Inelastic electron scattering mechanisms in clean aluminum films

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Magnetoresistance data for clean aluminum films ($R_\square = 0.2$–$4\ \Omega$) are analyzed in terms of localization and superconducting fluctuations. The inferred inelastic-scattering rate τ_i^{-1} is interpreted as the sum of electron-phonon and dirty-limit electron-electron processes. Extrapolation of these results for τ_i^{-1} shows good agreement with results of superconducting nonequilibrium studies of Chi and Clarke for $R_\square \gtrsim 1\ \Omega$. For lower $R_\square$, another inelastic mechanism is evident, possibly clean-limit electron-electron scattering.

Inelastic scattering is a fundamental concept in the modern theory of electron excitations in metals. The inelastic-scattering time τ_i is also an important parameter in two fields of significant current interest—electron localization[1] and quasiparticle nonequilibrium effects in superconductors.[2] In addition, inelastic processes determine the fundamental limits on power dissipation and speed for many devices. A full understanding of these processes is thus essential in various areas of research. For pure crystals, inelastic mechanisms are well understood. Electron-phonon scattering is dominant at low temperatures (1–10 K), though in certain metals other mechanisms may contribute.[3]

Inelastic processes in polycrystalline metal films are at present not fully understood. With localization experiments τ_i can be measured, and a variety of metal films have been studied. Film sheet resistances $R_\square$ were typically 100 Ω. An unexpectedly large scattering rate was observed in nearly all films studied; these large rates cannot be explained by current theories.[1] Results from superconducting nonequilibrium studies show that electron-phonon scattering is the predominant mechanism for Pb, Sn, and In films, but aluminum films show excess scattering,[2] which until now has not been explained. These aluminum films had $R_\square < 10\ \Omega$.

We have studied the inelastic-scattering rate τ_i^{-1} in clean Al films ($R_\square \sim 1\ \Omega$), using magnetoresistance measurements above T_c. Clean Al films were chosen in order to make contact between the nonequilibrium and localization studies. We find that the inelastic mechanisms can indeed be identified. Mechanisms are electron-electron and electron-phonon scattering. Other experiments on Al films of higher resistance have recently been reported, and are discussed at the end of this article. These experiments draw conclusions which differ in large part from ours.

Films studied were 150–800 Å thick, patterned by standard photolithography into strips of width $W = 10$, 40, or 200 μm on glass substrates. Resistance changes in perpendicular fields were measured with a three-terminal ac bridge. The fractional resistance resolution was $< 10^{-6}$, at current levels low enough to avoid self-heating. Table I lists essential film parameters. The electron mean free path l and diffusion constant $D = \frac{1}{3} v_F l$ were determined from the superconducting upper critical-field slope,[4] dH_{c2}/dT, with $v_F = 1.3 \times 10^8$ cm/sec.

The magnetoresistance of the films studied here, $\delta R = R(H) - R(H=0)$, is largely due to two-dimensional (2D) localization effects and Maki-Thompson superconducting fluctuations. The localization contribution[5] in the absence of magnetic scattering[6] is given as

$$\frac{\delta R^{\mathrm{loc}}}{R} = \frac{e^2 R_\square}{2\pi^2 \hbar}\left[-\frac{3}{2}\psi\left(\frac{1}{2}+\frac{H_2}{H}\right)+\frac{1}{2}\psi\left(\frac{1}{2}+\frac{H_i}{H}\right)+\ln\frac{H_2}{H}-\frac{1}{2}\ln\frac{H_i}{H_2}\right], \qquad (1)$$

TABLE I. Sample parameters. The electron-electron and electron-phonon coefficients, A_1^{th} and A_3^{th}, are given in the text. $R_\square$ is at 4.2 K.

Sample	$R_\square$ (Ω)	d (Å)	T_c (K)	l (Å)	A_1/A_1^{th}	A_3/A_3^{th}
1	0.17	780	1.27	258	$\cdots$ [a]	1.7
2	0.85	250	1.34	107	0.76	1.3
3	1.86	250	1.44	62	0.66	1.7
4	1.87	150	1.40	80	$\cdots$ [b]	2.0
5	3.95	150	1.46	59	0.66	1.6
BR-A[c]	8.15	95	1.82	52	0.56	1.5

[a] This sample shows excess scattering at 1.5 K (see text).
[b] At low temperatures this 10-μm strip shows precursors of a dimensional crossover effect, as $l_i \approx W$. This precludes an accurate determination of A_1.
[c] Sample A of Ref. 6.

