Symmetry breaking, Ward identities, and the two-fluid model

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We derive several important features of a Bose-condensed system, based on a diagrammatic analysis at finite temperatures. Firstly, the symmetry-breaking term necessary to describe Bose condensation is shown to give a rigorous basis for the fictitious gap argument of Gavoret and Nozières (1964). This puts their well-known results on a firm footing. Secondly, in the presence of a moving condensate with velocity v_S, the current carried by the noncondensate atoms is shown to be $(\rho_S - mn_o)v_S$. The results of this paper are direct consequences of various Ward identities which have been recently discussed for Bose-condensed systems.

I. INTRODUCTION

Recently, careful analysis of measurements[1] on the momentum distribution of atoms in superfluid ^{4}He have given reliable estimates of the condensate fraction n_0 (roughly 15% at $T=0$ K). However, equally strong evidence for the existence of a Bose condensate lies in the close connection between the latter and the characteristic superfluid properties of liquid ^{4}He, as first discussed with some generality by Hohenberg and Martin,[2] as well as Bogoliubov.[3] In this paper, we give rigorous proof of several important properties of Bose-condensed liquids. Our diagrammatic calculations show in a new and more transparent manner how the existence of the broken symmetry associated with a Bose condensate leads *directly* to a description consistent wtih two-fluid hydrodynamics.[4]

Our work is based on the infinite-order, finite-temperature field-theoretic analysis of Bose-condensed liquids using proper, irreducible diagrams. This approach is due to Ma and Woo[5] and has been extensively developed by Wong and Gould,[6] and others.[7,8] The great advantage of this approach over all other methods is that it ensures the coincidence of the single-particle spectrum and the density-fluctuation spectrum, which is probably the most important feature of a Bose-condensed system. We take care to include explicitly an infinitesimal symmetry-breaking term, which allows us to describe the Bose-condensed mode fully quantum mechanically. As we have recently emphasized,[8] this is crucial in the derivation of Ward identities in the long-wavelength, zero-frequency limit. These Ward identities are the basis of the present paper.

II. SYMMETRY BREAKING AND WARD IDENTITIES

We first give a brief summary of how one may write various correlation functions in terms of irreducible, proper contributions.[6,8,9] These results will be needed later.

The density and longitudinal-current correlation functions can be expressed in terms of their *irreducible* parts (denoted by an overbar) as follows:

$$\chi_{nn}(\vec{Q},\omega)=\frac{\bar{\chi}_{nn}(\vec{Q},\omega)}{\epsilon(\vec{Q},\omega)}\ , \tag{2.1}$$

$$\chi^{\ell}_{JJ}(\vec{Q},\omega)=\bar{\chi}^{\ell}_{JJ}(\vec{Q},\omega)+\bar{\chi}_{Jn}(\vec{Q},\omega)\frac{V(Q)}{\epsilon(\vec{Q},\omega)}\bar{\chi}_{Jn}(\vec{Q},\omega)\ , \tag{2.2}$$

where

$$\epsilon(\vec{Q},\omega)=1-V(Q)\bar{\chi}_{nn}(\vec{Q},\omega)\ . \tag{2.3}$$

Here, $V(Q)$ is the Fourier transform of the interatomic potential and $\bar{\chi}_{Jn}$ is the irreducible part of the longitudinal-current-density correlation function. The irreducible correlation functions can in turn be split into improper and proper parts,

$$\begin{aligned}\bar{\chi}_{nn}&=\bar{\Lambda}_\mu\bar{G}_{\mu\nu}\bar{\Lambda}_\nu+\bar{\chi}^R_{nn}\ ,\\ \bar{\chi}_{nJ}&=\bar{\Lambda}_\mu\bar{G}_{\mu\nu}\bar{\Lambda}^{\ell}_\nu+\bar{\chi}^R_{nJ}\ ,\\ \bar{\chi}^{\ell}_{JJ}&=\bar{\Lambda}^{\ell}_\mu\bar{G}_{\mu\nu}\bar{\Lambda}^{\ell}_\nu+\bar{\chi}^{\ell R}_{JJ}\ .\end{aligned} \tag{2.4}$$

Here, $\bar{G}_{\mu\nu}(\vec{Q},\omega)$ is the Beliaev 2×2 matrix Green's function defined in terms of irreducible self-energies $\bar{\Sigma}_{\mu\nu}(\vec{Q},\omega)$ $(\mu,\nu=+,-)$. The summation convention is assumed. The density $(\bar{\Lambda}_\mu)$ and the longitudinal-current $(\bar{\Lambda}^{\ell}_\mu)$ vertex functions are unique to a Bose-condensed system (they vanish if the condensate $n_o=0$).

