Low-temperature magnetoresistance in two-dimensional magnesium films

Alice E. White, R. C. Dynes, and J. P. Garno
AT&T Bell Laboratories, Murray Hill, New Jersey 07974
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We have measured the low-temperature (0.1–10 K) magnetoresistance of thin two-dimensional films of magnesium. A crossover from positive to negative magnetoresistance is observed at low fields. This allows a unique determination of both the spin-orbit scattering length and the inelastic scattering length at each temperature. Below 0.2 K in the 22.3 Ω/□ magnesium film, we find that the inelastic diffusion length of the electron is greater than 1 μm.

There is now a wealth of experiments on the two-dimensional (2D) electron gas associated with silicon metal-oxide-semiconductor field-effect transistors (MOSFET's) where detailed measurements have yielded information about the elastic and inelastic scattering rates at low temperatures.[1] The purpose of this work is to study these effects in thin "metallic" films. The films differ from the MOSFET's in that they are not strictly two dimensional (the film thicknesses are on the order of 100 Å while the electron wavelength is on the order of a few Å), but the results indicate that when $k_F l_e > 1$, where l_e is the electron mean free path for elastic scattering, the 2D theories can be successfully applied, yielding values for the inelastic and spin-orbit scattering rates.

The logarithmic divergence of resistance with temperature observed in MOS inversion layers and thin metallic films[2–5] is well modeled by incorporating weak localization[6] and Coulomb interaction effects.[7] In the low perpendicular field region, however, interaction effects are negligible.[8] This work demonstrates the facility with which the various scattering rates in metals can be directly determined using detailed fits to weak localization theory. It also clearly illustrates the crossover of inelastic and spin-orbit scattering rates and the resultant "antilocalization."

Magnesium was chosen for this study for two reasons. First, because it does not superconduct, magnetoresistance studies can be made to low temperatures and second, because of its low atomic number ($Z=12$), it has relatively weak spin-orbit scattering. In contrast, the magnetoresistance of gold, a larger-Z material, is dominated by spin-orbit scattering. Earlier measurements on Mg at higher temperatures by Bergmann[9] showed little evidence of spin-orbit scattering, indicating that the low-field magnetoresistance is determined primarily by the inelastic scattering rate ($1/\tau_i$). We find that at lower temperatures, the spin-orbit scattering rate ($1/\tau_{so}$) becomes larger than $1/\tau_i$ and therefore dominates the magnetoresistance. In these studies of Mg, inelastic diffusion lengths ($L_i=\sqrt{D\tau_i}$) greater than 1 μm are found at $T<0.2$ K.

The samples studied ranged from 22 to 1657 Ω per square ($R_\square$). These are all in the logarithmically localized regime ($R_\square \ll 10$ kΩ/□). The variation of magnetoresistance with temperature is clearly illustrated in Fig. 1(a) where a series of curves for the lowest $R_\square$ magnesium film studied is plotted. At 0.2 K, the resistance first increases sharply with field. $R(H)$ turns around at 100 G and decreases more slowly. This dip at zero field, which is due to spin-orbit scattering, becomes less pronounced as the temperature (and therefore the inelastic scattering rate) increases. In the high $R_\square$ film, the positive magnetoresistance due to spin-orbit scattering dominates the curves up to the highest experimental temperatures as shown in Fig. 1(b).

Hikami, Larkin, and Nagaoka[10] have calculated the magnetoresistance using localization theory and, in the limit $k_F l_e \gg 1$, the change in the conductance with temperature and perpendicular field is

$$\Delta\sigma(H,T_{\text{const}}) = \frac{-\alpha e^2}{2\pi^2\hbar}\{\psi(\tfrac{1}{2}+a_1)-\psi(\tfrac{1}{2}+a_2) + \tfrac{1}{2}[\psi(\tfrac{1}{2}+a_3)-\psi(\tfrac{1}{2}+a_4)]\} + \frac{\alpha e^2}{2\pi^2\hbar}\ln\left(\frac{a_1 a_3^{1/2}}{a_2 a_4^{1/2}}\right) . \quad (1)$$

