr{H}\{\psi,\phi\}} \ , \tag{1}$$

where $\phi(\vec{x})$ is the random field which describes the quenched impurities, $Z\{\phi\}$ is the (grand canonical) partition function for a given $\{\phi\}$ configuration, and $\mathscr{H}\{\psi,\phi\}$ is the quantum generalized dimensionless functional

$$\mathscr{H}\{\psi,\phi\} = \sum_{j=1}^{n/2} \sum_{\substack{\vec{k},\omega_l \\ 0<|\vec{k}|<1}} [r_0 + k^\sigma + f(\omega_l)]\,|\psi^j(\vec{k},\omega_l)|^2 + \frac{1}{V^{1/2}} \sum_{j=1}^{n/2} \sum_{\substack{\vec{k}_1,\vec{k}_2,\omega_l \\ 0<|\vec{k}_i|<1}} \phi(\vec{k}_1-\vec{k}_2)\,\psi^{j*}(\vec{k}_1,\omega_l)\,\psi^j(\vec{k}_2,\omega_l)$$
$$+ \frac{u_0 T}{4V} \sum_{i,j=1}^{n/2} \sum_{\substack{\{\vec{k}_\nu,\omega_{l_\nu}\} \\ 0<|\vec{k}_\nu|<1}} \psi^{i*}(\vec{k}_1,\omega_{l_1})\,\psi^{j*}(\vec{k}_2,\omega_{l_2})\,\psi^i(\vec{k}_3,\omega_{l_3})\,\psi^j(\vec{k}_1+\vec{k}_2-\vec{k}_3,\omega_{l_1}+\omega_{l_2}-\omega_{l_3}) \ . \tag{2}$$

In (2), $\phi(\vec{k})$ is the Fourier transform of $\phi(\vec{x})$ and standard notations have been used.[3,4,8-12,17,18] The function $f(\omega_l)$ ($\omega_l = 2\pi l T$; $l=0, \pm1, \pm2, \ldots$) and the definition of the parameters r_0 and u_0 depend on the particular system under study.[11] Here we consider the two possibilities: $f(\omega_l) = -i\omega_l$ for bosonized systems[8] (e.g., Bose gas and X-Y model in a transverse field) and $F(\omega_l)=\omega_l^2$ for the structural phase transitions.[17,18] The differences in the definitions of the parameters are not relevant for our investigation.

We now assume that $\phi(\vec{k})$ is a Gaussian random variable satisfying the average relations

$$[\phi(\vec{k})]_{\rm av} = 0, \quad [\phi(\vec{k})\phi(\vec{k}')]_{\rm av} = \Delta_0 \delta_{\vec{k},-\vec{k}'} \ , \tag{3}$$

where, for definition, Δ_0 is a non-negative quantity and the square bracket $[\cdots]_{\rm av}$ indicates an average over possible impurity configurations. Then, by using the replica trick, the original problem is reduced to an "effective" one characterized by a functional $\mathscr{H}_{\rm eff}(\{\psi_\alpha\})$ of m replications $\{\psi_\alpha; \alpha=1,\ldots,m\}$ of the field ψ of the form

$$\mathscr{H}_{\rm eff}(\{\psi_\alpha\}) = \mathscr{H}_{\rm eff}^{(0)}(\{\psi_\alpha\}) + \mathscr{H}_{{\rm eff}\,I}^{(1)}(\{\psi_\alpha\}) + \mathscr{H}_{{\rm eff}\,I}^{(2)}(\{\psi_\alpha\}) \ , \tag{4}$$

where

$$\mathscr{H}_{\rm eff}^{(0)}(\{\psi_\alpha\}) = \sum_{\alpha=1}^{m} \sum_{j=0}^{n/2} \sum_{\substack{q \\ 0<|\vec{k}|<1}} [r_0 + k^\sigma + f(\omega_l)]\,|\psi_\alpha^j(q)|^2 \ , \tag{5}$$

$$\mathscr{H}_{{\rm eff}\,I}^{(1)}(\{\psi_\alpha\}) = \frac{u_0 T}{4V} \sum_{\alpha=1}^{m} \sum_{i,j=1}^{n/2} \sum_{\substack{\{q_\nu\} \\ 0<|\vec{k}_\nu|<1}} \delta_{q_1+q_2;q_3+q_4}\, \psi_\alpha^{i*}(q_1)\,\psi_\alpha^{j*}(q_2)\,\psi_\alpha^i(q_3)\,\psi_\alpha^j(q_4) \ , \tag{6}$$