The key role of the vertex functions is shown by splitting χ^{ℓ}_{JJ} directly into proper and improper parts.[2,10,11] Calculation shows that the proper part is given by

$$\chi^{\ell P}_{JJ}(Q,\omega)=\bar{\chi}^{\ell R}_{JJ}+\bar{\chi}^R_{Jn}\frac{V}{\epsilon^R}\bar{\chi}^R_{Jn}\ , \tag{2.5}$$

while the improper part is

$$\chi^{\ell C}_{JJ}(Q,\omega)=\left[\bar{\Lambda}^{\ell}_\mu+\bar{\Lambda}_\mu\frac{V}{\epsilon^R}\bar{\chi}^R_{Jn}\right]\mathscr{G}_{\mu\nu}\left[\bar{\Lambda}^{\ell}_\nu+\bar{\Lambda}_\nu\frac{V}{\epsilon^R}\bar{\chi}^R_{Jn}\right]\ , \tag{2.6}$$

where

$$\epsilon^R\equiv 1-V(Q)\bar{\chi}^R_{nn}\ ,$$

and $\mathscr{G}_{\mu\nu}$ is the full Beliaev matrix Green's function. Equation (2.6) shows explicitly how the vertex functions directly couple the longitudinal current into the single-particle fluctuations described by $\mathscr{G}_{\mu\nu}$. In view of this and the fact that[11,8]

$$\lim_{Q\to 0}\chi_{JJ}^{\ell P}(Q,\omega=0)=-\rho_N,\quad \lim_{Q\to 0}\chi_{JJ}^{\ell C}(Q,\omega=0)=-\rho_S\ , \tag{2.7}$$

it has been suggested[11,12] that Eq. (2.5) can be interpreted as the "normal-fluid" part, and Eq. (2.6) as the "superfluid" part. However, model calculations show[8] that this division does not have much physical usefulness in that *both* the proper and improper parts are strongly affected by the structure arising from ϵ^R.

To complete this summary of the dielectric formalism, we list some important symmetry properties

$$\begin{aligned}
&\overline{\Lambda}_{\mu}^{\ell}(Q,\omega)=-\overline{\Lambda}_{-\mu}^{\ell}(Q,-\omega)\ ,\\
&\overline{\Lambda}_{\mu}(Q,\omega)=\overline{\Lambda}_{\mu}(Q,-\omega)\ ,\\
&\overline{\chi}_{Jn}^{R}(Q,\omega)=-\overline{\chi}_{Jn}^{R}(Q,-\omega)=\overline{\chi}_{nJ}^{R}(Q,\omega)\ ,\\
&\mathscr{G}_{\mu\nu}(Q,\omega)=\mathscr{G}_{\nu\mu}(Q,\omega)=\mathscr{G}_{-\mu,-\nu}(Q,-\omega)\ .
\end{aligned} \tag{2.8}$$

We also recall the definitions of the longitudinal functions

$$\overline{\Lambda}_{\mu}^{\ell}(Q,\omega)\equiv\frac{Q_i}{Q}\overline{\Lambda}_{\mu}^{i},\quad \overline{\chi}_{Jn}^{R}(Q,\omega)\equiv\frac{Q_i}{Q}\overline{\chi}_{J_in}^{R}\ , \tag{2.9}$$

where the correlation functions $\overline{\Lambda}_{\mu}^{i}$ and $\overline{\chi}_{J_in}^{R}$ involve the ith component of the momentum current.

It is generally accepted that the easiest way of dealing with Bose condensation is to introduce into the Hamiltonian a symmetry-breaking term[2,3,8]

$$-\xi(N_o)^{1/2}[\hat{a}_0+\hat{a}_0^{\dagger}-2(N_0)^{1/2}]\ , \tag{2.10}$$

where ξ is an infinitesimal positive energy. This allows one to treat Bose condensation in the usual sense,

$$\langle\hat{a}_0\rangle=(N_0)^{1/2}\neq 0\ , \tag{2.11}$$

but at the same time allows us to also keep the operator nature of $\hat{a}_0$ and $\hat{a}_0^{\dagger}$ which enables us to take into consideration all nonmacroscopic fluctuations of the condensate. As a result, the equation of continuity is found only to be modified by terms of order ξ. The equation of continuity is no longer satisfied in the usual treatments,[6,10] where the condensed mode is treated as a classical c number [i.e., if one makes the usual substitution $\hat{a}_0,\hat{a}_0^{\dagger}\to(N_0)^{1/2}$]. In this approximation, the equation of continuity must be imposed in an *ad hoc* manner.

