# Theory of collective excitations in semiconductor superlattice structures

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(Received 11 October 1983)

In this paper electronic collective excitations of type-I and -II superlattices are examined in detail. Type-I superlattices consist of quasi-two-dimensional layers of electrons, while type-II superlattices consist of alternating quasi-two-dimensional layers of electrons and holes. We use a simple model of the electronic structure and linear-response theory to calculate the density response of the system to an external perturbation. From this, we obtain an expression for the dielectric tensor, the zeros of which yield the dispersion relations of the collective modes. The theory is such that one can take into account many-body effects (depolarization and excitonic shifts), magnetic fields, and electron-phonon coupling in a simple way. A rich spectrum of excitations is found: quasi-two-dimensional plasmons, intersubband plasmons, magnetoplasmons, phonon-plasmon modes, and so on. Some interesting features of the excitations are examined, and their relevance to experiment is discussed.

## I. INTRODUCTION

Superlattices are a novel class of material composed of alternating layers of two (or more) different constituents. The development of molecular-beam epitaxy (MBE) has made it possible to produce high-quality superlattices made from two different semiconducting materials (e.g., InAs/GaSb, GaAs/AlAs, Ge/GaAs, etc.) with similar lattice structure and matching lattice parameters. In the direction of superlattice growth (called the superlattice axis, and taken to be the $z$ direction), a number of atomic monolayers of semiconductor $A$ are deposited in an atomically sharp way on atomic monolayers of semiconductor $B$ to form new superlattice unit cells. A macroscopic sample of such an $A/B$ superlattice is a new bulk material with properties intermediate between those of materials $A$ and $B$.

There are two types of superlattices whose properties have been studied in some detail. These are known as type-I and -II superlattices. Type-I superlattices are typified by the GaAs/$Al_xGa_{1-x}As$ system, in which the band gap of GaAs is smaller than, and contained within, that of $Al_xGa_{1-x}As$, giving rise to band-gap discontinuities in both the valence and conduction bands of the resultant superlattice system. If we dope the $Al_xGa_{1-x}As$ layers with donor impurities (which is done by "modulation-doping" methods), electrons will be released from the donors to drop into GaAs sides of the band-gap discontinuities. The resulting one-dimensional potential well quantizes the motion of the electrons in the $z$ direction, and so the conduction band of GaAs will be split into a series of subbands (if the electron wave functions in adjacent potential wells do not overlap) or minibands (if they do), each of which represents a continuum of free-electron-like states in the $x$-$y$ plane. Thus as far as electronic properties are concerned, type-I superlattices consist of a periodic array of quasi-two-dimensional electron gases.

In type-II superlattices, as typified by the InAs/GaSb system, the band matchup is such that the conduction-band minimum of InAs is below the valence-band maximum of GaSb. In this case there is a transfer of electrons from one layer (GaSb) to the other (InAs), resulting in a spatial separation of electrons and holes into adjacent potential wells, with the formation of electron and hole subbands (or minibands). We see that in contrast to type-I superlattices which consist purely of electron layers, type-II superlattices consist of alternating electron and hole layers.

Many aspects of the physics of superlattices have been studied in the past decade.[1–3] The structure of the subbands and cyclotron resonance have been investigated by far-infrared-absorption spectroscopy[4–6] and resonant light scattering techniques.[7–9] (These experiments were first performed on surface inversion layers; subsequent measurements have also been made on semiconductor superlattices.) Considerable work has been performed on transport processes in these systems, since the spatial separation of the carriers and impurities promises to yield high-mobility carriers, and because of the negative-resistance regions in their $I$-$V$ characteristics (with the voltage gradient applied in the direction of the superlattice axis). Some work has also been performed on the possibility of laser action in these structures.[10]

In this paper we examine the electronic collective modes which occur in these systems. Although a good deal of experimental work has been done on optical absorption and light scattering, for example, there is relatively little theoretical work on the interpretation of the results in terms of the excitation of various sorts of collective modes. A good deal is known about the collective oscillations of electrons in single quantum wells [as typified by surface inversion layers in metal-oxide—semiconductor field-effect transistors (MOSFET) devices] but, up to now, little of this has been applied to the elucidation of electronic collective oscillations in multiple—quantum-well systems (superlattices).

Plasmon collective modes in a two-dimensional electron gas (2D EG) were first discussed by Stern,[11] who calculated the dynamical polarizability of a two-dimensional (2D) electron layer, as a model for the ground state of an electron gas in a surface inversion layer. He found that in the nonretarded limit, the frequency of a 2D plasmon $\omega \sim \sqrt{q}$, where $q$ is the wave vector in the plane. The 2D plasmon, associated with the ground subband as it is, is essentially an intrasubband collective mode.

Intersubband collective modes, associated with the transitions between subbands (the quantized motions of the electrons in the $z$ direction), were discussed by Chen, Chen, and Burstein[12] in a simplified model. More complete treatments of the intersubband modes were given by Dahl and Sham[13] and by Eguiluz and Maradudin,[14] who considered different polarizations and effects of retardation. They showed that the effect of resonant screening is to shift the resonance energy above the subband separation; this is the depolarization shift. In a discussion of many-body effects, Vinter[15] stressed the importance of vertex corrections. Ando[16] showed that in a static local approximation the vertex correction introduced another shift in the resonance energy which almost exactly canceled the effects of the depolarization shift for typical inversion-layer electron densities. This other shift is known as the "final-state interaction" or "excitonic shift," and is associated with the interaction of the excited electron in the higher subband with the hole left in the lower subband. Tselis and Quinn[17] gave a unified model of collective modes in surface inversion layers in which the effects of dispersion in the $x$-$y$ plane on intersubband modes were given to second order in $q$, along with the effects of resonant screening and the vertex corrections (the latter evaluated in a static local approximation). They also included the effects of magnetic fields and obtained the 2D analog of the hybrid magnetoplasmon mode in three dimensions, with $\omega^2=\omega_c^2+\omega_p^2(q)$, where $\omega_c$ is the cyclotron frequency and $\omega_p(q)$ is the 2D plasmon.

The plasmon modes of a two-layer system were discussed by Chiu, Quinn, Lee, and Eguiluz, and by Das Sarma and Madhukar,[18] who found, in addition to the usual 2D plasmon, an acoustical plasmon mode with $\omega\sim q$; the latter was also found to be undamped under appropriate circumstances.

Considerations of multiple-layer systems have usually been restricted to one-dimensional (1D) arrays of purely 2D EG's. Visscher and Falicov[19] have discussed the static dielectric function of such systems in the random-phase approximation (RPA). Fetter[20] has given an extensive discussion of the plasmon modes of such systems in a hydrodynamic approximation. Apostol[21] has done a similar calculation using the equation-of-motion method, obtaining results in RPA. Caille[22] *et al.* considered the effects of LO phonons on interface plasmons in multilayer systems. Mizuno *et al.*[23] calculated the effects of magnetic fields and obtained magnetoplasmon modes in these systems.

Recently, Das Sarma and Quinn[24] have given a fairly complete discussion of plasma modes in type-I and -II superlattices with two-dimensionally confined carriers. They showed explicitly the existence of quasi-2D plasmons and magnetoplasmons for type-I superlattices, and coupled quasi-2D electron and hole plasmons and magnetoplasmons for type-II superlattices; these modes reduced to the correct behavior in the appropriate limits. In addition, they considered a hydrodynamical model of the modes in a magnetic field, and pointed out the possibility of transverse modes such as helicons in type-I superlattices and helicon and Alfven waves in type-II superlattices. Some of the results of Das Sarma and Quinn were reproduced by Bloss, and Bloss and Brody,[25] who also considered the effects of electron-phonon coupling. A few papers have included the effects of subband structure, in which the finite widths of the wells are taken into account, on the collective excitation spectrum.[26]

In this paper we shall present a unified picture of the electronic collective modes in single— and multiple—quantum-well structures by using a relatively simple linear-response formalism. We will show that the subband structure of the wells can have important effects on the light-scattering and optical-absorption resonances observed in superlattices. The theory accommodates intrasubband and intersubband modes on an equal footing in both types of superlattice structures and allows an easy inclusion of the effects of magnetic fields and the electron-phonon interaction. By using a self-consistent-field method, we are able to include many-body effects such as resonant screening and vertex corrections, leading to depolarization and excitonic shifts.