$$\mathscr{H}_{{\rm eff}\,I}^{(2)}(\{\psi_\alpha\}) = -\frac{\Delta_0}{2V} \sum_{\alpha,\beta=1}^{m} \sum_{i,j=1}^{n/2} \sum_{\substack{\{q_\nu\} \\ 0<|\vec{k}_\nu|<1}} \delta_{\vec{k}_1+\vec{k}_2;\vec{k}_3+\vec{k}_4}\, \delta_{\omega_{l_1},\omega_{l_3}}\, \delta_{\omega_{l_2},\omega_{l_4}}\, \psi_\alpha^{i*}(q_1)\,\psi_\beta^{j*}(q_2)\,\psi_\alpha^i(q_3)\,\psi_\beta^j(q_4) \tag{7}$$

with $q \equiv (\vec{k}, \omega_l)$.

The expressions (4)–(7) clearly show that the effective action $\mathcal{H}_{\text{eff}}$ for the quantum problem, due to the presence of quenched impurities, is translationally invariant in the space but presents an anisotropy in the "timelike" direction. A similar situation has been studied by Boyanovsky and Cardy[5] for n-component classical magnets with perfect correlations in the disorder along ϵ_d-dimensional "lines" of impurities and no correlations in the other directions. As we shall see, for this peculiarity, the quantum systems under study appear to have a very strange behavior at $T=0$ according to the predictions of the RG approach.

The quantum RG procedure,[7–9,12,20] where also the frequencies are to be scaled as $\omega_l' = b^z \omega_l \rightarrow T' = b^z T$ ($b > 1$ is the spatial rescaling factor and z is the dynamical critical exponent), can now be applied to the "effective problem" and the RG equations will be obtained taking $m \rightarrow 0$ in the final results.

The T-dependent RG differential equations to second order in the coupling parameters are

$$\begin{aligned}
\frac{dr}{dl} &= \sigma r + \frac{n+2}{4} k_d u F_1(r,T) - K_d \frac{\Delta}{1+r} \ , \\
\frac{du}{dl} &= [(2\sigma - z) - d] u - \frac{K_d}{4} u^2 [(n+6) F_2(r,T) + 2F_3(r,T)] + 6K_d \frac{u\Delta}{(1+r)^2} \ , \\
\frac{d\Delta}{dl} &= (2\sigma - d)\Delta - \frac{n+2}{2} K_d u \Delta F_2(r,T) + 4K_d \frac{\Delta^2}{(1+r)^2} \ , \\
\frac{dT}{dl} &= zT \ ,
\end{aligned} \tag{8}$$

where $K_d = 2^{1-d} \pi^{-d/2} / \Gamma(d/2)$. In (8) we have assumed $\eta = 2 - \sigma$ for the exponent which enters the rescaling of the fields $\{\psi_\alpha\}$ and z is given by the equations

$$\left.\begin{matrix} e^{(\sigma - z)l} \\ e^{(\sigma - 2z)l} \end{matrix}\right\} \times \left[1 + \left\{ K_d \int_{e^{-1}}^{1} dp \frac{p^{d-1}}{(r+p^\sigma)^2} \right\} \Delta \right] = 1 , \quad l \rightarrow \infty \tag{9}$$

for bosonized systems and structural phase transitions, respectively. The exponents η and z have been determined as usual by imposing that in $\mathcal{H}'_{\text{eff}}(\{\psi'_\alpha\})$ the coefficients of k'^σ and of the term in ω_l' are identical to those in the original effective action. The functions $F_i(r,T)$ $(i=1,2,3)$ are given by

$$F_1(r,T) = \begin{cases} (e^{(1+r)/T} - 1)^{-1}, & f(\omega_l) = -i\omega_l \\ \frac{1}{2}(1+r)^{-1/2} \coth\left[\frac{1}{2T}(1+r)^{1/2}\right], & f(\omega_l) = \omega_l^2 \ , \end{cases} \tag{10}$$

$$F_2(r,T) = \begin{cases} \frac{1}{4T} \sinh^{-2}\left[\frac{1}{2T}(1+r)\right], & f(\omega_l) = -i\omega_l \\ \frac{1}{4}(1+r)^{-3/2} \coth\left[\frac{1}{2T}(1+r)^{1/2}\right] + \frac{1}{8T}(1+r)^{-1} \sinh^{-2}\left[\frac{1}{2T}(1+r)^{1/2}\right], & f(\omega_l) = \omega_l^2 \ , \end{cases} \tag{11}$$

$$F_3(r,T) = \begin{cases} \frac{1}{2}(1+r)^{-1} \coth\left[\frac{1}{2T}(1+r)\right], & f(\omega_l) = -i\omega_l \\ F_2(r,T), & f(\omega_l) = \omega_l^2 \ . \end{cases} \tag{12}$$

Of course, the physical region in the parameter space is defined by $u \geq 0$ and $\Delta \geq 0$.