In this expression, ψ is the digamma function, α is a constant of order 1, and the a_n's are linear combinations of the scattering rates:

$$a_1 = \frac{\hbar c}{6eDH}\left(\frac{1}{\tau_e}+\frac{1}{\tau_{so}}+\frac{1}{\tau_s}\right) ,$$

$$a_2 = \frac{\hbar c}{6eDH}\left(\frac{4}{3}\frac{1}{\tau_{so}}+\frac{2}{3}\frac{1}{\tau_s}+\frac{1}{\tau_i}\right) ,$$

$$a_3 = \frac{\hbar c}{6eDH}\left(\frac{1}{\tau_i}+\frac{2}{\tau_s}\right) ,$$

$$a_4 = a_2 .$$

Here, $1/\tau_e$ is the elastic-scattering rate, $1/\tau_s$ is the spin-flip scattering rate, and the equivalence of a_2 and a_4 follows from Maekawa and Fukuyama.[11] If $1/\tau_{so} \ll 1/\tau_i$, $R(H=0)$ diverges logarithmically with T; however, if $1/\tau_{so}$ becomes comparable with $1/\tau_i$ the spin-orbit term acts to invert the logarithmic divergence. In the low-field limit, spin-orbit scattering results in a positive magnetoresistance, but as H is increased, the last term dominates and the magnetoresistance is negative and logarithmic in H.

The temperature dependence arises from τ_i which varies at T^{-p} and p is determined by the dominante inelastic scattering mechanism. For clean 2D systems, p is expected to be 2 for electron-electron scattering and 3 for electron-phonon scattering. Abrahams, Anderson, Lee, and Ramakrishnan[12] have calculated τ_i for electron-electron scattering in the dirty quasi-2D case ($k_F t > 1$ where t is the film thick-