This paper is organized as follows. After a brief discussion of supercells and electron miniband structure, we consider the linear response of the system to an external perturbation and the resulting dielectric function, the real and imaginary parts of which describe the reactive and dissipative aspects of the elementary excitations. A qualitative discussion of the nature of the collective modes follows. We then examine the particular case of superlattices with flat minibands in detail, calculating the density response and obtaining the dispersion relations for longitudinal intra- and intersubband modes in both type-I and -II superlattices, taking into account effects of electron-phonon interactions, magnetic fields, and vertex corrections.

## II. SUPERLATTICES

### A. Minibands and supercells

As described in the Introduction, superlattices consist of alternating layers of two (or more) semiconductors. Consider, for simplicity, a superlattice made up of two semiconductors, $A$ and $B$. The superlattice is made by depositing (usually by MBE) $n_A$ atomic layers of $A$ on $n_B$ atomic layers of $B$, and repeating the process until a macroscopic sample is obtained. The properties of the final crystal can be tailored by selectively doping the different layers with appropriate impurities. Assuming that the $A$ and $B$ layers are doped in a periodic way (i.e., all the $A$ and $B$ layers doped the same way, but the $A$ doping not necessarily the same as the $B$ doping), one obtains a new unit cell along the superlattice axis (the direction of the superlattice growth, here taken to be the $z$ axis), the supercell, consisting of $n_A$ atomic layers of $A$ and $n_B$ atomic layers of $B$.

One can readily see that the band structure of the $AB$ superlattice is quite different from that of bulk samples of $A$ and $B$. An approximate picture of the band structure can be given, if the layers are not too thin, as follows: Well within a particular layer, the bands are the same as in the bulk material, but when one reaches the interface with the next layer, the band structure discontinuously changes and assumes the character of the other bulk material. Of course, the presence of carriers will modify this picture; some of the modifications, such as Schottky barriers and local-field effects, have been described by Ruden and Döhler.[27] Nevertheless, the basic picture described above is useful for understanding the behavior of these systems.

In type-I superlattices, such as $GaAs/Al_xGa_{1-x}As$, the band gap of one of the semiconductors is smaller than and contained within that of the other. If the material with the larger band gap, $B$, for example, is doped with donor impurities, electrons are released, and then drop into the potential well formed by the part of the conduction band of the other material, $A$, which is below that of the first. Thus, just as in the case of the inversion layers, the $x$-$y$ motion of the electrons is free while motion in the $z$ direction is quantized. If the electron wave functions of the adjacent $A$ layers do not overlap, then the quantized wells are called subbands. The appropriate wave function for an electron with ($x$-$y$) momentum $\vec{k}$ in the $n$th subband in the $l$th layer is

$$|n,l,\vec{k}\rangle = e^{i\vec{k}\cdot\vec{r}}\xi_n(z-la) .$$

The electron is essentially regarded as being bound in the $l$th layer. On the other hand, if electron wave functions in adjacent $A$ layers do overlap, then it would be more appropriate to use a tight-binding sum of the states given above, so that the electron is regarded as having a quasi-free-motion in the $z$ direction. The tight-binding wave functions are of the form

$$|n,k_z\vec{k}\rangle = \sum_l e^{ik_z la}|n,l,\vec{k}\rangle ,$$

where $a$ is the superlattice "lattice parameter," the length of the supercell. These "minibands" have a nonzero bandwidth and allow electron motion along the whole length of the superlattice. Subbands may be regarded as a special case of minibands with zero bandwidth. We shall restrict our attention in this thesis to the case of subbands. (Much of this discussion applies to type-II superlattices as well.)

### B. Dielectric function $\epsilon(\omega)$

The optical properties of a superlattice, as for any other solid, are given by the dielectric functions. As is well known, the dispersion relations of the collective modes are given by the zeros of the dielectric functions,

$$\epsilon(\vec{q},\omega)=0 .$$

In a preceding publication[17] we showed that the equivalent condition for the inversion electron gas is that the determinant of the dielectric matrix (in "subband space") vanishes.

In the case of a superlattice, we obtain an expression similar to that obtained for the inversion layers,[17] except that we have structure factors multiplying terms related to $V_{nm}$. If we expand our quantities to $O(q^2)$, then we obtain the quasi-2D EG (Q2D EG) result, with $V_{nm}$ multiplied by a structure factor.

The collective modes are well-defined excitations when the real part of $\epsilon(q,\omega)$ has a zero in a region where the imaginary part of $\epsilon(q,\omega)$ vanishes; the latter describes the dissipative aspects of the excitations, while the former describes the reactive part. The problem of the damping of the excitations is not a simple one, and we shall not treat it here.

### C. Qualitative description of the collective modes

The collective modes which we examine will have wave vectors which can be decomposed into an in-plane part $q$ and an out-of-plane part $k_z$. The modes depend on $q$ in a different way than on $k_z$. The $k_z$ dependence occurs only in the structure factors. The reason for this is physically evident, since $k_z$ is directly related to the three-dimensional (3D) aspects of the system, and the structure factors precisely describe these aspects. In fact, physical variables, such as electron-density fluctuations and self-consistent perturbed potentials, are related in adjacent layers by a phase factor $e^{ik_z a}$, where $a$ is the superlattice parameter (distance between adjacent layers). The limiting cases of propagation in the $x$-$y$ plane or in the $z$ direction ($q\neq 0, k_z=0$ or $q=0, k_z\neq 0$, respectively) present some subtleties which will be examined in due course.

Two special cases are of particular interest; namely, those of strong and weak coupling between adjacent layers. In the weak-coupling limit, the superlattice parameter is so large (in the sense that $qa \gg 1$) that the layers act independently of each other. In the strong-coupling limit, on the other hand, the layers are so close together (in the sense that $qa \ll 1$) that the modes in one layer strongly affect those in the others. We shall examine this in more detail below.

The longitudinal modes, in which there is electron motion parallel to the wave vector of the excitation, will in general consist of the oscillations of the electron layers about their equilibrium positions along the $z$ axis, in addition to density oscillations in the $x$-$y$ plane. In the presence of a dc magnetic field parallel to the superlattice axis, the oscillations in the $x$-$y$ plane change somewhat. Upon introducing electron-phonon coupling, we find combined plasmon—optical-phonon modes, which are analogous to the 3D-plasmon—optical-phonon modes. In the case of type-II superlattices, we find the existence of coupled electron-hole modes; these have some new features associated with the relative phases of the electron- and hole-density oscillations.

### D. Flat minibands: Type-I superlattices

We proceed to discuss the linear response of a superlattice to an external potential in the flat miniband limit, in which the carriers are confined to their quantum wells. (The reason that the minibands are called flat is that the energy of an electron does not depend on the momentum $p_z$, so that the velocity in the $z$ direction $\partial\epsilon/\partial p_z$ vanishes.) Thus, the minibands become subbands, and electron wave functions in adjacent layers do not overlap.

The electronic wave functions are given by

$$|n,\vec{k},l\rangle = e^{i\vec{k}\cdot\vec{r}}\xi_n(z-la) , \tag{1a}$$

where $n$ is the subband index and $l$ is a layer index. We shall assume that the subband structure is the same in all the wells. The energy eigenvalues are

$$\epsilon_{nkl}=\epsilon_n+\frac{\hbar^2k^2}{2m}\ , \tag{1b}$$

where $\epsilon_n$ is the energy at the bottom of the $n$th subband. We assume an isotropic effective mass and no valley degeneracy. (This is valid for GaAs, but not for silicon.)