As discussed elsewhere,[6,8] one can derive a whole series of exact Ward identities relating to various correlation functions, which are direct consequences of the equation of continuity together with the broken-symmetry condition in Eq. (2.11). Two of these identities are

$$\omega\overline{\Lambda}_{\mu}=\frac{Q}{m}\overline{\Lambda}_{\mu}^{\ell}+(n_0)^{1/2}\beta_{\nu}\overline{G}_{\nu\mu}^{-1}+\xi(n_0)^{1/2}\beta_{\mu}\ , \tag{2.12}$$

$$\omega\overline{\chi}_{Jn}^{R}=\frac{Q}{m}(\overline{\chi}_{JJ}^{\ell R}+\rho)-(n_0)^{1/2}\beta_{\mu}\overline{\Lambda}_{\mu}^{\ell}\ , \tag{2.13}$$

where $\beta_{\nu}\equiv\operatorname{sgn}\nu$. We assume that the chemical potential $\overline{\mu}$ takes on its physical value, as determined by the condition (2.11).

For $\vec{Q}=0$ and $\omega=0$, Eq. (2.12) reduces to

$$(n_o)^{1/2}\beta_{\nu}\overline{G}_{\nu\mu}^{-1}+\xi(n_0)^{1/2}\beta_{\mu}=0\ ,$$

or equivalently,[8]

$$\overline{\Sigma}_{++}(0,0)-\overline{\Sigma}_{+-}(0,0)=\Sigma_{++}(0,0)-\Sigma_{+-}(0,0)=\overline{\mu}+\xi\ . \tag{2.14}$$

This corresponds to the well-known Hugenholtz-Pines relation. However, the fact that the difference between the diagonal and off-diagonal Beliaev self-energies is given by $\overline{\mu}+\xi$ instead of $\overline{\mu}$ immediately implies[3,13] that there will be an infinitesimal energy gap in the excitation spectrum of $\mathscr{G}_{\mu\nu}$ at $\vec{Q}=0$ (as well as the other correlation functions χ_{nn} and χ_{JJ}^{ℓ}, since these share the same poles).

The preceding result calls to mind the work of Gavoret and Nozières (GN),[10] who were able to derive a number of results for a Bose liquid at $T=0$ K, to all orders in perturbation theory. Their analysis, however, was plagued with various infrared divergences, which they removed by introducing an energy gap Δ at $(\vec{Q},\omega)=0$. While GN believed (see p. 369 of Ref. 10) that the correct procedure should be $\Delta\to 0$ and *then* $(Q,\omega)\to 0$, reversing the order of these limits gave well-defined results without any divergences. All self-energies and vertex functions could be expanded in powers of Q and ω in a well-defined way and GN were able to calculate the long-wavelength poles of various correlation functions. They proved the following (at $T=0$ K).

(a) The lowest excited state is longitudinal, i.e., a density fluctuation. This means that the transverse-current correlation function χ_{JJ}^{t} $(Q=0,\omega=0)$ vanishes, and so does the normal-fluid density ρ_N [this follows from Eq. (3.1)].

(b) This long-wavelength fluctuation is a phonon with the isothermal speed of sound, and is a pole of the longitudinal-current correlation function as well as the single-particle Green's functions.

However, GN were not happy with their *ad hoc* treatment of the infinitesimal energy gap Δ. What we have shown here (see also pp. 67 and 68 of Ref. 3) is that the gap GN introduced is *not* fictitious, but is an inevitable consequence of the symmetry breaking, which, in one way or the other, one must introduce into the Hamiltonian if one is to allow for Bose condensation.[14] The *correct* limiting procedure is precisely that used by GN, i.e., $(Q,\omega)\to 0$ *before* $\Delta\to 0$. Beginning with GN themselves, this procedure has been viewed as a major weakness of their analysis (for a recent example of such criticism, see Ref. 15). Our present work gives a firm footing to the infinite-order perturbation results of Ref. 10. Moreover, it casts some doubt on the results of Nepomnyashchii and Nepomnyashchii,[15,16] who argued that the removal of the infrared divergence required that $\overline{\Sigma}_{+-}(Q=0,\omega=0)=0$. For a further discussion of Ref. 15, we refer to Chap. 6 of Ref. 17.

