Fully-spin-polarized ^{3}He

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The properties of fully-spin-polarized ^{3}He (^{3}He$^\uparrow$) are calculated from first principles within the Galitskii-Feynman T-matrix and Hartree-Fock approximation with the use of the HFDHE2 pair potential of Aziz *et al.* The ground-state energy agrees well with variational calculations, the Landau parameters with model calculations, and the single-particle energy with results expected from nuclear matter. Although the total pair interaction in ^{3}He$^\uparrow$ is not weaker than that in normal ^{3}He it appears to be dominated by the direct part via the pair potential and by the Pauli exclusion repulsion with induced interactions playing a minor role. Since the effective mass is ≈ 1 and the Landau parameters are small, ^{3}He$^\uparrow$ should respond nearly like an ideal Pauli paramagnet close to full polarization.

I. INTRODUCTION

Spin-polarized quantum systems form an exciting new area of condensed-matter physics.[1] Electron spin-polarized hydrogen, which is a Bose system, has received much of the attention.[2] There is a parallel interest in spin-polarized Fermi systems, both theoretically[3–27] and experimentally.[28–33] Examples are nuclear spin-polarized pure ^{3}He, of interest both in the gas[17,18,30,31] and liquid[3–14,28–29] phases, dilute solutions of ^{3}He in liquid ^{4}He,[15,16,32] and electron spin-polarized deuterium D$^\downarrow$, which may have a variety of nuclear spin states.[19–27,33] Much of the theoretical work on spin-polarized Fermi liquids has focused on fully-spin-polarized liquid ^{3}He (Refs. 6–13) which we denote here by ^{3}He$^\uparrow$.

In this paper, we calculate the properties of ^{3}He$^\uparrow$ within a Brueckner-Hartree-Fock (BHF) approximation.[34–37] The aim is twofold. The first is to evaluate the properties of liquid ^{3}He$^\uparrow$ from first principles beginning with the pair potential between He atoms. For the pair potential we use the HFDHE2 potential developed by Aziz *et al.*,[38] which presently provides the best overall description of gaseous helium. The second is to test how well a BHF theory can describe a fully spin-polarized Fermi fluid such as ^{3}He$^\uparrow$.

In fact, we use the less-well-known Galitskii-Feynman-Hartree-Fock (GFHF) theory[39] in which the Galitskii-Feynman (GF) T matrix[40] replaces the Brueckner T matrix.[34] The difference between them is that the GF T matrix includes scattering to both intermediate particle (P) and hole (H) states symmetrically when two particles interact. The Brueckner T matrix allows scattering to particle states only. While the numerical difference between the two is not large, it is important to include scattering to both P and H states in order to describe some properties qualitatively correctly,[41] such as the single-particle lifetime near the Fermi surface. Also the GF T matrix fits naturally into Green-function theory which has provided the microscopic foundation[42] of Fermi-liquid theory. We test the GFHF theory by comparing predicted results with the results of variational,[6] correlated basis function[12,13] (CBF) and model calculations.[10] The present work also represents the first application of BHF or GFHF theory to a spin-polarized Fermi liquid. A preliminary report of this work using the pair potential developed by Beck[43] has been reported.

To see why a GFHF theory may describe ^{3}He$^\uparrow$ well, we turn to the effective interaction in normal ^{3}He. This effective interaction has three strong components of approximately equal importance.[44,45] Firstly, there is the strong short-range repulsion between pairs of atoms arising from the steeply repulsive hard core of the bare pair potential. Secondly, there is repulsion between like spin particles due to the Pauli exclusion principle. In normal ^{3}He this operates between only one-half of the particles. Thirdly, there are the molecular-field-like interactions between pairs induced via the excitations in the fluid: the density and spin-density fluctuations.

Particularly, the interaction induced via the spin-density fluctuations seems most important in determining the values of the Landau parameters and the effective mass m^* in normal ^{3}He. The GFHF theory takes account of the first and second components but ignores the third component entirely; that is, it describes pair interactions via the bare interaction and the pair Fermi statistics and ignores any many-body terms, which may be viewed as induced interactions.

In ^{3}He$^\uparrow$ all spins are aligned so that spin fluctuations are "frozen out." This part of the induced interaction operating via spin fluctuations is therefore absent in ^{3}He$^\uparrow$. Also, with all spins aligned, the Pauli exclusion principle operates between all spins. The Fermi statistic correlations therefore increase in relative importance. Since the wavelength in ^{3}He$^\uparrow$ below 1 K is ~ 10 Å, this correlation operates beyond nearest neighbors and, as Lhuillier and Levesque[6] point out, may simulate many-body, long-range

correlations introducing an almost crystallinelike order. The remaining interaction induced via density correlations may be relatively much less important. Since GFHF theory takes account of the short-range correlations induced via the hard core and the Fermi statistics, it may provide a reasonable description of ^{3}He$^\uparrow$.

In the next section we outline the elements of the present GFHF theory. In Sec. III we present results for the ground-state energy, inverse compressibility, Landau parameters, and the effective mass. These results are discussed in Sec. IV.

II. THEORY BACKGROUND

A. GFHF Approximation

In the GFHF theory we begin with N free ^{3}He atoms (fermions) in a box of volume Ω, $\Omega_0=\Omega/N=n^{-1}$. Using the standard Green-function[39] method, we evaluate the single-particle self-energy Σ to first order in the interaction between the atoms. This leads to the Hartree-Fock (HF) approximation which incorporates Fermi statistics. Next, we evaluate contributions to Σ from a series of interaction diagrams, the ladder diagrams, which account for the interaction between pairs of atoms, particularly via the hard core of the pair potential, to all orders. The contribution of the ladder diagrams to Σ can be included by replacing the pair potential in the Hartree-Fock (HF) approximation by the Galiskii-Feynman T matrix. All higher-order interactions, chiefly three- and four-body interactions are ignored.