An external perturbing potential of the form

$$v^{\text{ext}}(\vec{r},z,t)=v^{\text{ext}}(\vec{q},\omega;z)e^{i(\omega t-\vec{q}\cdot\vec{r})} \tag{2}$$

will induce a perturbed electron density, which in turn induces perturbed Hartree and exchange-correlation potentials. The total perturbation

$$v=v^{\text{ext}}+v^H+v^{\text{xc}} \tag{3}$$

is also of the form (2). We use the Ehrenreich-Cohen[28] self-consistent-field prescription to calculate the induced electron density (as in Ref. 17)

$$\delta n(\vec{r},z;t)=\sum_{\alpha,\alpha'}\frac{f(\epsilon_{\alpha'}-f(\epsilon_\alpha)}{\epsilon_{\alpha'}-\epsilon_\alpha-\hbar\omega}\langle\alpha\,|\,v\,|\,\alpha'\rangle \times\langle\alpha'\,|\,\delta(r-r')\delta(z-z')\,|\,\alpha\rangle\ . \tag{4}$$

Here, $\alpha$ is the composite index $n,l,\vec{k}$. The electrons are assumed to feel the total self-consistent potential (3), so that we are constructing the RPA here. Although the RPA is a high-density approximation, it has been used with success in the inversion-layer problem where the densities are only moderate. This turns out to be due to the fact that the effective coupling constant $r_s=r_0/a_0$ (where $r_0$ is the mean distance between electrons and $a_0$ is the effective Bohr radius) is small for low-effective-mass systems since $a_0$ is inversely proportional to the effective mass. (In GaAs, $m=0.068m_0$.)

Assuming no overlap between electrons in different subbands, we have

$$\langle\alpha\,|\,v\,|\,\alpha'\rangle=\delta_{ll'}\delta_{\vec{k}',\vec{k}+\vec{q}}\langle n\,|\,v_l(\vec{q},\omega)\,|\,n'\rangle\ , \tag{5}$$

where

$$\langle n\,|\,v_l(\vec{q},\omega)\,|\,n'\rangle=\int dz\,\xi_n(z-la)v(\vec{q},\omega;z)\xi_{n'}(z-la). \tag{6}$$

Using (5) in Eq. (4), we obtain

$$\delta n(\vec{q},\omega;z)=\sum_{n,n',l}\Pi_{nn'}(\vec{q},\omega)\langle n\,|\,v_l(\vec{q},\omega)\,|\,n'\rangle \times\xi_n(z-la)\xi_{n'}(z-la)\ , \tag{7}$$

where

$$\Pi^*_{nn'}(\vec{q},\omega)=2\sum_k\frac{f(\epsilon_{n'},\vec{k}+\vec{q})-f(\epsilon_n,\vec{k})}{\epsilon_{n'},\vec{k}+\vec{q}-\epsilon_{n,\vec{k}}-\hbar\omega} \tag{8}$$

is the irreducible polarization insertion.

The perturbed Hartree potential is given, as before, by

$$v^H(\vec{q},\omega;z)=\frac{2\pi e^2}{\epsilon_s q}\int_{-\infty}^{\infty}dz'e^{-q\,|z-z'|}\delta n(\vec{q},\omega;z'). \tag{9}$$

The exchange-correlation perturbation is given by

$$v^{\text{xc}}(\vec{r},z,t)=\frac{\delta v_{\text{xc}}[n]}{\delta n}\delta n(\vec{r},z,t)\ . \tag{10}$$

The matrix elements of $v^H$ are

$$\langle n,l\,|\,v^H(\vec{q},\omega;z)\,|\,n',l\rangle=\sum_{\substack{m,m'\\l'}}\Pi_{mm'}(\vec{q},\omega)V_{nn'}^{mm'}(\vec{q};l,l') \times\langle m\,|\,v_{l'}(\vec{q},\omega)\,|\,m'\rangle\ , \tag{11}$$

where

$$V_{nn'}^{mm'}(\vec{q};l,l')=\frac{2\pi e^2}{\epsilon_s q}\int dz\,dz'\xi_n(z)\xi_{n'}(z) \times e^{-q\,|z-z'+(l-l')a\,|}\xi_m(z')\xi_{m'}(z')\ . \tag{12}$$

The matrix elements of $v^{\text{xc}}$ are

$$\langle n,l\,|\,v^{\text{xc}}(\vec{q},\omega;z)\,|\,n',l\rangle =-\sum_{m,m',l'}\Pi_{mm'}(\vec{q},\omega) \times V^{\text{xc}}_{nn',mm'}(l,l')\langle m\,|\,v_{l'}(\vec{q},\omega)\,|\,m'\rangle\ , \tag{13}$$

where

$$V^{\text{xc}}_{nn',mm'}(l,l')=-\int dz\,\xi_n(z-la)\xi_{n'}(z-la)\frac{\delta v_{\text{xc}}[n]}{\delta n} \times\xi_m(z-l'a)\xi_{m'}(z-l'a)\ . \tag{14}$$

Since the wave functions in different layers do not overlap, we must have $l=l'$ in (14) in order for it not to vanish. In that case, we can shift the origin and use the periodicity of $\delta v_{\text{xc}}/\delta n$ to write

$$V^{\text{xc}}_{nn',mm'}(l,l')=\delta_{l,l'}V^{\text{xc}}_{nn',mm'}\ , \tag{15}$$

with

$$V^{\text{xc}}_{nn',mm'}=-\int dz\,\xi_n(z)\xi_{n'}(z)\frac{\delta v_{\text{xc}}[n]}{\delta n}\xi_m(z)\xi_{m'}(z)\ . \tag{16}$$

Setting (11) and (13) in (3), taking the electric quantum limit, and defining

$$V_{n0}^{m0}(\vec{q};l,l')=V_{nm}(\vec{q};l,l'),\quad V^{\text{xc}}_{n0m0}(l,l')=V^{\text{xc}}_{nm}(l,l')\ ,$$

we have

$$\langle n\,|\,v_l\,|\,0\rangle=\langle n\,|\,v_l^{\text{ext}}\,|\,0\rangle+\sum_{m,l'}\chi_{m0}(\vec{q},\omega) \times[V_{nm}(\vec{q};l,l')-V^{\text{xc}}_{nm}(l,l')] \times\langle m\,|\,v_{l'}\,|\,0\rangle\ , \tag{17}$$

$$\chi_{m0}=\Pi_{m0}+\Pi_{0m}\ ,$$

which determines the response of the system to an external perturbation $v^{\rm ext}$.

In view of the translational symmetry, we make the ansatz

$$\langle n\,|\,v_l\,|\,0\rangle = e^{ik_z la}\langle n\,|\,v_0\,|\,0\rangle\ . \tag{18}$$

To see what this ansatz means physically, note that the electron-density oscillation $\delta n$ and perturbing potential $v$ are proportional, so that Eq. (18) indicates that the phase of $\delta n$ is advanced by an amount $k_z a$ from one layer to the next. We shall assume that all relevant $z,z'$ in the integral for $V_{nm}$ are much smaller than $la$ and $l'a$. Then, substituting (15) and (18) into (17) yields

$$\begin{aligned}\langle n\,|\,v_0\,|\,0\rangle = \langle n\,|\,v_0^{\rm ext}\,|\,0\rangle &+ \sum_m \chi_{m0}(\vec{q},\omega)[\,V_{nm}(q) - V_{nm}^{\rm xc} + S_+\widetilde{V}_{mn}(q)\\ &+ S_-\widetilde{V}_{nm}(q)]\langle m\,|\,v_0\,|\,0\rangle\ .\end{aligned} \tag{19}$$

where

$$V_{nm}(q) = \frac{2\pi e^2}{\epsilon_2 q}\int dz\,dz'\xi_n(z)\xi_0(z) \times e^{-q\,|z-z'|}\epsilon_m(z')\xi_0(z') \tag{20a}$$

and

$$\widetilde{V}_{nm}(q) = \frac{2\pi e^2}{\epsilon_s q}\int dz\,dz'\xi_n(z)\xi_0(z) \times e^{-q(z-z')}\xi_m(z')\xi_0(z')\ . \tag{20b}$$

The structure factors $S_\pm$ are defined by

$$1+S_\pm(\vec{q},k_z) = \frac{1}{1-e^{-qa}e^{\pm ik_z a}}\ . \tag{21}$$

The condition for the collective modes of the system is that self-sustaining oscillations in the electron density occur. This means that $v^{\rm ext}=0$, while $v\neq 0$. From (19), the condition for nonvanishing $v$ is

$$\begin{aligned}\det|\,\delta_{nm} - \chi_{m0}(\vec{q},\omega)[\,V_{nm}(q) - V_{nm}^{\rm xc} &+ S_-\widetilde{V}_{nm}(q)\\ &+ S_+\widetilde{V}_{mn}(q)]\,|=0\ .\end{aligned} \tag{22}$$