ψ is the digamma function. $H_2 = H_i + \frac{4}{3}H_{so}$, with H_i the inelastic scaling field $= \hbar c/(4eD\tau_i)$, and $H_{so} = \hbar c/(4eD\tau_{so})$, where τ_{so} is the spin-orbit scattering time. Our films are in the 2D limit for localization, since the inelastic diffusion length $l_i = (D\tau_i)^{1/2}$ is greater than the film thickness d, but less than W. The contribution due to Maki-Thompson fluctuations is[7]

$$\frac{\delta R^{\rm MT}}{R} = \frac{e^2 R_\square}{2\pi^2\hbar}\beta(T/T_c)\left[\psi\left(\frac{1}{2}+\frac{H_i}{H}\right)+\ln\frac{H}{H_i}\right] . \quad (2)$$

$\beta(T/T_c)$ is the parameter introduced by Larkin[7] to describe interactions between electrons. β diverges as $T \to T_c$. Aslamasov-Larkin (A-L) superconducting fluctuations[8] are smaller, but may be included in the analysis.

Other terms which can contribute to δR include classical magnetoresistance, $\propto H^2$, which is temperature independent since $\tau^{-1} >> \tau_i^{-1}$; τ is the elastic-scattering time. This classical term is observed only for $H > 1$ kG in samples with low $R_\square$, and it is not significant at lower fields ($\lesssim 200$ G). Interaction effects[9(a)] and related spin effects[9(b)] are negligible at low fields, since the (quantum) time for these effects, $\hbar/k_B T$, is much less than τ_i.

Figure 1 shows the normalized magnetoresistance for sample 3. The theoretical expression for $\delta R/R$, the sum of Eqs. (1) and (2), is shown by a solid line. Fitting this table to the data was done at low fields, by choosing H_{so} and H_i. The H_{so} values are independent of temperature. $\beta(T/T_c)$ was taken from the table by Larkin.[7] $\beta(T/T_c)$ may be depressed in a magnetic field comparable to H_{c2}.[9(a)] We therefore also include in Fig. 1 a plot of $\delta R/R$ with the proposed field-dependent $\beta(T,H)$.[10] In addition, we show the effect of also including the A-L term (dotted line). The theoretical curves all overlap in the low-field region. One can thus extract values of τ_i with confidence by fitting in this region. At large fields the theoretical curves differ from the experimental data. Further theoretical work will be required for this higher-field regime.

The inelastic-scattering rate determined from experiment is shown in Fig. 2. The solid lines are fits to the form

$$\tau_i^{-1} = A_1 T + A_3 T^3 . \quad (3)$$

The fit to Eq. (3) is excellent for all samples. Attempts to fit to forms involving any other two integral powers of T (e.g., T and T^4) were unsatisfactory. We estimate an accuracy of $\sim 15\%$ for the values of A_1 and A_3. The agreement of the measured rates with Eq. (3), and the magnitudes of A_1 and A_3 as discussed below, prove that the inelastic scattering is due to a combination of electron-electron and electron-phonon processes, such that

$$\tau_i^{-1} = \tau_{ee}^{-1} + \tau_{ep}^{-1} . \quad (4)$$

Lawrence and Meador[11] have calculated theoretically the electron-phonon scattering in Al, at E_F,[12] and give

$$\tau_{ep}^{-1} = (0.91\times 10^7\ {\rm sec^{-1}K^{-3}})T^3 = A_3^{\rm th} T^3 . \quad (5)$$

In the temperature range where the T^3 term is dominant, the films are three dimensional with respect to the typical phonon wavelength, $\lambda_{\rm ph} \approx (750/T)$ Å. (Transverse phonons contribute predominantly.[11]) Good adhesion to the substrate enhances the three dimensionality. The films are also "clean," in that $q_{\rm ph} l \gtrsim \pi$.