Several choices of the single-particle energy (SPE) are possible. We choose it as the energy in the single-particle Green function $G_1(1,\omega_1)$;

$$\epsilon(1,\omega_1)=k_1^2/2m+\Sigma(1,\omega_1)\ , \tag{1}$$

where[46]

$$\Sigma(1,\omega_1)=\frac{-i}{\Omega}\sum_{k_2,\sigma_2}\int\frac{d\omega_2}{2\pi}G_1^{\rm HF}(k_2,\omega_2)\Gamma^{\sigma_1\sigma_2}(k_{12},\omega_1+\omega_2), \tag{2}$$

and

$$\Gamma^{\sigma_1\sigma_2}=\Gamma(12,12)-\delta_{\sigma_1\sigma_2}\Gamma(21,12) \tag{3}$$

is the usual HF interaction. Here

$$\Gamma(12,34)=U_0(1-3)+\frac{1}{\Omega}\sum_{k_{56}}U_0(1-5)G_2^{\rm HF}(56,E_{12})\times\Gamma(56,34)\ , \tag{4}$$

is the GF T matrix. It describes the interaction between a pair of particles scattering from initial momentum states 1 and 2 to final momentum states 3 and 4 via intermediate states (either both particle states or both hole states) 5 and 6. U_0 is the Fourier transfer of the pair potential. Γ depends on the initial total energy of the pair, E_{12}, via the Fourier transform of the two-body Green function,

$$G_2^{\rm HF}(56,E_{12})=\frac{(1-n_5)(1-n_6)}{E_{12}-\epsilon_5-\epsilon_6+i\eta}-\frac{n_5n_6}{E_{12}-\epsilon_5-\epsilon_6-i\eta}\ , \tag{5}$$

where $\epsilon_5=\epsilon(5,\omega_5)$, $n_5=\Theta(\mu-\epsilon_5)$, and η is a small positive constant. In (1) only "on-energy-shell" values of Γ, $E_{12}=\epsilon_1+\epsilon_2$, are needed. The first term in G_2 represents scattering to two-particle intermediate states above the Fermi surface (μ) and the second term to two-hole intermediate states within the Fermi sea. In BHF theory, only the first term in G_2 is retained. Also, we use a continuous SPE given by (1) at all momentum in Γ, which is natural in the Green-function method, whereas in BHF theory a single-particle (SP) spectrum having a gap at k_F is normally used.[36,37] A continuous ϵ is needed to characterize excitations near k_F correctly.[37]

In ^{3}He$^\uparrow$ with all spins aligned ($\uparrow$) we have only the spin-triplet interaction $\Gamma^{\uparrow\uparrow}=\Gamma(12,12)-\Gamma(12,21)$. Since the spin-triplet interaction is symmetric in spin space, it must be antisymmetric in configuration space. Hence, if we expand $\Gamma^{\uparrow\uparrow}$ in partial wave components, this expansion can contain only odd angular momentum (L) components, i.e.,

$$\Gamma^{\uparrow\uparrow}=2\sum_{\substack{L\\ \text{odd}}}(2L+1)\Gamma_L\equiv 2a_0\ . \tag{6}$$

By contrast in normal ^{3}He, where both spin states ($\uparrow$ and $\downarrow$) are identical and equally weighted, the interaction in (1) would be

$$\sum_{\sigma_2}\Gamma^{\uparrow\sigma_2}=\Gamma^{\uparrow\uparrow}+\Gamma^{\uparrow\downarrow}\equiv 2\Gamma_s =\tfrac{1}{2}(3a_0+a_e)\ , \tag{7}$$

where Γ_s is the usual[44] spin-symmetric interaction and a_e is the corresponding sum in (6) over even L components. In ^{3}He$^\uparrow$, $k_F^\uparrow=(6\pi^2n)^{1/3}$, $\epsilon_F^\uparrow=\hbar^2k_F^2/2M$, $\epsilon_F^{*\uparrow}=\epsilon_F^\uparrow/m^{*\uparrow}$, and the density of single spin states per unit volume at the Fermi surface is

$$\left[\frac{dn}{d\epsilon}\right]^\uparrow=\frac{1}{2}\frac{m^{*\uparrow}k_F^\uparrow}{\pi^2\hbar^2}=\frac{3}{2\epsilon_F^{*\uparrow}\Omega_0}\ . \tag{8}$$

where $m^{*\uparrow}$ is the effective mass to be determined theoretically. Otherwise the GFHF theory in ^{3}He$^\uparrow$ is identical to that in normal ^{3}He.[47] Since Γ depends upon ϵ and ϵ depends upon Γ, the T matrix and SPE must be solved iteratively until self-consistent. We began the iteration using the free SPE $\epsilon_1=k_1^2/2m$ in Γ.

B. Self-energy

The integration over ω_2 along the real axis in (2) may be evaluated via contour integration. For this we need to know the analytic properties of the one-body Green function

$$G_1^{\rm HF}(k_2,\omega_2)=\frac{1-n_2}{\omega_2-\epsilon_2+i\delta}+\frac{n_2}{\omega_2-\epsilon_2-i\delta}$$

and of Γ. $G_1^{\rm HF}$ has a pole above the real axis at $\omega_2=\epsilon_2+i\delta$ in the hole (n_2) term and a pole below the real

axis at $\omega_2=\epsilon_2-i\delta$ in the particle $(1-n_2)$ term. By formally iterating Γ in (4) we can show that Γ has the same analytic properties as the full two-body Green function G_2. A Lehmann representation of G_2 shows that it has a line of poles (cut) just above the real axis for $\omega_1+\omega_2<2\mu$ (corresponding to two-hole excitations) and a line of poles (cut) just below the real axis (corresponding to two-particle excitations).