It is convenient to introduce the structure factor $S$ defined by

$$\begin{aligned}S(\vec{q},k_z) &= 1+S_+(\vec{q},k_z)+S_-(\vec{q},k_z)\\ &= \frac{\sinh(qa)}{\cosh(qa)-\cos(k_z a)}\ .\end{aligned} \tag{23}$$

### *1. Intrasubband modes*

For purely two-dimensional layers, only the $n=m=0$ element contributes. In this case the electron-density profile in the $z$ direction is a $\delta$ function, $\xi_n^2(z-la)=\delta(z-\delta a)$. We have $V_{nm}=\widetilde{V}_{nm}=2\pi e^2/\epsilon_s q$, and $\chi_{00}^*(\vec{q},\omega)=\Pi_{00}^*(\vec{q},\omega)$, the 2D polarizability; thus (22) becomes

$$1 = \frac{2\pi e^2}{\epsilon_s q}\Pi_{00}(\vec{q},\omega)S(q,k_z)\ . \tag{24}$$

This is just the dispersion relation of Das Sarma and Quinn.[24] Equation (24) can also be derived by considering the solutions to Maxwell's equations between the layers and using the appropriate boundary conditions. If the electric field in the space between the $l$th and $(l+1)$th layers is of the form

$$\vec{E} = e^{iqy-i\omega t} \times (0,\ E_l^+e^{i\beta z}+E_l^-e^{-i\beta z},\ -q\beta^{-1}(E_l^+e^{i\beta z}-E_l^-e^{i\beta z}))\ ,$$

with $\beta^2=\epsilon_s\omega^2/c^2-q^2$, then the boundary conditions that $E_y$ be continuous at $z=la$ and the discontinuity of $D_z=\epsilon E_z$ at $z=la$ be equal to the induced charge $4\pi\delta\rho_l$, yield

$$\frac{i\beta}{q}v_q\Pi_{00}(q,\omega)\frac{\sinh(i\beta a)}{\cosh(i\beta a)-\cos(k_z a)} = 1\ ,$$

where we have used the ansatz

$$E_l^\pm = e^{ik_z la}E_0^\pm\ .$$

In the nonretarded limit, $c\to\infty$, and this reduces to Eq. (24).

In the weak-coupling limit, the planes are well separated in the sense that $qa\gg 1$, so that the distance between the planes is much larger than the wavelength of the density oscillation in the plane. In this limit $S(q,k_z)\simeq 1$, and the dispersion relation is that of the 2D plasmon.

In the strong-coupling limit, the planes are close together in the sense that $qa\ll 1$. In this limit there are two cases to consider, $k_z\neq 0$ and $k_z=0$. We look at the second case first. For $k_z=0$ and $qa\ll 1$, we have from (23),

$$S(q,k_z)\simeq 2/qa\ ,$$

and Eq. (24) becomes

$$1 = \frac{4\pi e^2}{\epsilon_s a q^2}\Pi_{00}(\vec{q},\omega)\ .$$

Using the long-wavelength form of $\Pi_{00}^*$,

$$\Pi_{00}(\vec{q},\omega)\simeq\frac{n_s}{m}\frac{q^2}{\omega^2}\ ,$$

where $n_s$ is the electron concentration (cm$^{-2}$) and $m$ is the electron mass, we find

$$\omega^2 = \Omega_p^2 = \frac{4\pi n_{\rm eff}e^2}{m\epsilon_s}\ ,$$

which is a 3D plasmon, with an effective 3D electron density $n_{\rm eff}=n_s/a$.

The physical origin of this mode is evident. The electron-density oscillations are in phase ($k_z=0$) in all the layers which are close together, so that effectively the mode is indistinguishable from a 3D plasmon propagating perpendicular to the superlattice axis.

For the case $k_z\neq 0$ and $qa\ll 1$, we find

$$S(q,k_z)\simeq\frac{qa}{1-\cos(k_z a)}\ ,$$

and thus the mode has the frequency

$$\omega_{00}^2=\frac{2\pi n_s e^2}{m\epsilon_s}\frac{aq^2}{1-\cos(k_z a)} .$$

This is an acoustic plasmon with $\omega\sim q$. The mode is softened because the electron-density oscillations in adjacent modes are no longer in phase, and the restoring force is decreased from what it would be in the 3D case.

If we include the effects of coupling between the intra- and intersubband modes, then we have to keep the off-diagonal matrix elements in Eq. (22). If we include coupling between the intrasubband mode and first intersubband mode only, we find that the effects of coupling are important only in the strong-coupling regime. The intersubband mode is affected only in $O(q^2)$, while the intrasubband mode becomes[29]

$$\omega=\omega_{00}(1-\delta) , \tag{25}$$

with $\omega_{00}$ the intrasubband acoustical plasmon defined above, and

$$\delta=\frac{1}{2}\left[1+\frac{E_{10}^2}{\omega_{01}^2}\right]^{-1}\left[\frac{L_{10}}{a'}\right]\left[\frac{L_{10}}{L_{11}}\right]\times\left[1+\left[\frac{\sin k_z a}{1-\cos(k_z a)}\right]^2\frac{z_{10}^2}{L_{10}^2}\right][1-\cos(k_z a)] ,$$

where

$$a'=a+L_{00}[1-\cos(k_z a)] .$$

Here, $E_{10}$ is the separation between the ground and first excited subbands, $z_{10}$ is the dipole matrix element, $\omega_{D1}$ is the depolarization shift of the $0\rightarrow1$ transition, and

$$L_{nm}=-\int dz\,dz'\xi_n(z)\xi_0(z)\,|z-z'|\,\xi_m(z')\xi_0(z') .$$

If we include effects of coupling to the first and second excited subbands, we again obtain Eq. (25), but now the quantity $\delta$ is given by

$$\delta=-\frac{1}{2a'}\frac{2L_{12}(L_{10}L_{20}+s^2z_{10}z_{20})-L_{11}(L_{10}^2+s^2z_{10}^2)(1+E_{10}^2/\omega_{01}^2)-L_{22}(L_{20}^2+s^2z_{20}^2)(1+E_{20}^2/\omega_{02}^2)}{L_{11}L_{22}(1+E_{10}^2/\omega_{D1}^2)(1+E_{20}^2/\omega_{D2}^2)-L_{12}^2}(1-\cos k_z a) ,$$

with $a'$, $L_{nm}$ as defined above, and $s=\sin(k_z a)/[1-\cos(k_z a)]$. The factor $1-\delta$ essentially corresponds to a renormalization of the electron's mass.

### 2. *Intersubband modes*

We now consider the higher roots of (22). It is difficult to obtain exact dispersion relations from (22), as in the single—quantum-well case. We make two approximations. Firstly, we truncate the determinant by neglecting off-diagonal elements: This is equivalent to neglecting mixing between different intersubband excitations. Secondly, we expand all quantities to $O(q^2)$. In doing so, however, we shall fix the quantity $qa$, so that $S$ can be treated formally as a constant. We then obtain the dispersion relation

$$\omega^2=\Omega_{n0}^2(1+\alpha_{nn}-\beta_{nn})-\Omega_{n0}^2\mu_{nn}S(q,k_z)q+\{(\hbar\Omega_{n0}/m)[1+\tfrac{1}{2}(\alpha_{nn}-\beta_{nn})]+v^2[\tfrac{3}{4}+1/(\alpha_{nn}-\beta_{nn})]-\Omega_{n0}^2\gamma_{nn}\}q^2 . \tag{26}$$

The symbols $\alpha_{nn}$, $\beta_{nn}$, $\gamma_{nn}$, and $\mu_{nn}$ are given by

$$\alpha_{nn}=\frac{2n_s}{\hbar\Omega_{n0}}\frac{2\pi e^2}{\epsilon_s}\times\left[-\int dz\,dz'\xi_n(z)\xi_0(z)\,|z-z'|\,\xi_n(z')\xi_0(z')\right] ,$$

$$\beta_{nn}=\frac{2n_s}{\hbar\Omega_{n0}}V_{nn}^{\mathrm{xc}},\quad \mu_{nn}=\frac{2n_s}{\hbar\Omega_{n0}}\frac{2\pi e^2}{\epsilon_s}|z_{n0}|^2 ,$$

$$\gamma_{nn}=\frac{2n_s}{\hbar\Omega_{n0}}\frac{2\pi e^2}{\epsilon_s}\frac{1}{6}\int dz\,dz'\xi_n(z)\xi_0(z)\,|z-z'|^3\times\xi_n(z')\xi_0(z') .$$

Here $\alpha_{nn}$ and $\beta_{nn}$ give rise to the depolarization and excitonic shifts, respectively, and are well known in single-layer intersubband transitions. We note that Eq. (26) is the same as the dispersion relation for the intersubband collective modes in a single quantum well which were considered in Ref. 17, except that the term linear in $q$ is modified by the structure factor $S(q,k_z)$. The constant term and the coefficient of $q^2$ are exactly the same as in the single—quantum-well case, and they are interpreted physically in Ref. 17. In the weak-coupling case, with $qa\gg1$, $S=1$ and each layer supports its own intersubband mode.