For $T \neq 0$, since $T(l) \rightarrow \infty$ by iteration of the RG transformation, in terms of the new coupling parameter $v = uT$, Eqs. (8) reduce, in any case, to the ones for the corresponding classical n-vector model with quenched impurities appropriate to a discussion of the static properties to first order in $\epsilon = 2\sigma - d$.[21–23] However, in our quantum scheme, there is the additional possibility of having information also about the critical dynamics. For instance, for the Bose gas, from (9), it follows that

$$z = \sigma + K_{2\sigma} \Delta^* \tag{13}$$

at the fixed point. This is just the result derived in Refs. 22 and 24 for a classical system with quenched random impurities and nonconserved order parameter.

We now restrict ourselves to the quantum limit $T=0$.

Bosonized systems. In this case, Eqs. (8) reduce to

$$\begin{aligned}
\frac{dr}{dl} &= \sigma r - K_d \frac{\Delta}{1+r} \ , \\
\frac{du}{dl} &= [(2\sigma - z) - d] u - \frac{K_d}{4} \frac{u^2}{1+r} + 6K_d \frac{u\Delta}{(1+r)^2} \ , \\
\frac{d\Delta}{dl} &= (2\sigma - d)\Delta + 4K_d \frac{\Delta^2}{(1+r)^2} \ ,
\end{aligned} \tag{14}$$

where the number n of order-parameter components is absent. A peculiarity of Eqs. (14) is that, due to the anisotropy in the temporal direction, two alternative RG expansion parameters $\epsilon_q = (2\sigma - z) - d$ and $\epsilon = 2\sigma - d$ appear in the problem. This gives the possibility of having information about the $T=0$ behavior near the two borderline dimensions $d_q^*=2\sigma-z$ and $d^*=2\sigma$ to first order in ϵ_q and ϵ, respectively.

If one uses ϵ_q as the expansion parameter, Eqs. (14) have the Gaussian fixed point (GFP) $r^*=u^*=\Delta^*=0$ and the pure fixed point (PFP) $r^*=0$, $u^*=4\epsilon_q/K_d$, $\Delta^*=0$. In both cases we have, from (9), $z=\sigma$ so that $\epsilon_q=\sigma-d$ and $d_q^*=\sigma$. The corresponding eigenvalues of the linearized RG equations are ($\lambda_r^{(G)}=\sigma$, $\lambda_u^{(G)}=\epsilon_q$, $\lambda_\Delta^{(G)}=\epsilon_q+\sigma$) and ($\lambda_r^{(P)}=\sigma$, $\lambda_u^{(P)}=-\epsilon_q$, $\lambda_\Delta^{(P)}=\epsilon_q+\sigma$) for $|\epsilon_q|<<1$. From these it follows that (i) the GFP is "doubly" unstable with respect to u and Δ perturbations for $d<\sigma$ and "simply" unstable with respect to Δ for $d>\sigma$; and that (ii) the PFP is simply unstable with respect to Δ perturbation for $d<\sigma$ and doubly unstable with respect to u and Δ for $d>\sigma$. Thus, even if the PFP and the GFP are unstable near $d=\sigma$, they are characterized by the same degree of instability with respect to Δ perturbation for $d<\sigma$ and $d>\sigma$, respectively.

If we assume $\epsilon=2\sigma-d$ as expansion parameter, two fixed points of Eqs. (14) are found: the GFP with eigenvalues ($\lambda_r^{(G)}=\sigma$, $\lambda_u^{(G)}=\epsilon-\sigma$, $\lambda_\Delta^{(G)}=\epsilon$) and the random fixed point (RFP) $r^*=-1\epsilon/4\sigma$, $u_*=0$, $\Delta^*=-\epsilon/4K_d$ with eigenvalues $[\lambda_r^{(R)}=\sigma-\epsilon/4$, $\lambda_u^{(R)}=-(\sigma+\epsilon/4)$, $\lambda_\Delta^{(R)}=-\epsilon]$. Thus, (iii) the GFP is simply unstable with respect to Δ for $d<2\sigma$ and stable for $d>2\sigma$; (iv) the RFP is unphysical because it has a negative value of Δ^* for $\epsilon>0$, whereas for $\epsilon<0$ it lies in the physical region of parameter space, but it is unstable. Therefore this fixed point should be rejected and only the Gaussian one is to be taken into account.