We choose a contour to avoid the cuts in G_2 (Wick rotation) as shown in Fig. 1 and consider the integral $(\omega_2\to z)$

$$I_c=-i\int\frac{d^3z}{2\pi}G_1(k,z)\Gamma^{\uparrow\uparrow}(k_{12},\omega_1+z)$$

around this contour. Specifically, the path down the imaginary axis must be set to that $\omega_1+z=2\mu$ (or $z=2\mu-\omega_1$) to avoid the cuts. The pole in G_1 at $z=\epsilon_2+i\delta$ will lie within the contour if $\epsilon_2>2\mu-\omega_1$ [residue proportional to $\Theta(2\mu-\omega_1-\epsilon_2)$] and that at $z=\epsilon_2-i\delta$ will lie within the contour if $\epsilon_2<2\mu-\omega_1$ [residue proportional to $1-\Theta(2\mu-\omega_1-\epsilon_2)$]. Direct integration then gives[48,49]

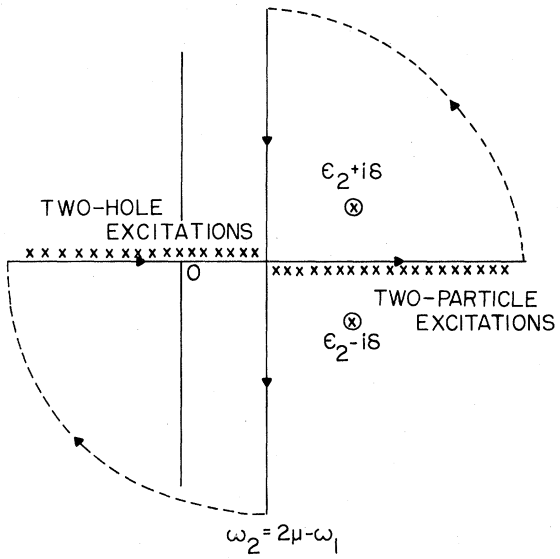

FIG. 1. Contour used to evaluate integral over ω_2 in the self-energy.

$$\Sigma(1,\omega_1)=\frac{1}{\Omega}\sum_{k_2}\left[[\Theta(\mu-\epsilon_2)-\Theta(2\mu-\omega_1-\epsilon_2)]\Gamma^{\uparrow\uparrow}(k_{12},\omega_1+\epsilon_2)\right.$$
$$\left.+\int_{-\infty}^{\infty}\frac{dy}{2\pi}\left[\frac{1-\Theta(\mu-\epsilon_2)}{iy+2\mu-\omega_1-\epsilon_2+i\delta}+\frac{\Theta(\mu-\epsilon_2)}{iy+2\mu-\omega_1-\epsilon_2-i\delta}\right]\Gamma^{\uparrow\uparrow}(k_{12},2\mu+iy)\right]. \quad (9)$$

The first term comes from the residues at the poles in G_1 and the second term from the integration down the imaginary axis ($z=x+iy$). Here $\Gamma^{\uparrow\uparrow}$ depends upon the relative momentum $k_{12}=\frac{1}{2}(\vec{k}_1-\vec{k}_2)$, the c.m. momentum $P=\vec{k}_1+\vec{k}_2$ and the energies as shown.

The first term in (9) has both a real and imaginary part proportional to the real and imaginary parts of $\Gamma^{\uparrow\uparrow}$. The second term can be shown to be purely real. Hence ImΣ comes entirely from the first term of (9),

$$\mathrm{Im}\Sigma(1,\omega_1)=\frac{1}{\Omega}\sum_{k_2}[\Theta(\mu-\epsilon_2)-\Theta(2\mu-\omega_1-\epsilon_2)]$$
$$\times\mathrm{Im}\Gamma^{\uparrow\uparrow}(k_{12},\omega_1+\epsilon_2)\,, \quad (10)$$

while the real part has a contribution from both terms. ImΣ clearly vanishes at $\omega_1=\mu$ and to obtain this behavior it is necessary to retain the contribution from both the particle and hole terms of G_1. In conventional BHF theory only the first term of (10) is usually retained.

We have not been able to evaluate the second term of Σ satisfactorily. Therefore for ReΣ we return to the usual BHF expression. This may be obtained[46] from the GFHF Σ in (1) or (9) by ignoring the hole contribution to $\Gamma^{\uparrow\uparrow}$ and choosing a contour in ω_2 closed entirely in the upper half-plane. We then capture only the hole term of G_1 giving

$$\mathrm{Re}\Sigma(1,\omega_1)=\frac{1}{\Omega}\sum_{k_2}\Theta(\mu-\epsilon_2)\mathrm{Re}\Gamma^{\uparrow\uparrow}(k_{12},\omega_1+\epsilon_2)\,. \quad (11)$$

The real and imaginary parts of Σ were evaluated directly from (10) and (11), respectively.

The above Re$\epsilon(1,\epsilon_1)$ is identical to that obtained from $[n_1=\Theta(\mu-\epsilon_1)]$

$$\mathrm{Re}\epsilon(1,\epsilon_1)=\frac{\delta E}{\delta n_1}\,,$$

where

$$E=\sum_1\frac{k_1^2}{2m}+\frac{1}{\Omega}\sum_{1,2}\Gamma^{\uparrow\uparrow}(k_{12},\epsilon_1+\epsilon_2)n_1n_2 \quad (12)$$

is the GFHF ground-state energy, but rearrangement terms in the differentiation are ignored. In normal ^{3}He we found[47] the rearrangement terms made a negligible contribution to ϵ when the GF T matrix was used.

III. RESULTS

A. Single-particle and ground-state energies

In Fig. 2 we show the SPE $\epsilon(k_1,\epsilon_1)$ in ^{3}He$^\uparrow$ at volume $V=35.1$ cm^3/mol, calculated by using the GF T matrix in (1), (10), and (11). This is the final self-consistent SPE obtained by iterating Eqs. (1), (4), and (5) for the T matrix and ϵ. The ϵ in Fig. 2 gives a ground-state energy (GSE) (12) in ^{3}He$^\uparrow$ of $E=-1.46$ K. For comparison we show some SPE spectra in normal ^{3}He in Fig. 3.