In the strong-coupling limit ($qa\ll1$), we have, as before, two cases to consider: $k_z=0$ and $k_z\neq0$. For $k_z=0$, we have

$$\omega^2=\Omega_{n0}^2(1+\alpha_{nn}-\beta_{nn}-2\mu_{nn}/a)+O(q^2) , \tag{27}$$

where the coefficient of $q^2$ is exactly the same as in (26). In this case, the term proportional to $S(q,k_z)q$ becomes independent of $q$ and softens the constant part. To see physically why this happens, note that the wavelength of the electron-density oscillations in the $x$-$y$ plane is long, and that the oscillations are in phase in all the layers. The electrons in any particular layer will then "feel" the (attractive) potential in the adjacent layer (since it is unscreened by its own electrons), so that the restoring force due to the original layer is decreased.

For the $k_z\neq0$ case, we find that

$$\omega^2=\Omega_{n0}^2(1+\alpha_{nn}-\beta_{nn})+\left[-\frac{\Omega_{n0}^2\mu_{nn}a}{1-\cos(k_z a)}+\frac{\hbar\Omega_{n0}}{m}\left[1+\frac{\alpha_{nn}-\beta_{nn}}{2}\right]+v_F^2\left[\frac{1}{\alpha_{nn}-\beta_{nn}}+\frac{3}{4}\right]-\Omega_{n0}^2\gamma_{nn}\right]q^2 ,$$

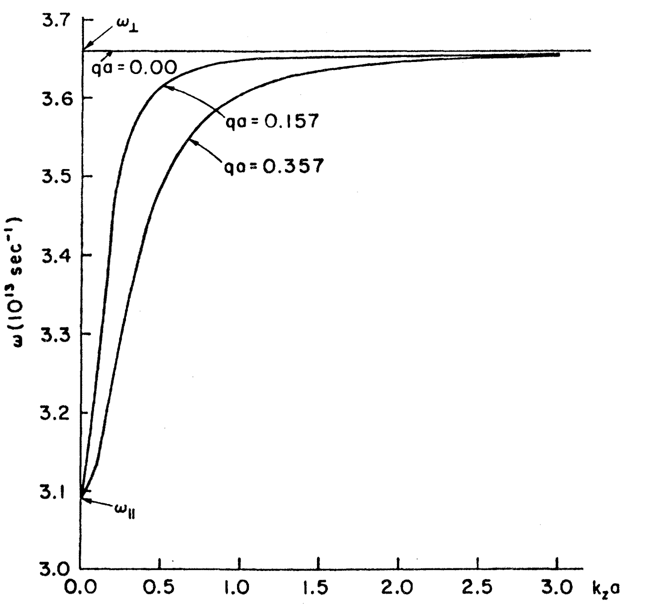


FIG. 1. Plot of $\omega$ [given by Eq. (26)] as a function of $k_z a$ for different values of $qa$. Typical values of the parameters from Ref. 30 have been used; $\omega_\perp = 3.66\times10^{13}$ sec$^{-1}$ and $\omega = 3.09\times10^{13}$ sec$^{-1}$.

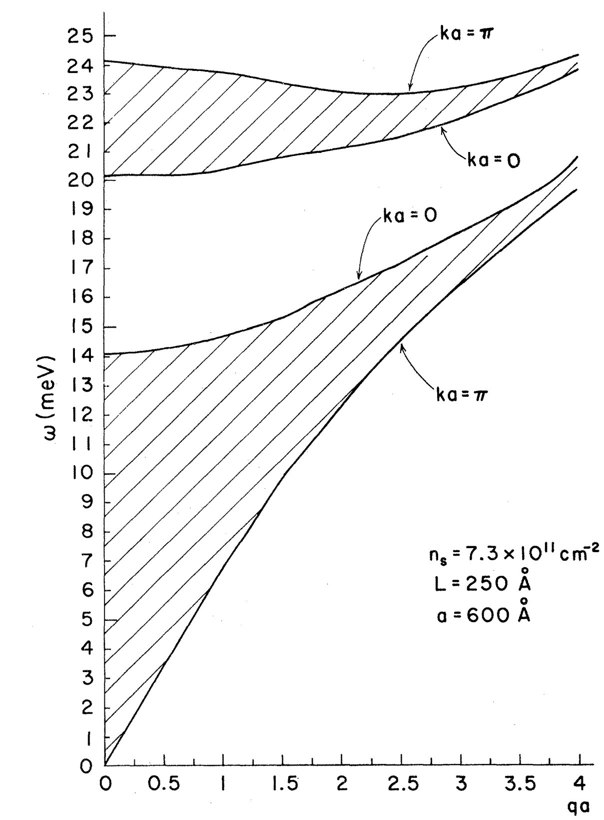


FIG. 2. Plot of $\omega$ given by Eqs. (24) and (26), as a function of $qa$ slowing the band structure of the intrasubband and intersubband plasmon bands.

so that here the term proportional to $S(q,k_z)q$ is softened to $O(q^2)$. Physically, this is because the density oscillations in adjacent layers are no longer in phase, and thus the sort of decrease in restoring force discussed previously is no longer as effective.

In Fig. 1 we have plotted the frequency $\omega$ given by Eq. (26) for the $0\rightarrow1$ transition as a function of $k_z a$ for different values of $qa$. In the numerical calculation, a model system has been used in which the quantum wells have infinite barriers, and thus

$$\xi_n(z)=\sqrt{2/L}\,\sin[(n+1)\pi z/L]$$

and

$$\epsilon_n=(\hbar^2/2mL^2)(n+1)^2\pi^2 .$$

The parameters in the model have been chosen to correspond to the systems studied by Olego *et al.*[30] For $q=0$, $\omega=\omega_\perp=\omega_{10}(1+\alpha_{11})=3.66\times10^{13}$ sec$^{-1}$ (we have neglected vertex corrections in the numerical calculation) and the intersubband mode propagates perpendicular to the layers. For $q\neq0$, but $k_z=0$, $\omega=\omega_{\|}=3.09\times10^{13}$ sec$^{-1}$, and the mode propagates parallel to the layers. Note that, with $k_z a$ fixed, the mode softens as $qa$ increases, as expected from the discussion above.

The peculiar behavior of the intersubband mode as a function of $k_z$ and $q$ can be traced to the fact that the structure factor $S$ defined in Eq. (23) is nonanalytic at the origin of the $q$-$k_z$ plane. For any $q\neq0$, the frequency of the mode drops to $\omega$, while at $q=0$, $\omega=\omega_\perp>\omega_{\|}$. To see this, consider the case when $qa\ll1$. If we expand $\cosh qa$ in the denominator of $S$ to $O(q^2)$, then Eq. (26) becomes

$$\omega^2=\Omega_{n0}^2(1+\alpha_{nn}-\beta_{nn})+\left[\gamma_n-\frac{2a\mu_{nn}\Omega_{n0}^2}{(qa)^2+4\sin^2(\frac{1}{2}k_z a)}\right]q^2 ,$$

where $\gamma_n$ is the coefficient of $q^2$ in Eq. (26). Suppose now that both $qa$ and $k_z a$ are very small compared to unity (but are of the same magnitude). Then the second term within the large parentheses is $-2\mu_{nn}\Omega_{n0}^2/Q^2a$, where $Q^2=k_z^2+q^2$. In this case,

$$\omega^2=\Omega_{n0}^2[1+\alpha_{nn}+\beta_{nn}-(2\mu_{nn}/a)\sin^2\theta]+\gamma_n q^2 ,$$

where $\theta$ is the angle between $\vec{Q}=\vec{q}+k_z\hat{z}$ and the $z$ axis. This explicitly displays the character of the nonanalyticity. Thus in this limit, $\omega$ starts at a constant value which depends upon the angle of the propagation of the collective mode relative to the electron layer, and its dispersion is proportional to $q^2$. In Fig. 2 we have plotted Eqs. (24) and (26) as functions of $qa$ for a model system in which the electrons are confined to their layers by infinite square-well potentials, with $n_s=7.3\times10^{11}$ cm$^{-2}$, $a=600$ Å, and $L=250$ Å. Note that for small $qa$, the bandwidth is at a maximum. This is because the layers are close together and are therefore strongly coupled: If one layer is excited, the excitation is not localized to that layer, but spreads throughout the system. For large $qa$, the bandwidth becomes very small, and individual layers support their own plasmons; the excitation in any layer will remain localized at that layer.