Structural phase transitions. The ($T=0$) RG equations are

$$\frac{dr}{dl}=\sigma r+\frac{n+2}{8}K_d\frac{u}{(1+r)^{1/2}}-K_d\frac{\Delta}{1+r}\ ,$$

$$\frac{du}{dl}=[(2\sigma-z)-d]u-K_d\frac{n+8}{16}\frac{u^2}{(1+r)^{3/2}}+6K_d\frac{u\Delta}{(1+r)^2}\ ,$$

$$\frac{d\Delta}{dl}=(2\sigma-d)\Delta-\frac{n+2}{8}K_d\frac{u\Delta}{(1+r)^{3/2}}+4K_d\frac{\Delta^2}{(1+r)^2}\ . \tag{15}$$

Note that, in contrast to the bosonized case, n now enters the equations. Owing to the presence of two possible but unsimultaneous expansion parameters ϵ_q and ϵ also in the structural case, it is easy to see that the only acceptable fixed points of Eqs. (15) are the GFP and the PFP

$$r^*=-(2/\sigma)[(n+2)/(n+8)]\epsilon_q,$$

$$u^*=[16/K_d(n+8)]\epsilon_q,\ \ \Delta^*=0,$$

for $|\epsilon_q|=|\frac{3}{2}\sigma-d|<<1$ with eigenvalues ($\lambda_r^{(G)}=\sigma$, $\lambda_u^{(G)}=\epsilon_q$, $\lambda_\Delta^{(G)}=\epsilon_q+\sigma/2$) and $[\lambda_r^{(P)}=\sigma-(n+2)/(n+8)\epsilon_q$, $\lambda_u^{(P)}=-\epsilon_q$, $\lambda_\Delta^{(P)}=(4-n)/(n+8)\epsilon_q+\sigma/2]$, respectively; the GFP with eigenvalues ($\lambda_r^{(G)}=\sigma$, $\lambda_u^{(G)}=\epsilon-\sigma/2$, $\lambda_\Delta^{(G)}=\epsilon$) and the unphysical RFP (the same as for bosonized systems) for $|\epsilon|<<1$. It is now apparent that all the statements (i)–(iv) for the bosonized case are valid also for the structural one with $d_q^*=\sigma$ replaced by $d_q^*=\frac{3}{2}\sigma$ ($z=\frac{1}{2}\sigma$). We incidentally note that, surprisingly, whereas at $T\neq0$ two RFP's exist, in the quantum limit $T\to0$ only the unphysical RFP survives.

We now draw some physical conclusion from the previous $T=0$ RG analysis. It turns out that the randomness does not create stable fixed points for any of the studied quantum models at $T=0$ when $d<2\sigma$, as it should be whenever the transition is second order.[21] Thus we predict that the transition, associated with two different fixed points for $d<d_q^*$ and $d_q^*<d<2\sigma$ characterized by the same degree of instability, is not a sharp second-order transition. Only for $d>2\sigma$ is the behavior of random quantum systems governed by a stable GFP and we should have an ordinary Gaussian second-order transition with $\nu=1/\lambda_r^{(G)}=1/\sigma$, $\eta=2-\sigma$, and $z=\sigma$ or $\sigma/2$. It must be stressed that the runaway for $d<2\sigma$ in the parameter space carries out of the range where our approximations are valid and RG-independent calculations are needed to identify the nature of the transition. Superficially, due to the lack of stable fixed points, one might be tempted to conclude that, for $d<2\sigma$, the $T=0$ transition should be of first order. However, as pointed out by Aharony[25] and Lubensky[26] for classical systems, the runaway for random systems is of a fundamentally different nature than the runaway for the pure systems. Therefore, one should rather expect that the randomness induces a crossover from the pure second-order quantum transition to a "smeared or rounded" transition.[27] Of course, in contrast with the pure situation, any dimensional crossover[10–12,17] for $T\to0$ is now absent.

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