From Fig. 2 we see Reϵ suggests strong binding at $k=0$, which rises rapidly to Re$\epsilon(k_F)\approx-3.7$ K at the Fermi surface. If the Hugenholtz–van Hove equality were satisfied identically, we would have Re$\epsilon(k_F)=E$. The

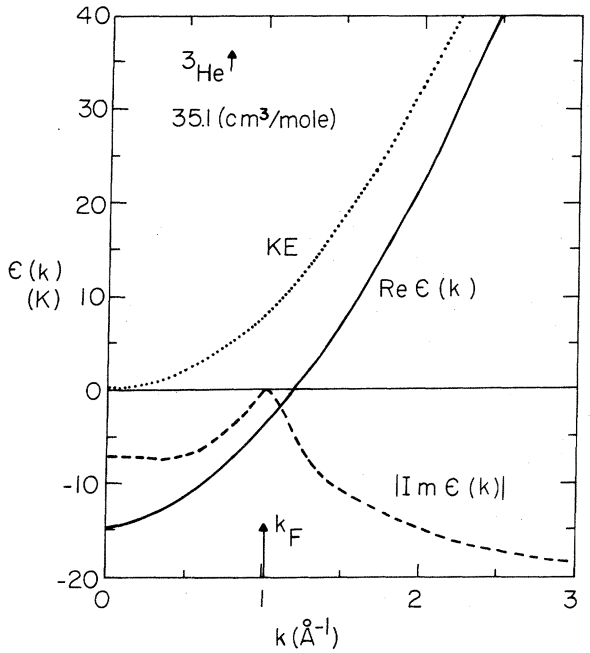

FIG. 2. Single-particle energy spectrum $\epsilon(k)$ in ^{3}He$^\uparrow$ at $\Omega=35.1$ cm^3/mol.

difference between $\mathrm{Re}\epsilon(k_F)$ and $E=-1.46$ K indicates outstanding contributions to E or to Re $\epsilon(k)$ not included in the GFHF approximation or errors in the calculation.

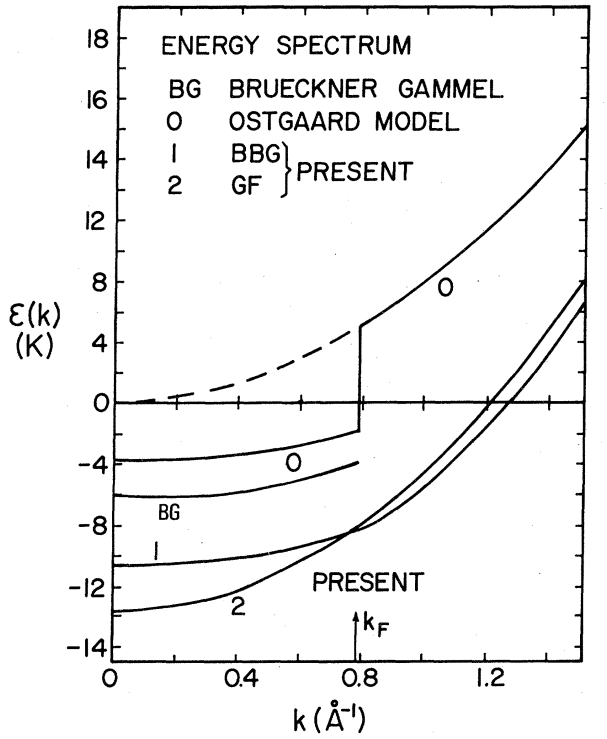

FIG. 3. SPE spectra in normal ^{3}He at $\Omega=36.83$ cm^3/mol: GF is $\epsilon(k)$ calculated using the present Galitskii-Feynman T matrix, BBG is calculated using the present T matrix but retaining only particle intermediate states (denoted the Brueckner-Bethe-Goldstone approximation), BG is the Brueckner and Gammel (Ref. 34) self-consistent $\epsilon(k)$, and 0 is Østgaard's model spectrum (Ref. 35).

Since $\mathrm{Re}\epsilon(k_F)$ is changing so rapidly with k near k_F high precision in interactions is required to get $\mathrm{Re}\epsilon(k_F)$ precisely (within ± 0.2 K). Outstanding contributions are more likely to change $\mathrm{Re}\epsilon(k)$ rather than E significantly.

Comparing the Reϵ in Fig. 2 with the corresponding GF Reϵ in normal ^{3}He shown in Fig. 3, we see that at $k=0$ the two $\epsilon(k)$ are effectively the same. At $k\simeq 1$ Å^{-1} (near $k_F^\uparrow$), however, $\epsilon(k)$ is significantly higher in ^{3}He$^\uparrow$ than in normal ^{3}He.

To identify the difference in the interaction between a pair of atoms in ^{3}He and ^{3}He$^\uparrow$ in the GFHF model we show the partial wave components $\Gamma_L(k,k)$ of the T-matrix pair interaction in Fig. 4 for $L=0$–3 in ^{3}He. The components $\Gamma_L(k,k)$ will be the same in ^{3}He$^\uparrow$ and ^{3}He except for a scaling of k_F. The first difference between ^{3}He and ^{3}He$^\uparrow$ is that only the odd-L components exist in the spin triplet interaction $\Gamma^{\uparrow\uparrow}=2a_0$ in ^{3}He$^\uparrow$ [see Eq. (6)]. However, the odd-L components also dominate the spin-symmetric interaction $\Gamma^s=\frac{1}{2}(3a_0+a_e)$ in ^{3}He which enters the self-energy Σ_1. Hence the absence of even components in ^{3}He$^\uparrow$ makes less difference to Σ than might be anticipated at first sight.