III. TWO-FLUID MODEL

Around 1963, Martin as well as Pines and Nozières emphasized[12] that one of the most fundamental ways of defining the normal-fluid density ρ_N arising in the two-fluid hydrodynamic description was in terms of the static *transverse*-current—current correlation function

$$\lim_{Q\to 0}\chi^{t}_{JJ}(Q,\omega=0)=-\rho_N \ . \tag{3.1}$$

With the use of the dielectric formalism,[17] χ^{t}_{JJ} can be shown to involve the same diagrams as the regular part $(\overline{\chi}^{\,\ell R}_{JJ})$ of the longitudinal-current—current correlation function, and thus one has a diagrammatic proof that Eq. (3.1) is equivalent to[18]

$$\lim_{Q\to 0}\overline{\chi}^{\,\ell R}_{JJ}(Q,\omega=0)=-\rho_N \ . \tag{3.2}$$

The usefulness of this last result has been emphasized recently by the authors.[11,8] In the present section we shall combine Eq. (3.2) with the zero-frequency limit of the Ward identity in Eq. (2.13) to show that the total mass current density associated with a moving condensate (of velocity $v_S=Q/m$) is given by

$$\langle \vec{J}\,\rangle_Q=\rho_S\vec{v}_S \ , \tag{3.3}$$

where the superfluid density is $\rho_S\equiv\rho-\rho_N$. This is a key relation of the two-fluid model of Landau.

Before proceeding, we should comment briefly on Eq. (3.1). As Pines and Nozières[19] have discussed with great clarity, Eq. (3.1) is based on considering a superfluid at rest and describing its linear response to an external transverse probe. The response will involve the elementary excitations of the system in which the condensation is into the $\vec{p}=0$ state. This approach is essentially perturbative and is less general than one which allows for completely different forms of the condensate structure, e.g., those which are only stable in the presence of rotation, such as vortex formation. The ensuing analysis is rigorous within this limitation of characterizing superfluidity in terms of response functions. This means we are restricted to describing superfluid flow in which the condensate undergoes uniform translation.

We first show that the zero-frequency vertex function $\overline{\Lambda}^{\ell}_{\mu}(Q,\omega=0)$ is closely related to superfluid flow. Consider ^{4}He atoms condensed into a state with momentum

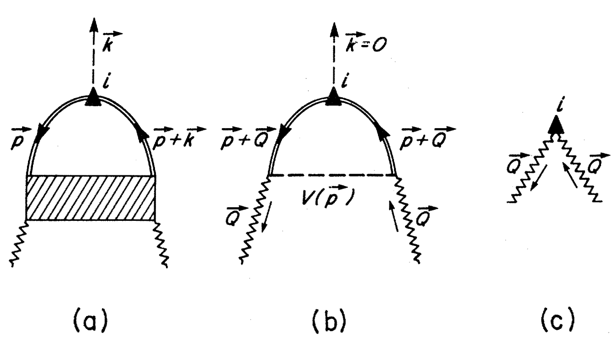

FIG. 1. (a) Diagrams which contribute to the ith component of the current $\tilde{J}^{\,i}_{k}$ carried by noncondensate atoms. The double lines represent the single particle propagator when the condensate atoms have momentum $\vec{Q}$. The jagged lines are the condensate-atom propagators. (b) Simplest diagram which contributes to $\langle \tilde{J}^{\,i}_{o}\rangle_Q$. (c) Diagram which yields the current carried by the condensate atoms.

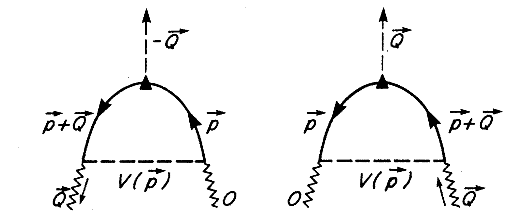

FIG. 2. Lowest-order diagrams for the excited-atom current $\langle \tilde{J}^{\,i}_{o}\rangle_Q$ to first order in Q. Lines represent the single-particle propagators when the condensate is stationary.