### 3. *Effects of dc magnetic field*

If we impose a uniform dc magnetic field parallel to the superlattice axis, the solutions to the Schrödinger equation in the Landau gauge are

$$|n,l,k,j\rangle = e^{iky}u_l(x+l_H^2k)\xi_n(z-ja) ,$$

where $l$ is now the Landau-level quantum number and $j$ is a subband index. A perturbation of the form Eq. (2.3) induces an electron-density oscillation

$$\delta n(q,\omega;z) = \sum_{n,n',j} \Pi_{nn'}^{(H)}(\vec{q},\omega)\langle n\,|\,v_j(\vec{q},\omega)\,|\,n'\rangle \times \xi_{n'}(z-ja)\xi_n(z-ja) ,$$

where $\Pi^{(H)}$ is the irreducible polarization insertion defined by

$$\Pi_{nn'}^{(H)}(\vec{q},\omega) = \frac{1}{\pi l_H^2}\sum_{ll'}\left[\frac{f(E_{n'l'})-f(E_{nl})}{E_{n'l'}-E_{nl}-\hbar\omega}\right]|J_{l'l}(\vec{q})|^2 , \tag{28}$$

where

$$J_{l'l}(\vec{q}) = \int_{-\infty}^{\infty} dx\, e^{iq_x x}u_{l'}(x+l_H^2 q_y)u_l(x) .$$

We assume the electric quantum limit, and go through precisely the same steps as before to obtain the condition for collective modes,

$$\det|\delta_{nm} - \chi_{m0}^{(H)}(\vec{q},\omega)[V_{nm}(q) - V_{nm}^{\rm xc} + S_-\tilde{V}_{nm}(q) + S_+\tilde{V}_{mn}(q)]| = 0 . \tag{29}$$

In the 2D case, only the $n=m=0$ matrix element contributes, thus Eq. (29) becomes

$$1 = \Pi_{00}^{(H)}(\vec{q},\omega)S(q,k_z)[(2\pi e^2/\epsilon_s q) - V_{00}^{\rm xc}] . \tag{30a}$$

This is just the result of Das Sarma and Quinn,[24] if we put $V^{\rm xc}=0$. Equation (30a) contains a large number of resonances (higher cyclotron modes and Bernstein modes). To first order in $q$, Eq. (30a) becomes

$$\omega^2 = \omega_c^2 + \omega_p^2(q)S(q,k_z) ,$$

where $\omega_p(q)$ is the 2D plasmon frequency. Note that in the weak-coupling limit, $qa \gg 1$, $S=1$, and each layer supports its own 2D magnetoplasmon. The higher cyclotron (or Bernstein) modes occur for higher values of $\omega$,

$$\omega \simeq n\omega_c ,$$

in addition to higher-order terms in $q$, with $n$ an integer. These are obtained by expanding the polarization insertion in Eq. (30a) to higher order in $q$. The Bernstein modes have minimal spectral weight at long wavelengths, and thus we do not discuss them any more.

In the strong-coupling case with $k_z \neq 0$,

$$\omega^2 = \omega_c^2 + \left[\frac{2\pi n_s e^2}{m\epsilon_s}\frac{1}{1-\cos(k_z a)} - \frac{n_s V_{00}^{\rm xc}}{m}\right]q^2 , \tag{30b}$$

which is a cyclotron-acoustic plasmon mode. For the case of $k_z=0$, on the other hand, we find

$$\omega^2 = \omega_c^2 + \Omega_p^2 . \tag{30c}$$

Here, $\Omega_p$ is the effective 3D plasmon frequency. The physical interpretation of these modes is essentially that given for the zero-magnetic-field intrasubband modes, except that now there is an extra restoring force due to the motion of the electrons perpendicular to the magnetic field.

The higher roots of Eq. (29) are obtained, upon truncating the determinant to diagonal form in the usual manner, as

$$\omega^2 = \Omega_{n0}^2(1+\alpha_{nn}-\beta_{nn}) - \Omega_{n0}^2\mu_{nn}S(q,k_z)q + \Omega_{n0}^2\left[\Omega_{n0}\left[\frac{l_H(\alpha_{nn}-\beta_{nn})}{2}\right]^2 \times\left[\frac{(\pi l_H^2 n_s+3)\Omega_+}{\Omega_{n0}^2(\alpha_{nn}-\beta_{nn})-\omega_c^2-2\omega_c\Omega_{n0}} - \frac{(\pi l_H^2 n_s+1)\Omega_-}{\Omega_{n0}^2(\alpha_{nn}-\beta_{nn})-\omega_c^2+2\omega_c\Omega_{n0}}\right] - \gamma_{nn}\right]q^2 . \tag{31}$$

This is the same result as in the case of a single quantum well, except that the term linear in $q$ is modified by the structure factor $S(q,k_z)$. Note that as in the single—quantum-well case, only the $O(q^2)$ term is affected by the magnetic field.

In the weak-coupling limit, $S=1$, and each layer supports its own intersubband plasmon. In the strong-coupling limit, we have two cases: $k_z \neq 0$ and $k_z=0$. These give the following results.

*Case 1: $qa \ll 1$, $k_z \neq 0$.* Here, $S = qa[1-\cos(k_z a)]^{-1}$, and thus

$$\omega^2 = \Omega_{n0}^2(1+\alpha_{nn}-\beta_{nn}) + \Omega_{n0}^2\left[\frac{-\mu_{nn}a}{1-\cos(k_z a)} + \Omega_{n0}\left[\frac{l_H(\alpha_{nn}-\beta_{nn})}{2}\right]^2\left[\frac{(\pi l_H^2 n_s+3)\Omega_+}{\Omega_{n0}^2(\alpha_{nn}-\beta_{nn})-\omega_c^2-2\omega_c\Omega_{no}} - \frac{(\pi l_H^2 n_s+1)\Omega_-}{\Omega_{n0}^2(\alpha_{nn}-\beta_{nn})-\omega_c^2+2\omega_c\Omega_{n0}}\right] - \gamma_{nn}\right]q^2 . \tag{32a}$$

As before, the term proportional to $S(q,k_z)q$ becomes $O(q^2)$, and the constant term is unaffected.

*Case 2: $qa \ll 1$, $k_z=0$.* In this limit, $S(q,k_z) \simeq 2/qa$, and the dispersion relation simplifies to

$$\omega^2=\Omega_{n0}^2(1+\alpha_{nn}-\beta_{nn}-2\mu_{nn}/a)+O(q^2) . \tag{32b}$$

The $O(q^2)$ term is unaffected. The physical interpretation of these results has been discussed above.

#### 4. *Effects of electron-phonon coupling*

Since $GaAs/Ga_{1-x}Al_xAs$ (our prototypical semiconductor superlattice) is a polar semiconductor, we expect that the effects of electron-phonon coupling are important. To include the effects of phonons, we modify the background dielectric constant $\epsilon_s$ into a frequency-dependent dielectric function

$$\epsilon_s(\omega)=\epsilon_\infty \left[ \frac{\omega^2-\omega_{\rm L}^2}{\omega^2-\omega_{\rm T}^2} \right] , \tag{33}$$

where $\omega_{\rm L}$ and $\omega_{\rm T}$ are the longitudinal- and transverse-optical phonon frequencies. Then the quantity $V_{nm}$ we defined in Eq. (17) becomes

$$V_{nm}(\vec{q},\omega;l,l')=\frac{\omega^2-\omega_{\rm T}^2}{\omega^2-\omega_{\rm L}^2} V_{nm}(\vec{q};l,l') , \tag{34}$$

where $V_{nm}(q,l,l')$ is the quantity defined in Eq. (12), but with $n'=m'=0$, and with $\epsilon_s$ replaced by $\epsilon_\infty$.