Secondly, the $\Gamma_L(k,k)$ appear in Σ heavily weighted by a density of states proportional to k^2. This means that $\Gamma_L(k,k)$ at high k is emphasized. Particularly, the s-wave Γ_0 shown in Fig. 4, which is strongly negative at low k and appears to be important, actually makes rather little contribution to $\epsilon(k)=k^2/2m+\Sigma_1(k)$ in ^{3}He. Thus $\epsilon(k)$ is dominated by Γ_1 in both cases; as has been understood for normal ^{3}He a long time.[34,35] The dominant difference appears to be the absence of Γ_2 in ^{3}He$^\uparrow$ and, in Figs. 2 and 3, this will be noticed in $\epsilon(k)$ at higher k near k_F, where $\epsilon(k)$ in ^{3}He$^\uparrow$ is higher than in normal ^{3}He. This appears to account for the higher GSE in ^{3}He$^\uparrow$ than in ^{3}He, at least in the GFHF picture.

In Fig. 2, we see that Im$\epsilon(k)$ vanishes at $k=k_F$, as it should, but rapidly becomes large away from k_F. This suggests the quasiparticle lifetime away from the Fermi surface rapidly becomes short; i.e., a $\mathrm{Im}\epsilon(k)\approx 10$ K corresponds to a lifetime $\sim 4\times 10^{-12}$ sec. The Im$\epsilon(k)$ we find is substantially larger than that for ^{3}He$^\uparrow$ obtained by Krotscheck *et al.*,[12] who used the CBF method. This is

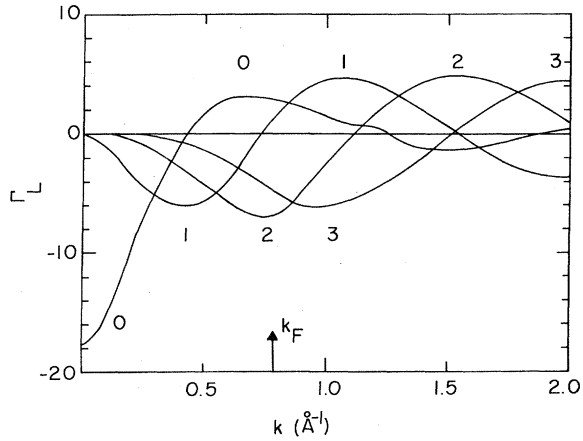

FIG. 4. Partial wave components of the T matrix, $L=0$–3 in normal ^{3}He at $\Omega=36.83$ cm^3/mol; $(dn/d\epsilon)\Gamma_L$.

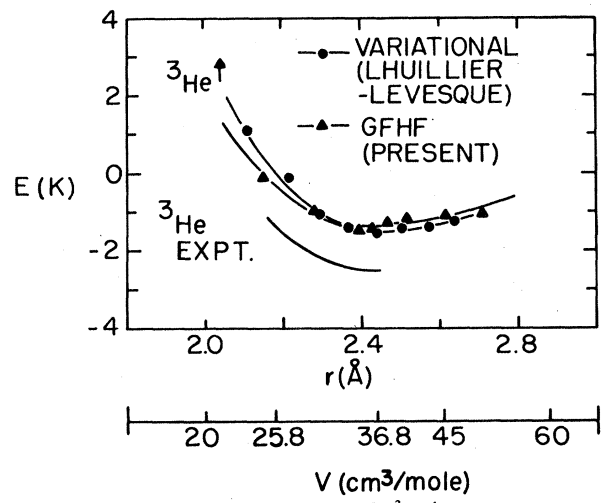

FIG. 5. Ground-state energy of ^{3}He↑: GFHF, present Galitskii-Feynman-Hartree-Fock approximation; variational, indicates the variational results of Lhuillier and Levesque (Ref. 6); ^{3}He expt., observed valued for normal ^{3}He (Ref. 50).

the chief disagreement between the CBF and GFHF approaches; otherwide the predicted results of the two methods agree well. The Im$\epsilon(k)$ in Fig. 2 agrees very well in form [i.e., Im$\epsilon(k)$ rises rapidly away from k_F] with that deduced from experiment in nuclei.[37,49]

In Fig. 5 we show the GSE of ^{3}He↑ calculated in the GFHF approximation using the self-consistent $\Gamma^{\uparrow\uparrow}$ along with the variational values obtained by Lhuillier and Levesque[6] (LL). Between 25 and 50 cm^3/mole, the GFHF energy (given in K) is well described by

$$E=-1.45+9.0x^2-14.5x^3\,, \tag{13}$$

where $x=(\Omega-\Omega_s)/\Omega_s$ and $\Omega_s=35.7$ cm^3/mol. This gives a minimum energy of $E_0=-1.45\pm0.05$ K at a saturation volume $\Omega_s=35.7\pm0.8$ cm^3/mol$=59.25$ Å^3/atom ($n=16.9\times10^{-3}$ Å^{-3}), while LL find a minimum of $E_0=-1.56\pm0.06$ K at a saturation volume of $\Omega_s=37.9\pm0.8$ cm^3/mol$=63.0$ Å^3/atom. The present GFHF E_0 can be lowered ~0.2 K if we set Im$\epsilon(k)=0$, as is usually done in nuclear matter calculations, throughout the iterations. Similarly, if we keep only the contributions to Im$\epsilon(k)$ from the hole states ($k<k_F$) only, as was done in previous GFHF calculations,[11] E_0 is shifted by ~0.1 K. Clearly, the agreement between the present GFHF and the variational GSE results of LL, which both use the HFDHE2 potential of Aziz *et al.*,[38] is good. In a preliminary report,[11] we used the Beck potential and found an $E_0=-1.2$ K at $\Omega_s=35.1$ cm^3/mol for ^{3}He↑. We believe the potential of Aziz *et al.* does give a lower GSE for ^{3}He↑ outside the combined errors in the preliminary[11] and present calculations.