$\vec{Q}=m\vec{v}_S$. This will require a symmetry-breaking term

$$-\xi(n_0)^{1/2}\int d\vec{r}\,[\hat{\psi}(\vec{r})e^{-i\vec{Q}\cdot\vec{r}}+\hat{\psi}^{\dagger}(\vec{r})e^{i\vec{Q}\cdot\vec{r}}-2(n_0)^{1/2}] \ , \tag{3.4}$$

such that the condensate mode is now described by [compared with Eq. (2.11)]

$$\langle\hat{\psi}(\vec{r})\rangle_Q\equiv\psi_0(\vec{r})=(n_0)^{1/2}e^{i\vec{Q}\cdot\vec{r}} \ . \tag{3.5}$$

For the case of a moving condensate described by Eq. (3.5), the diagrams have exactly the same Feynman rules as for a stationary condensate, except that now all condensate lines have momentum $\vec{Q}$. The condensate mode has an associated current density given by

$$\vec{J}_C(\vec{r})=-\frac{i}{2}(\langle\hat{\psi}^*\rangle_Q\langle\vec{\nabla}\hat{\psi}\rangle_Q-\langle\vec{\nabla}\hat{\psi}^*\rangle_Q\langle\hat{\psi}\rangle_Q)=n_0\vec{Q} \ . \tag{3.6}$$

The atoms not in the condensate (i.e., with $\vec{p}\neq\vec{Q}$) also contribute to the current density according to

$$\langle\tilde{\vec{J}}(\vec{r})\rangle_Q=-\frac{i}{2}\langle\tilde{\psi}^*(\vec{\nabla}\tilde{\psi})-(\vec{\nabla}\tilde{\psi}^*)\tilde{\psi}\rangle_Q \ , \tag{3.7}$$

where $\tilde{\psi}(r)\equiv\hat{\psi}(\vec{r})-\langle\hat{\psi}(\vec{r})\rangle_Q$. We can write Eq. (3.7) as

$$\langle\tilde{J}^{\,i}(r)\rangle_Q=\frac{1}{\Omega}\sum_{\vec{k}}e^{i\vec{k}\cdot\vec{r}}\langle\tilde{J}^{\,i}_{k}\rangle_Q \ , \tag{3.8}$$

where $\langle\tilde{J}^{\,i}_{k}\rangle_Q$ is shown in Fig. 1(a). Each vertex still conserves momentum and thus $\langle\tilde{J}^{\,i}_{k}\rangle_Q=0$ unless $\vec{k}=0$. The incoming (or outgoing) momentum of the condensate lines

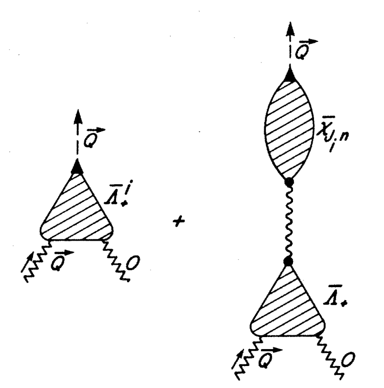

FIG. 3. Diagrammatic representation of Eq. (3.10). Solid circles represent a density vertex (n) and solid triangles represent a current vertex (J).

changes the momentum of the other lines from the case of the condensate at rest. An example of this is shown in Fig. 1(b).

To evaluate the diagrams for $\langle \tilde{J}^i_0 \rangle_Q$ to lowest order in Q, we let all the condensate lines have zero momentum except for one.[10] (If all condensate lines had zero momentum, we would simply obtain the excited-atom current in a system with a stationary condensate, which vanishes.) Enumerating all such diagrams (a simple example is shown in Fig. 2), we find that the result can be expressed as

$$\langle \tilde{J}^i_0 \rangle_Q = \langle \tilde{J}^i_Q \rangle_0 + \langle \tilde{J}^i_{-Q} \rangle_0 , \tag{3.9}$$

to lowest order in Q. Thus, we only need to evaluate the Qth Fourier component of the excited-atom current in a frame of reference in which the condensate is stationary, except for one condensate propagator (see Fig. 2). Diagrammatically one finds (see Fig. 3)

$$\langle \tilde{J}^i_0 \rangle_Q = (n_0)^{1/2} \left[\overline{\Lambda}^i_+(\vec{Q},0) - (n_0)^{1/2}\frac{Q_i}{2} + \overline{\chi}^R_{J_i n}(\vec{Q},0)\frac{V(Q)}{\epsilon^R(Q,0)}\overline{\Lambda}_+(\vec{Q},0) + \overline{\Lambda}^i_-(-\vec{Q},0) - (n_0)^{1/2}\frac{Q_i}{2} + \overline{\chi}^R_{J_i n}(-\vec{Q},0)\frac{V(Q)}{\epsilon^R(Q,0)}\overline{\Lambda}_-(-\vec{Q},0) \right] , \tag{3.10}$$