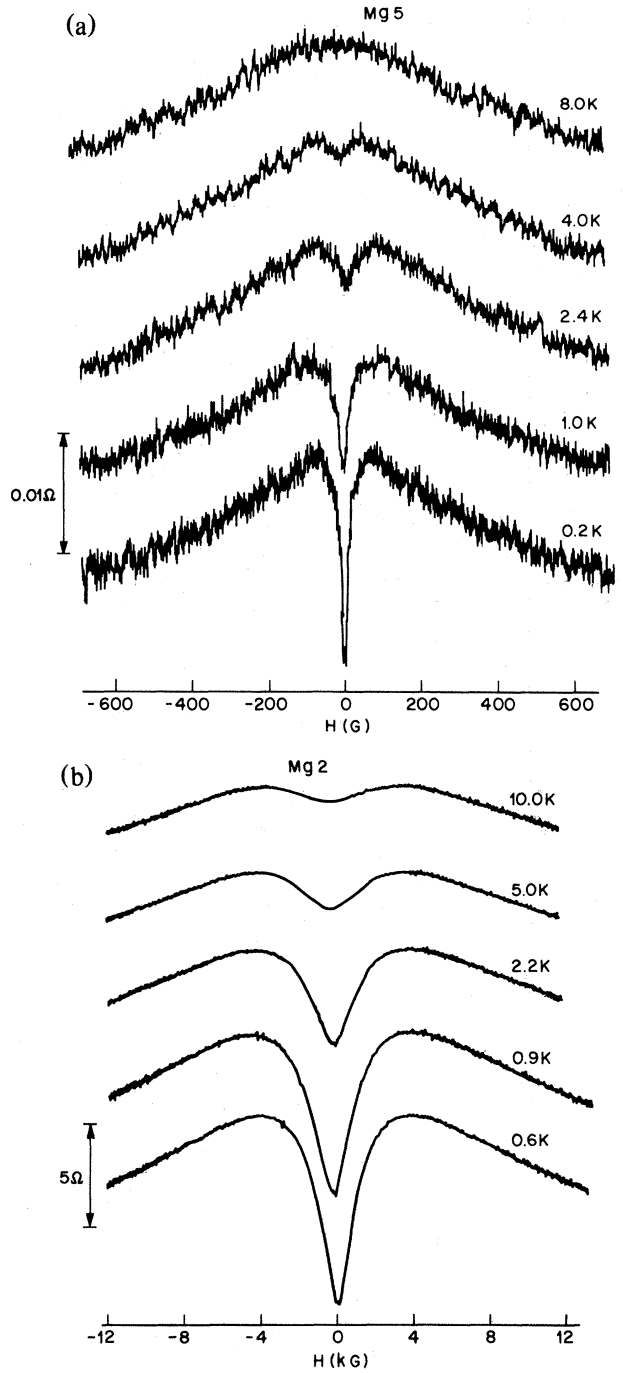

FIG. 1. (a) Magnetoresistance curves for Mg5 (22.3 Ω/□) at various temperatures. The sharp dip at zero magnetic field is due to spin-orbit scattering. (b) A similar set of curves for Mg2 (1657 Ω/□). Here, the spin-orbit scattering dominates.

ness) and find

$$1/\tau_i = \left(\frac{\pi}{k_F l}\right)\frac{k_B T}{2Dm}\left|\ln\frac{T}{T_1}\right| , \tag{2}$$

with

$$T_1 = \frac{32}{27}\frac{me^4}{2\hbar^2}(k_F l_e)^3 .$$

In addition to the reasons cited, magnesium was chosen because of a suggestion that it would have a rather long inelastic diffusion length (> 1 μm at 1.2 K).[13] In order to obtain clean films covering a range of resistances per square, 99.99% magnesium was evaporated onto glass substrates at 4.2 K.[14] It was found that it was essential to do the magnetoresistance measurements in a four-probe configuration, as spurious contact and lead resistances can completely obscure the behavior of the film resistance. Observation of the logarithmic dependence of $R_\square(H=0)$ on T indicates that the samples were being adequately cooled down to 0.1 K.

Extensive measurements were made for five magnesium films. The first two (Mg1 and Mg2) were evaporated in separate runs, while the last three (Mg3, Mg4, and Mg5) were made in stages in the same run. Mg5 was later transferred to a dilution refrigerator to extend the temperature range of the data. Since higher $R_\square$ films degrade upon warming to room temperature, it was only possible to obtain the low-temperature data for Mg5. Measured sample parameters are listed in Table I. The estimate of τ_e was obtained from the measured film resistivities and the diffusivity was then calculated using $D=\frac{1}{3}v_F^2\tau_e$. The 3D form of the diffusivity was employed because the films were three dimensional with respect to l_e. In each case, however, L_i was much greater than the thickness of the film, justifying the application of the 2D theories. The variation of D with film thickness does imply that the films are not strictly uniform in thickness; therefore the quoted values for t (as well as l_e) represent an average over the distances probed in the experiment. However, as long as the length scale of the fluctuations is small compared with L_i, the use of the localization theories remains valid.

In fitting to (1), several simplications were possible. First, we found that the choice of $\alpha=1$ gave the best fit. With the additional assumptions that interaction effects are negligible at these low fields and that $1/\tau_s \ll 1/\tau_e$, unique fits to the data from Mg3, Mg4, and Mg5 were obtained; however, those films that were in the regime $k_F l_e \lesssim 1$ (where the theory is not valid) could *not* be satisfactorily fitted by (1) although the curves looked qualitatively similar to those where $k_F l_e > 1$.

Since τ_{so} is roughly independent of temperature at these low temperatures,[15] we chose the best-fit value for τ_{so} from

TABLE I. Sample parameters for the five magnesium films discussed in the text. It was not possible to fit the data for the two films having $k_F l_e < 1$, so the values of τ_{so} for these films are not known.

Film	Thickness (Å)	$R_\square$ (Ω/□)	D (cm²/sec)	τ_{so} (sec)	$k_F l_e$
Mg1	303	519	0.2190		0.57
Mg2	170	1657	0.1231		0.32
Mg3	140	392	0.6282	9.6×10^{-11}	1.63
Mg4	262	96.5	1.364	1.1×10^{-10}	3.55
Mg5	401	22.3	3.855	1.27×10^{-10}	10.0

the lowest-temperature curve in each film, checked that it was consistent with the best-fit value at the highest temperature, and used it to determine τ_i as a function of T. In general, the chosen values for τ_{so} varied slightly with $R_\square$ and varied by $< 10\%$ from the best-fit values over the entire temperature range. The feature at zero field in the data makes it possible to uniquely determine both τ_i and τ_{so}.