The fit (13) gives an inverse compressibility (in units of K)

$$(n\kappa)^{-1}=\Omega^2\frac{\partial^2E}{\partial\Omega^2}=18 \tag{14}$$

at saturation and a first-sound velocity $c_1=(mn\kappa)^{-1/2}=220$ m/sec. Since κ varies a great deal along the GSE curve, κ is not precise ($\pm15\%$). This $(n\kappa)^{-1}$ agrees well the value $(n\kappa)^{-1}=17.4$ obtained by LL. Previously,[11] using the Beck potential, we obtained $(n\kappa)^{-1}=24$ K and the difference between this and (14) reflects the precision to which $(n\kappa)^{-1}$ is determined. The inverse compressibility $(n\kappa)^{-1}$ in ^{3}He is, however, significantly greater than that observed in normal ^{3}He [$(n\kappa)^{-1}=12.1$ K].

The fit (13) suggests $(n\kappa)^{-1}$ in ^{3}He↑ does not increase as rapidly with density as it does in ^{3}He. Also from (13) the calculated pressure at $\Omega_0=25$ cm^3/mol, where solid ^{3}He is expected, is only ~15 atm whereas the corresponding solidification pressure in ^{3}He is 34 atm. However, the pressure and $(n\kappa)^{-1}$ at $\Omega_0\approx25$ cm^3/mol are not well determined. Since liquid ^{3}He↑ cannot exist for $\Omega_0>\Omega_s$, the predicted existence range is $25<\Omega_0<36$ cm^3/mol, much the same as for normal liquid ^{3}He.

B. Landau parameters

The Landau parameters represent the interaction between two particles in momentum states 1 and 2 on the Fermi surface. In ^{3}He↑ only spin-triplet Landau parameters, $F_L^{\uparrow\uparrow}$, exist. In lowest-order approximation, these can be obtained directly from the GF $\Gamma^{\uparrow\uparrow}$ by using the standard[34,35] result:

$$F_L^{\uparrow\uparrow}=\left[\frac{dn}{d\epsilon}\right]^{\uparrow}\frac{2L+1}{2}\int_0^{\pi}d\theta\sin\theta P_L(\cos\theta)\Gamma^{\uparrow\uparrow}(\theta)\,. \tag{15}$$

Since the Landau parameters are "forward scattering" interactions, the diagonal $\Gamma^{\uparrow\uparrow}(k_1k_2,k_1k_2)$ ($k_3=k_1$, $k_4=k_2$) is used in (15). With $|k_1|=|k_2|=k_F$, this diagonal $\Gamma^{\uparrow\uparrow}$ depends only upon the angle θ between $\vec{k}_1$ and $\vec{k}_2$ given by $\vec{k}=\frac{1}{2}(\vec{k}_1-\vec{k}_2)=k_F\sin(\theta/2)$ with c.m. momentum $\vec{P}=\vec{k}_1+\vec{k}_2=2k_F\cos(\theta/2)$. The resulting $F_L^{\uparrow\uparrow}$ at three volumes are listed in Table I along with the effective mass $m^{*\uparrow}=(1+F_1^{\uparrow\uparrow}/3)$.

The $F_L^{\uparrow\uparrow}$ in Table I are generally a factor of 10 smaller than the F_L in ^{3}He, for two reasons. Firstly, because $k_F^{\uparrow}$ is somewhat larger and $m^{*\uparrow}$ is considerably smaller, the Fer-

TABLE I. Lowest-order Landau parameters calculated directly from the Galitskii-Feynman T matrix using Eq. (15).

Volume (cm^3/mol)	$F_0^{\uparrow\uparrow}$	$F_1^{\uparrow\uparrow}$	$F_2^{\uparrow\uparrow}$	$F_3^{\uparrow\uparrow}$	$F_4^{\uparrow\uparrow}$	$F_5^{\uparrow\uparrow}$	$m^{*\uparrow}$
30	−0.46	−0.55	−0.12	0.63	0.56	0.11	0.82
35	−0.61	−0.49	0.08	0.57	0.50	0.07	0.84
40	−0.71	−0.41	0.19	0.56	0.43	0.06	0.86

TABLE II. Properties of ^{3}He$^\uparrow$ at saturation: $m^{*\uparrow}=(1+F_1^{\uparrow\uparrow}/3)$; $F_0^{\uparrow\uparrow}$ calculated from the GSE using $(\kappa n)^{-1}=(2\epsilon_F^{*\uparrow}/3)(1+F_0^{\uparrow\uparrow})$, the Fermi velocity $v_F=k_F/m^{*\uparrow}$ and the first- (c_1) and zero- (c_0) sound velocities.

Ω (cm^3/mol)	$m^{*\uparrow}$	$F_0^{\uparrow\uparrow}$	$v_F^{*\uparrow}$ (m/sec)	$c_0^{\uparrow}$ (m/sec)	$c_0^{\uparrow}$ (m/sec)	$\left[\frac{dn}{d\epsilon}\right]^{\uparrow}$ (K Å^3)$^{-1}$
35.7	0.84	1.8	255	220	270	0.0027

mi energy $\epsilon_F^{*\uparrow}=\epsilon_F^{0\uparrow}/m^{*\uparrow}$ is much larger in ^{3}He$^\uparrow$. The normalizing density of states $(dn/d\epsilon)^\uparrow$ in (8) is then a factor of 5 smaller in ^{3}He$^\uparrow$ [e.g., $(dn/d\epsilon)^\uparrow=0.002\,65$ (K Å^3)$^{-1}$ compared with $(dn/d\epsilon)=0.015$ (K Å^3)$^{-1}$ in ^{3}He, both at saturation). This reduces the overall magnitude of all the $F_L^{\uparrow\uparrow}$. Also the calculated $\Gamma^{\uparrow\uparrow}(\theta)$ are smaller in ^{3}He$^\uparrow$.