where the functions on the right-hand side are evaluated for a condensate at rest. The quantity $(n_0)^{1/2}Q_i/2$ has been subtracted out from $\overline{\Lambda}^i_\mu(\vec{Q},0)$, because the diagram shown in Fig. 1(c) corresponds to the condensate current $\vec{J}_C$ given in Eq. (3.6). Taking into account the symmetries given in Eq. (2.8), one sees that $\overline{\chi}^R_{Jn}(\vec{Q},\omega=0)=0$, and hence Eq. (3.10) reduces to

$$\langle \tilde{J}^i_0 \rangle_Q = 2(n_0)^{1/2}\overline{\Lambda}^i_+(Q,0) - n_0 Q_i . \tag{3.11}$$

We emphasize that Eq. (3.11) is exact to lowest order in Q.

We next recall that the zero-frequency limit of the Ward identity in Eq. (2.13) gives

$$\overline{\chi}^{\ell R}_{JJ}(\vec{Q},\omega=0) = -\rho + (n_0)^{1/2}\frac{2m}{Q}\overline{\Lambda}^\ell_+(\vec{Q},\omega=0) . \tag{3.12}$$

Combining this with the long-wavelength result in Eq. (3.2), we obtain[8]

$$\lim_{Q\to 0}\overline{\Lambda}^i_+(Q,\omega=0) = \frac{Q_i}{2m(n_0)^{1/2}}(\rho-\rho_N) . \tag{3.13}$$

Using this rigorous result in Eq. (3.11), we find

$$\langle \tilde{J}^i_0 \rangle_Q = (\tilde{n}m - \rho_N)\frac{Q_i}{m} , \tag{3.14}$$

where the density of excited atoms is $\tilde{n} \equiv n - n_0$. Clearly the sum of this excited-atom current and the condensate-atom current in Eq. (3.6) yields Eq. (3.3) since

$$\langle \vec{J} \rangle = n_0 m \vec{v}_S + (\tilde{n}m - \rho_N)\vec{v}_S = (\rho - \rho_N)\vec{v}_S . \tag{3.15}$$

Recalling that[10] $\rho_N = 0$ at $T=0$ K, the result in Eq. (3.15) proves that the lowest-energy eigenstate of a Bose liquid with a moving condensate corresponds to the *entire* liquid moving uniformly with velocity v_S.

Our derivation of Eq. (3.14) shows very clearly that it is through the interaction-dependent part of the vertex function $\overline{\Lambda}^\ell_\mu(\vec{Q},\omega=0)$ that the condensate atoms of momentum $\vec{Q}=m\vec{v}_S$ cause the noncondensate atoms to contribute to the total current. We also recall from Eq. (2.6) that it was $\overline{\Lambda}^\ell_\mu(\vec{Q},\omega)$ which determined the characteristic features of the current response function unique to a Bose-condensed system. In the work of Bogoliubov,[3] and Hohenberg and Martin,[2] ρ_S was effectively defined through the expression (3.3), and then it was shown that with this definition, one was led to Eq. (3.1). We emphasize that our derivation of Eqs. (3.14) and (3.15) is quite general and does *not* require a simple quasiparticle description to be valid. Within the quasiparticle picture, of course, it is a straightforward exercise[20] to derive Eq. (3.15) with ρ_N given by Landau's quasiparticle formula.

ACKNOWLEDGMENTS

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New York, 1965), p. 61 (see especially Table IV).

[13]By expanding the denominator of the single-particle matrix Green's function $\mathscr{G}_{\mu\nu}$ in powers of ω and Q, at $T=0$ K the energy gap is found to be of the order $(2\xi m c_I^2 N_o/N)^{1/2}$, where c_I is the isothermal sound velocity.

[14]This symmetry-breaking term is necessary for perturbation theory and statistical mechanics to be valid (Refs. 2 and 3). As the gap becomes infinitesimal $(\Delta \rightarrow 0+)$, the region in Q space where the acoustic dispersion relation $\omega_Q = c_I Q$ is *not* valid becomes an arbitrarily small region near $\vec{Q}=0$.

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