In Fig. 2, the results of the fitting procedure for Mg3, Mg4, and Mg5 are plotted. Below 4 K in samples Mg3 and Mg4, and for all temperatures in sample Mg5, the exponent p from fitting the temperature dependence of τ_i is close to 1, indicating that electron-electron scattering is dominant. In addition, $\tau_i \propto D$ for these three films and the absolute magnitude of the scattering rate agrees within a factor of 3 with the Abrahams *et al.* prediction (2).

From Fig. 2, it is apparent that, above 4 K, the values of $\tau_i(T)$ for the two larger $R_\square$ films show a steeper dependence on temperature ($p \approx 1.5$). This behavior is attributed to a crossover to electron-phonon scattering which is frozen out at lower temperatures. Consistent with this picture, a simple free-electron model calculation of the electron-phonon scattering rate shows that it is roughly equal to a linear extrapolation of the measured inelastic rate at ~ 20 K. In experiments on comparable magnesium films in the high-T regime, Bergmann[9] found $\tau_i \propto T^{-2}$ and he observed that τ_i was independent of D.[16] Although the data presented in Fig. 2 are not available over a sufficiently large temperature range to allow a quantitative determination of the exponent p, it is not consistent with $p = 2$ between 4 and 10 K. These discrepancies cannot be easily reconciled, except to emphasize that because we actually observe the effects of spin-orbit scattering in our data, our analysis allows a direct, unambiguous determination of both τ_{so} and $\tau_i(T)$ in magnesium. At the lowest temperatures our values of τ_i give $L_i \sim 1.7$ μm. Quantitative comparison of this measurement with that of Sharvin and Sharvin on comparable films is difficult because of lack of published data, but the inelastic length implied by their experiment seems to be an order of magnitude longer than our directly measured length.

In summary, we have measured the magnetoresistance of thin 2D magnesium films. At low temperatures (< 4 K), a change in sign of the magnetoresistance occurs as a result of spin-orbit scattering. This makes it possible to determine $\tau_i(T)$ and τ_{so} by fits to the model of Hikami *et al.* The resulting inelastic-scattering rate shows a linear dependence on T and a magnitude that agrees within a factor of 3 with the quantitative prediction of Abrahams *et al.* for electron-electron scattering in the dirty limit. In low $R_\square$ magnesium films, long inelastic-scattering times at temperatures below 0.2 K result in inelastic diffusion lengths of the order of a μm.

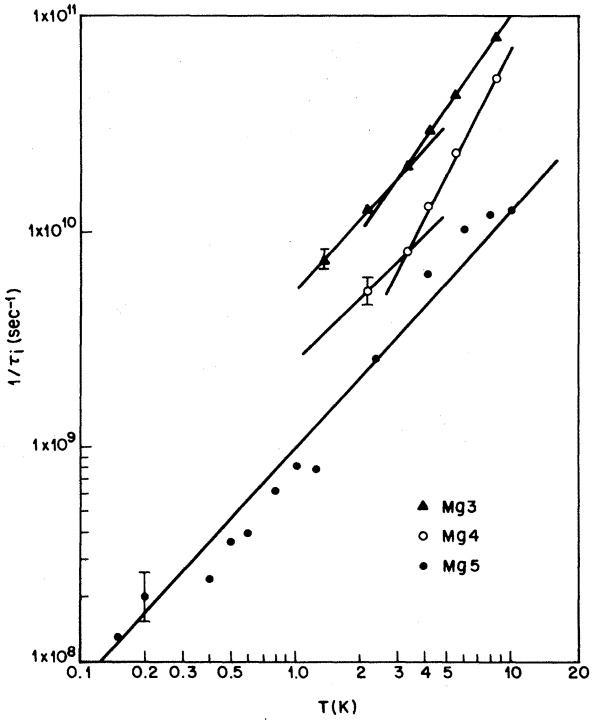

FIG. 2. $1/\tau_i$ vs T for the three Mg films with $k_F l_e > 1$. The data show a linear dependence on T at low temperatures and a steeper dependence above ~ 4 K. We interpret this as a crossover from electron-electron scattering to electron-phonon scattering as T increases.

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