The effect of the electron-phonon coupling on the exchange-correlation perturbation is somewhat more difficult to take into account, however. We include the effects of electron-phonon coupling in the simplest possible manner. We write the exchange-correlation perturbation as a sum of an exchange part and a correlation part. The latter is discarded. We note that the former depends linearly on the coupling constant $e^2$, and thus we can include the phonons by simply replacing $e^2$ by $e^2/\epsilon_s(\omega)$. This can be done, however, when the conduction band contains a single nondegenerate minimum (valley). In the case of a multivalley structure, exchange effects can have profound consequences for the electronic structure of Q2D EG's, as shown by Yi and Quinn.[31] Here, we will assume that only a single valley exists (this is true for GaAs, but not for silicon and germanium). $V_{nm}^{\rm xc}$ then becomes

$$V_{nm}^{\rm xc}(\omega;l,l')=\frac{\omega^2-\omega_{\rm T}^2}{\omega^2-\omega_{\rm L}^2} V_{nm}^{x}(l,l') , \tag{35}$$

where $V_{nm}^x$ is the quantity defined in (14), but with $n'=m'=0$ and $V_{\rm xc}[n]$ replaced by $V_x[n]$. A commonly used form of the latter (especially for numerical calculations) is the Slater $X\alpha$ form,

$$V_{\rm xc}[n]=-\alpha n^{1/3} .$$

By using (34) and (35), we can express our condition (22) for the collective modes as

$$\det \left| \delta_{nm} - \left[ \frac{\omega^2-\omega_{\rm T}^2}{\omega^2-\omega_{\rm L}^2} \right] \chi_{m0}(\vec{q},\omega)[V_{nm}(q)-V_{nm}^x+S_-\tilde{V}_{nm}(q)+S_+\tilde{V}_{mn}(q)] \right| =0 . \tag{36}$$

In the limit of 2D quantum wells, this yields

$$\omega^2=\tfrac{1}{2}[\omega_{\rm L}^2+\omega_p^2(q)S-\beta_{00}q^2]\pm\tfrac{1}{2}\{[\omega_{\rm L}^2+\omega_p^2(q)S-\beta_{00}q^2]^2-4\omega_{\rm T}^2[\omega_p^2(q)S-\beta_{00}q^2]\}^{1/2} . \tag{37}$$

Here we have defined $\beta_{00}=N_sV_{00}^x/m$. This is just the result of Bloss and Brody[25] if we neglect the effects of exchange ($\beta_{00}=0$). In the weak-coupling limit, $S=1$, and Eq. (37) yields coupled 2D-plasmon—optical-phonon modes.[32] In the strong-coupling limit with $k_z=0$, we have $S(q,k_z)\omega_p^2(q)=\Omega_p^2$, and Eq. (37) yields coupled effective 3D-plasmon—optical-phonon modes. In the strong-coupling limit with $k_z\neq 0$,

$$\omega_p^2(q)S(q,k_z)\simeq\frac{2\pi n_s e^2}{m\epsilon_\infty}\frac{aq^2}{1-\cos(k_za)} ,$$

and Eq. (37) yields coupled 3D-acoustical-plasmon—optical-phonon modes.

In the case of a uniform dc magnetic field pointing along the superlattice axis, the collective modes are given by (36), but with $\chi_{m0}$ replaced by $\chi_{m0}^{(H)}$. For 2D wells, Eq. (36) yields

$$\omega^2=\tfrac{1}{2}[\omega_{\rm L}^2+\omega_c^2+\omega_p^2(q)S]\pm\frac{1}{2}\{[\omega_{\rm L}^2+\omega_c^2+\omega_p^2(q)S]^2-4[\omega_{\rm L}^2\omega_c^2+\omega_{\rm T}^2\omega_p^2(q)S]\}^{1/2} , \tag{38}$$

where we have neglected the effects of exchange. In the weak-coupling limit, this yields coupled optical-phonon—2D-magnetoplasmon modes. In the strong-coupling limit, with $k_z=0$, we obtain coupled optical-phonon—3D-magnetoplasmon modes, where the effective 3D magnetoplasmon frequency is $(\omega_c^2+\Omega_p^2)^{1/2}$ with $\Omega_p$ as defined above. In the strong-coupling limit with $k_z\neq 0$, on the other hand, Eq. (38) yields coupled optical-phonon—"acoustical"-magnetoplasmon modes, where the acoustical-magnetoplasmon frequency is given by $\{\omega_c^2+\frac{1}{2}q^2a^2[1-\cos(k_za)]^{-1}\}^{1/2}$. In Fig. 3, we have plotted the two branches of Eq. (38) as a function of magnetic field for parameters corresponding to the systems studied by Olego *et al.*

The results for combined optical-phonon—intersubband modes [these are the higher roots of Eq. (36)] are much simplified by keeping only $O(q)$ terms. Keeping only the diagonal elements in (36) yields, to $O(q)$,

$$\omega^2=\tfrac{1}{2}[\omega_L^2+\Omega_{n0}^2(1+\alpha_{nn}-\beta_{nn}-\mu_{nn}Sq)]\pm\tfrac{1}{2}\{[\omega^2+\Omega_{n0}^2(1+\alpha_{nn}-\beta_{nn}-\mu_{nn}Sq)]^2 -4[\omega_L^2+\omega_T^2(\alpha_{nn}-\beta_{nn}-\mu_{nn}Sq)]\Omega_{n0}^2\}^{1/2}\,. \tag{39}$$

Equation (39) also persists in the presence of a magnetic field; only the coefficient of $q^2$ changes.

In the weak-coupling limit, we have the phonon-intersubband modes[32]

$$\omega^2=\tfrac{1}{2}[\omega_L^2+\Omega_{n0}^2(1+\alpha_{nn}-\beta_{nn}-\mu_{nn}q)]\pm\tfrac{1}{2}\{[\omega_L^2+\Omega_{n0}^2(1+\alpha_{nn}-\beta_{nn}-\mu_{nn}q)]^2-4[\omega_L^2+\omega_T^2(\alpha_{nn}-\beta_{nn}-\mu_{nn}q)]\Omega_{n0}^2\}^{1/2}\,.$$

In the strong-coupling limit, with $k_z\neq 0$, $Sq$ is of $O(q^2)$, so the terms in (39) involving $\mu_{nn}$ disappear. For $k_z=0$, $Sq\simeq 2/a$, and (39) becomes

$$\omega^2=\tfrac{1}{2}[\omega_L^2+\Omega_{n0}^2(1+\alpha_{nn}-\beta_{nn}-2\mu_{nn}/a)]\pm\tfrac{1}{2}\{[\omega_L^2+\Omega_{n0}^2(1+\alpha_{nn}-\beta_{nn}-2\mu_{nn}/a)]^2 -4[\omega_L^2+\omega_T^2(\alpha_{nn}-\beta_{nn}-2\mu_{nn}/a]\Omega_{n0}^2\}^{1/2}\,,$$

which shows the typical softening of the intersubband part of the mode in this limit.

### E. Flat minibands: Type-II superlattices

We now proceed to discuss the electronic collective modes of type-II superlattices. The model we use for the electronic structure of type-II superlattices is essentially the same as that adopted for type-I superlattices, except that every other layer contains holes instead of electrons. The layers are labeled by an integer; even-numbered layers will be taken to be electron layers, while odd-numbered layers are hole layers. The solutions of the Schrödinger equation are taken to be

$$\psi_{nj\vec{k}}(\vec{x})=e^{i\vec{k}\cdot\vec{r}}\psi_{nj}(z-ja)\,. \tag{40a}$$

where

$$\psi_{nj}(z-ja)=\begin{cases}\xi_n(z-ja) & \text{for } j \text{ even},\\ \eta_n(z-ja) & \text{for } j \text{ odd}.\end{cases} \tag{40b}$$

Here, $\xi_n(z)$ is the $n$th electron subband wave function, while $\eta_n(z)$ is the $n$th hole subband wave function. The electron-number density and mass are denoted by $n_e$ and $m_e$, respectively, and are taken to be the same in all electron layers, while the hole-number density and mass are denoted by $n_h$ and $m_h$, respectively. The eigenvalue belonging to the solution (40a) is

$$\epsilon_{nj\vec{k}}=\epsilon_{nj}+\frac{\hbar^2k^2}{2m_j}\,. \tag{40c}$$

Thus electron and hole layers have their own subband ladders. It should be noted that our model also includes the possibility that the odd layers have, instead of holes, electrons with a different density and effective mass than even layers.