In Landau theory,[44] the Landau parameters are defined as

$$f_{12}=\frac{\delta^2 E}{\delta n_1 \delta n_2}$$

where E is the total energy [$F=(dn/d\epsilon)f$]. If we use the GFHF energy (12) for E and differentiate only the explicit values of n_1 and n_2 in (12) we obtain the relation (15). We can readily obtain a higher order value of $F_0^{\uparrow\uparrow}$ by fully differentiating the GFHF E. In this case $F_0^{\uparrow\uparrow}$ is related to the GFHF ground-state compressiblity by

$$n^{-1}\Omega\left[\frac{\partial^2 E}{\partial\Omega^2}\right]=(n\kappa)^{-1}=(\tfrac{2}{3}\epsilon_F^{*\uparrow})(1+F_0^{\uparrow\uparrow})\ . \tag{16}$$

The value of $F_0^{\uparrow\uparrow}$ obtained from (16) includes the contribution resulting from the dependence of $\Gamma^{\uparrow\uparrow}$ on particle occupation. These contributions are often called rearrangement terms because they reflect the dependence of the interaction on the rearrangement of particle occupation when the density is changed. The difference between $F_0^{\uparrow\uparrow}$ obtained from (15) and from (16) is a measure of the interaction between a pair of particles induced via density changes[51,52] (and fluctuations).

In Table II we list $F_0^{\uparrow\uparrow}$ calculated from (16). Comparing with Table I we see the rearrangement contributions make $F_0^{\uparrow\uparrow}$ positive, but relative to ^{3}He ($F_0^s=10.07$) (Ref. 44) $F_0^{\uparrow\uparrow}$ remains small. Thus we expect the induced interactions via density excitations to be much smaller in ^{3}He$^\uparrow$ than in ^{3}He. The higher-order value of $F_0^{\uparrow\uparrow}$ in Table II agrees well with the value $F_0^{\uparrow\uparrow}=1.82$ obtained by Bedell and Quader.[10]

The Landau parameters in Table I are approximately twice the values we quoted previously[11] using the Beck potential. This is because we made an error by a factor of 2 in the density of states in our previous report.

The most dramatic diffrerence between ^{3}He and ^{3}He$^\uparrow$ is the effective mass $m^{*\uparrow}$, which we find is $m^{*\uparrow}=0.84$ at saturation. This sets the scale for most other properties. Also $m^{*\uparrow}$ tends to decrease slightly with increasing density while in ^{3}He m^* increases dramatically with density. The present value at saturation agrees well with the value $m^{*\uparrow}=0.82$ obtained by Bedell and Quader,[10] who used a model of both the direct and indirect interactions in ^{3}He$^\uparrow$, which worked well[53] in normal ^{3}He. Using the CBF method, Krotschek *et al.* calculated a total effective mass of $m^{*\uparrow}\approx 0.9$ at $\Omega=36.3$ cm^3/mol. On this basis, there is broad agreement that $m^{*\uparrow}$ is close to or somewhat less than 1 in ^{3}He$^\uparrow$.

C. Effective-mass enhancement

We may also obtain the effective mass from the real part of the SPE as[42,52]

$$m^*(k,E)=\left[\frac{M}{\hbar^2 k}\left[\frac{\partial\epsilon}{\partial k}\right]\right]^{-1}$$
$$=\left[1-\left[\frac{\partial\Sigma}{\partial E}\right]_k\right]\Big/\left[1+\left[\frac{\partial\Sigma}{\partial T_k}\right]_E\right], \tag{17}$$

where $T_k=\hbar^2k^2/2M$. The $m^*(k,E)$ is often[36,37] separated into a "k mass," $m^*(k)=[1+(\partial\Sigma/\partial T_k)_E]^{-1}$ and an "E mass" $m^*(E)=[1-(\partial\Sigma/\partial E)_k]$ with $m^*(k,E)=m^*(k)m^*(E)$. In Fig. 6 we show $m^*(k)$ and the total $m^*(k,E)$. The $m^*(k)$ at $k=k_F$ agrees well with the $m^{*\uparrow}=(1+F_1^{\uparrow\uparrow}/3)$. When the energy dependence of $\Sigma(k,E)$ is included an enhancement of $m^*(k,E)$ which is roughly independent of k, having a maximum value of $m^*(k,E)=1.5$, is obtained. This enhancement is much smaller than is found in normal ^{3}He.[47,54,55] Also, since spin fluctuations are not possible in ^{3}He$^\uparrow$ the enhancement obtained in the present model must come from density fluctuations. In this case, the enhancement shown in Fig. 5 is larger than might be expected.

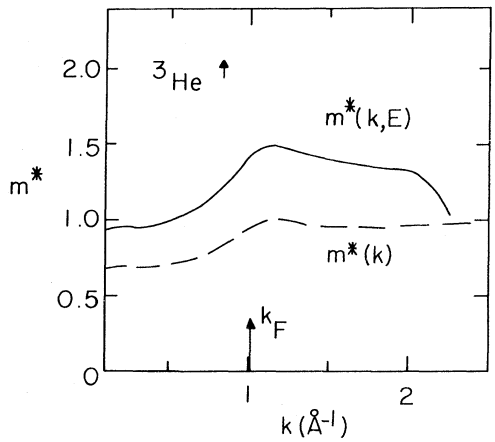

FIG. 6. The "k mass," $m^*(k)$, and the total effective mass $m^*(k,E)=m^*(k)m^*(E)$, given by Eq. (17) in ^{3}He$^\uparrow$ at $\Omega=35$ cm^3/mol.