Abrahams, Anderson, Lee, and Ramakrishnan[13] have calculated τ_{ee}^{-1} for a dirty ($\hbar/\tau > k_B T$), 2D system ($\hbar D/k_B T > d^2$), appropriate for our samples. They find that

$$\tau_{ee}^{-1} = \frac{e^2 R_\square}{2\pi\hbar^2} k_B T \ln(T_1/T) , \quad (6)$$

with $T_1 = 9\times 10^5 (k_F l)^3 \sim 10^{12}$. Since $T_1 >> T$,

$$\tau_{ee}^{-1} \approx 13.4\times 10^7 R_\square T = A_1^{\rm th} T .$$

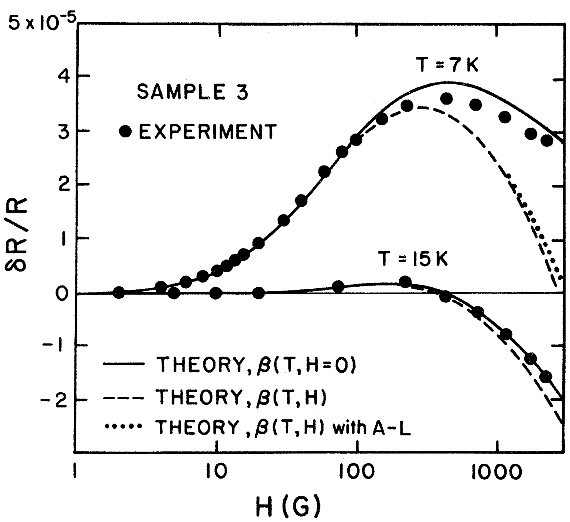

FIG. 1. Normalized magnetoresistance for sample 3. Fitting parameters are $H_{so} = 30$ G, and $H_i = 3.8$ G at 7 K and 33 G at 15 K. At 15 K, the Aslamasov-Larkin contribution to δR is negligible.

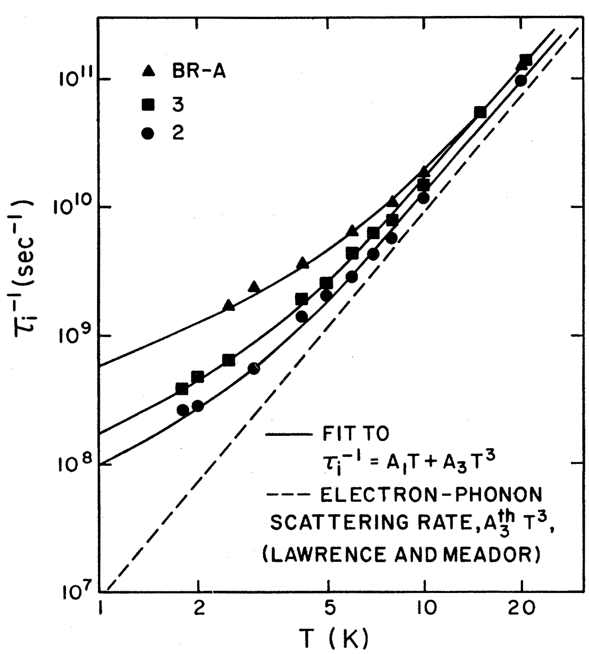

FIG. 2. Inelastic-scattering rate vs temperature. Rate for curve BR-A is from our analysis of magnetoresistance data of sample A, Ref. 6. Values of A_1 and A_3 are listed in Table I. Theoretical electron-phonon scattering rate, Ref. 11, is $\tau_{ep}^{-1} = 0.91\times 10^7 T^3$.

The experimental magnitudes A_1 and A_3 seen in Table I are in very good *quantitative* agreement with the relevant theoretical predictions.

We now turn to other magnetoresistance experiments on Al films, which treat films of larger resistance.[6,14,15] In Refs. 6 and 14, the theoretical analyses used to extract τ_i are incomplete. Both papers fitted data to a theoretical form which is correct only for very strong or very weak spin-orbit scattering. Thus the τ_i values and conclusions regarding inelastic mechanisms differ from ours (see note, Ref. 6.) Gordon, Lobb, and Tinkham[15] studied granular, high-resistivity Al films, with $l \sim 10$ Å. They fitted their data to a form like Eq. (3) for the one film with $R_\square = 15\ \Omega$, and used $\tau_{ep}^{-1} = A_4 T^4$ for films with $R_\square > 50\ \Omega$. They conclude that electron-phonon scattering is operative. However, they employ a value of A_3^{th} five times that of Lawrence and Meador.

A significant result of our work is the partial resolution of questions raised by superconducting charge-relaxation experiments of Chi and Clarke.[2] In Fig. 3 we plot their data for $(\tau_i T_c)^{-1}$ vs $R_\square$, along with our result for this quantity, using average *experimental* coefficients for $A_1/R_\square$ and A_3. We see that the rise of τ_i^{-1} with $R_\square$ is largely explainable as being due to dirty-limit electron-electron scattering.[13] This is the first *quantitative* explanation of the seemingly anomalous superconducting result. Also, our data verify that Chi and Clarke were, in fact, measuring charge relaxation by inelastic processes. A recent study of microwave gap enhancement,[16] received after completion of our analysis, comes to a similar conclusion regarding the dependence of τ_i on $R_\square$. These data are also included in Fig. 3.

In the low-resistance films of Chi and Clarke, $R_\square < 0.1\ \Omega$, both electron-phonon and dirty-limit electron-electron scattering are too small to account for the experimental rates. Our results for sample 1, with $R_\square = 0.17\ \Omega$, also show excess scattering. We make the admittedly speculative suggestion that the extra scattering may be due to clean-limit electron-electron scattering. A recent theoretical calculation of this effect for strictly 2D systems[17] yields $\tau_{ee}^{-1} = 1.7\times10^7 T^2$, within a factor of ~ 3 of the amount of extra scattering seen in the data of Fig. 3. Our data for sample 1 are also consistent with such a term. The question of electron-electron scattering in clean 2D systems merits further study; electron-phonon enhancement of the rate, as discussed for 3D systems,[3] may need to be considered.

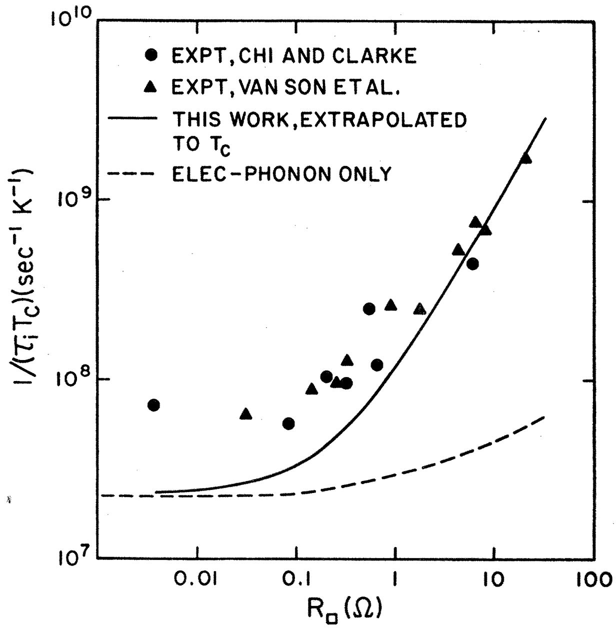

FIG. 3. Inelastic-scattering rate divided by T_c as a function of sheet resistance, $R_\square$.

In conclusion, we have used magnetoresistance measurements to identify the inelastic-scattering mechanisms in *clean* Al films at $T \geq 2$ K. This provides a basis for understanding some of the "anomalous" results on inelastic scattering in prior superconducting nonequilibrium studies. Our work also indicates that a new scattering mechanism is evident in very clean Al films ($R_\square < 0.1\ \Omega$). As a whole, these results show the potential of such magnetoresistance studies for providing insight into other areas of research.

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$$g^{-1} = \ln(T_c/T) + \psi(\tfrac{1}{2}) - \psi((\tfrac{1}{2}) + (DeH/2\pi ck_B T)) \ .$$

This agrees with the zero- and high-field limits in Ref. 9(a). Reference 7 gives β as a function of g.

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