Blaizot and Friman,[49] who studied m^* in nuclear matter, show that the GFHF approximation can overestimate the enhancement of $m^*(k,E)$ near k_F if the potential is sufficiently repulsive. For these reasons, while the present model does predict some enhancement of $m^*(k,E)$, its magnitude remains to be clarified. A large enhancement of $m^*(k,E)$ near k_F is expected in normal ^{3}He (Refs. 47, 54, and 55) and this enhancement has been proposed as an explanation of the large change in the specific heat with temperature observed in normal ^{3}He.

Finally, using $F_0^{\uparrow\uparrow}=1.8$ and $m^{*\uparrow}=0.84$, we predict a zero-sound velocity of $c_0\approx 270$ m/sec, while using $m^{*\uparrow}=1.5$ we find this velocity increases to $c_0=350$ m/sec. In this range of $F_0^{\uparrow\uparrow}$ values c_0 is very sensitive to the value of $m^{*\uparrow}$. The c_0 in ^{3}He$^\uparrow$ is, in any case, predicted to be substantially greater than c_0 in ^{3}He, due chiefly to the large value of the Fermi velocity in ^{3}He$^\uparrow$.

IV. DISCUSSION

The present results represent the first application of GFHF theory to a spin-polarized Fermi system. The good agreement of the GSE with variational results and of $m^{*\uparrow}$ with other values suggests the GFHF theory is a much better approximation for polarized than for unpolarized ^{3}He. Why should the GFHF theory work better in ^{3}He$^\uparrow$ than in normal ^{3}He?

The GFHF theory may be viewed as the first- (and second-) order approximation in the hole-line expansion.[56] This is roughly an expansion in the density, but identifying the expansion parameter κ precisely is difficult.[37,57] If κ is given by the "wound integral" we would expect this to be substantially the same in ^{3}He$^\uparrow$ and normal ^{3}He since κ is dominated by the hard-core radius relative to the interparticle spacing, which is the same in the two cases. An estimate[37] of κ is $\kappa=1-m^*(E)^{-1}$, where $m^*(E)$ is the "E mass" at $\kappa\approx 0.75\ k_F$. From Fig. 6 this gives $\kappa\approx 0.3$ in ^{3}He$^\uparrow$. In nuclear matter estimates of κ range from $\kappa\approx 0.15$–0.25 and there three- and four-body terms are important.[57] From this view we would not expect the GFHF theory to work well in ^{3}He$^\uparrow$. We believe, however, that the long-range Fermi statistical correlation due to the Pauli principle, which operates between all pairs in ^{3}He, simulate (and therefore reduce the need for additional) three-body and higher correlations. For example, Lhuillier and Levesque[6] find that adding three-body correlations to their Jastrow function in ^{3}He$^\uparrow$ does not lead to a significant lowering of the GSE, whereas a significant lowering is found in normal ^{3}He.

As noted above the total interaction between particles may be separated into a direct part, a statistical correlation and an induced part.[51] The latter represents the component induced via spin fluctuations and density excitations.[44,45] The spin fluctuations do not exist in ^{3}He$^\uparrow$. The difference between $F_0^{\uparrow\uparrow}$ shown in Tables I and II provides a measure of the interaction induced via density fluctuations. The $F_0^{\uparrow\uparrow}$ in Table I, obtained directly from the T matrix via (15), represents the $F_0^{\uparrow\uparrow}$ due to the direct term (strictly the T-matrix approximation to the direct term). The $F_0^{\uparrow\uparrow}$ in Table II includes the induced component to second order (or equivalently the rearrangement terms). In normal ^{3}He the first[35,47] F_0^s is $F_0^s\approx -8$ and the second[35] is $F_0^s\approx 7$. The corresponding difference in ^{3}He$^\uparrow$ is $F_0^{\uparrow\uparrow}=-0.6$ and $F_0^{\uparrow\uparrow}=1.8$. Clearly, while induced contributions are not negligible in ^{3}He$^\uparrow$, they are much smaller than in normal ^{3}He.

If induced interactions are relatively small in ^{3}He$^\uparrow$ this leaves the direct interaction and statistical correlations. If the T matrix is a good approximation to the direct part, which it should be, then we would expect the GFHF theory to work well in ^{3}He$^\uparrow$. It would be interesting to test this for other spin-polarized Fermi systems such as D$^\downarrow$.

The most recent variational[14] results which include three-body and momentum-dependent correlations find $E=-2.1$ K at $n=16.2\times 10^{-3}$ Å^{-3}. Most recent CBF (Ref. 13) and variational[14] calculations also now demonstrate that the GSE of normal ^{3}He lies below that of ^{3}He$^\uparrow$. While this is also true in the GFHF theory, the GFHF GSE in normal ^{3}He is quite unreliable. There also seems a general consensus that $m^{*\uparrow}$ in ^{3}He$^\uparrow$ is approximately 1 or slightly below 1. The present results and those of LL (Ref. 6) find the inverse compressibility of ^{3}He$^\uparrow$ greater than that of normal ^{3}He. This additional stiffness comes from the Pauli principle which, as pointed out by LL, tends to localized the atoms as if on a lattice. Since $m^{*\uparrow}\approx 1$ and all the Landau parameters are small, ^{3}He$^\uparrow$ acts approximately like an ideal noninteracting Pauli paramagnet. In this model the magnetic susceptibility will be less near full polarization than in normal ^{3}He.

With the smaller $m^{*\uparrow}$ the zero-sound velocity is also much higher than in normal ^{3}He, $c_0^\uparrow\approx 270$ m/sec using $m^{*\uparrow}=0.84$. Because $F_0^{\uparrow\uparrow}$ is small, $c_0^\uparrow$ is very sensitive to the value of the effective mass. Thus a measurement of $c_1^\uparrow$ and $c_0^\uparrow$ would help identify both $F_0^{\uparrow\uparrow}$ and $m^{*\uparrow}$, respectively. A measurement of the specific heat would also identify $m^{*\uparrow}$ in the usual way.

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