To obtain the electronic collective modes of the system, we use the same procedure as before. We assume an external perturbing potential of the form given in Eq. (2). This gives rise to some total self-consistent potential to which the carriers (electrons and holes) respond. Linear-response theory gives the carrier density perturbation in terms of the total self-consistent potential; this closes the self-consistency loop and yields conditions for the collective modes to occur.

We proceed essentially as before. We assume an external perturbation with only one Fourier component, and assume a total self-consistent potential additively composed of the external part, a Hartree perturbation, and an exchange-correlation potential. The density response is

$$\delta n(\vec{q},\omega;z)=\sum_{\substack{m,m'\\ j}}\Pi^{(j)}_{mm'}(\vec{q},\omega)\langle m\,|\,v_j(\vec{q},\omega)\,|\,m'\rangle \times\psi_{mj}(z-ja)\psi_{m'j}(z-ja)\,, \tag{41}$$

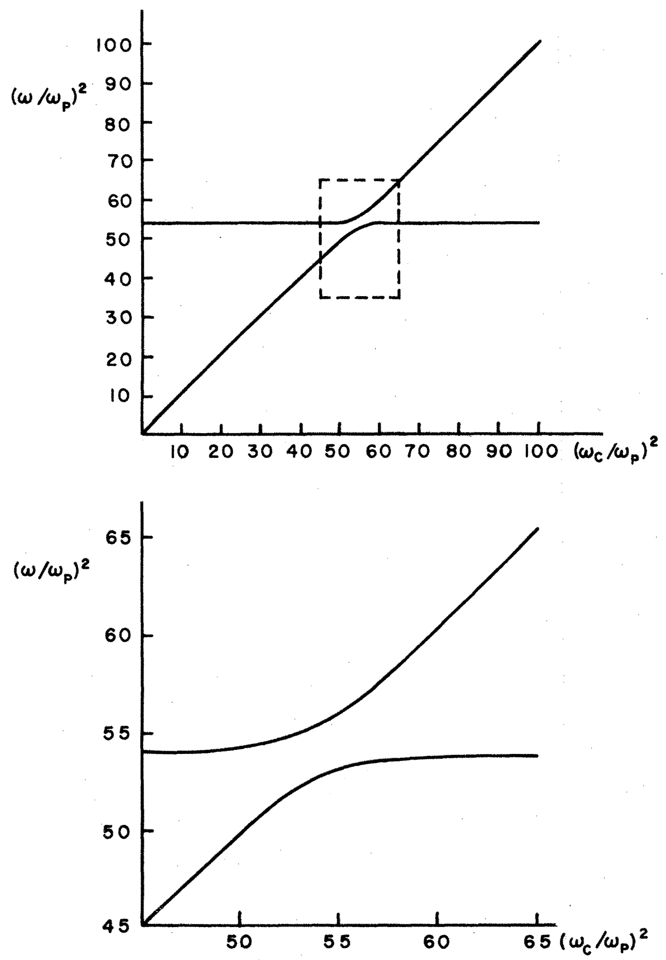


FIG. 3. Plot of Eq. (38) as a function of magnetic field. The two branches of the coupled optical-phonon-magnetoplasmon modes are clearly shown. The lower part of the figure is an enlargement of the interaction region of the two branches of the coupled optical-phonon—magnetoplasmon model.

where we have assumed that carriers in adjacent layers have zero overlap, and

$$\Pi_{mm'}^{(j)}(\vec{q},\omega)=2\sum_{k}\frac{f(\epsilon_{n',\vec{k}+\vec{q};j})-f(\epsilon_{n,\vec{k};j})}{\epsilon_{n',\vec{k}+\vec{q};j}-\epsilon_{n,\vec{k};j}-\hbar\omega} \qquad (42)$$

is the irreducible polarization for the $j$th layer. Note that $\Pi^{(j)}$ depends on whether $j$ is even or odd, in contrast to the case of the type-I superlattices. The Hartree and exchange-correlation perturbations are given, as usual, by

$$v^H(\vec{q},\omega;z)=\frac{2\pi e^2}{\epsilon_s q}\int_{-\infty}^{\infty}dz'e^{-q|z-z'|}\delta n(\vec{q},\omega;z') \qquad (43a)$$

and

$$V^{xc}(\vec{q},\omega;z)=\frac{\delta V_{xc}[n]}{\delta n}\delta n(\vec{q},\omega;z)\ . \qquad (43b)$$

The matrix elements of $V^H$ and $V^{xc}$ are

$$\langle n\,|\,v_j^H(\vec{q},\omega)\,|\,n'\rangle=\sum_{\substack{m,m'\\ j'}}\Pi_{mm'}^{(j')}(\vec{q},\omega)\langle m\,|\,v_{j'}(q)\,|\,m'\rangle \times V_{nn'}^{mm'}(\vec{q};j,j')\ , \qquad (44a)$$

where

$$V_{nn'}^{mm'}(\vec{q};j,j')=\frac{2\pi e^2}{\epsilon_s q}\int dz\,dz'\psi_{nj}(z)\psi_{n'j}(z) \times e^{-q|z-z'+(j-j')a|} \times \psi_{mj'}(z')\psi_{m'j'}(z') \qquad (44b)$$

and

$$\langle n\,|\,v_j^{xc}(\vec{q},\omega)\,|\,n'\rangle=-\sum_{\substack{m,m',\\ j'}}\Pi_{mm'}^{(j')}(\vec{q},\omega)\langle m\,|\,v_{j'}(\vec{q},\omega)\,|\,m'\rangle \times V_{nn',mm'}^{xc}(j,j')\ , \qquad (45a)$$

where

$$V_{nn',mm'}^{xc}(j,j')=\delta_{jj'}V_{nn',mm'}^{xc}(j)\ , \qquad (45b)$$

with

$$V_{nn',mm'}^{xc}(j)=-\int dz\,\psi_{nj}(z)\psi_{n'j}(z)\frac{\delta v_{xc}[n_0(j)]}{\delta n} \times \psi_{mj}(z)\psi_{m'j}(z)\ . \qquad (45c)$$

Note that $V^{xc}$ now depends on the layer index $l$; this is because the exchange-correlation energy will be different for electrons and holes, since they will have different densities and effective masses.

Taking the electric quantum limit and setting $n'=0$, we obtain Eqs. (44a) and (45a) in the form

$$\langle n\,|\,v_j^H(\vec{q},\omega)\,|\,0\rangle=\sum_{mj'}\chi_{m0}^{(j')}(\vec{q},\omega)\langle m\,|\,v_{j'}(\vec{q},\omega)\,|\,0\rangle \times V_{nm}(j,j')\ , \qquad (46)$$

$$\langle n\,|\,v_j^{xc}(\vec{q},\omega)\,|\,0\rangle=-\sum_{mj'}\chi_{m0}^{(j')}(\vec{q},\omega)\langle m\,|\,v_{j'}(\vec{q},\omega)\,|\,0\rangle \times V_{nm}^{xc}(j,j')\ , \qquad (47)$$

where $V_{nm}(\vec{q};j,j')$ is the expression in (44b) with $n'=m'=0$, and $V_{nm}^{xc}(j,j')$ is the expression in (45b) and (45c) with $n'=m'=0$. Thus the matrix elements of the total self-consistent potential satisfy

$$\langle n\,|\,v_j(\vec{q},\omega)\,|\,0\rangle=\langle n\,|\,v_j^{ext}(\vec{q},\omega)\,|\,0\rangle+\sum_{m,j'}\chi_{m0}^{(j')}(\vec{q})\langle m\,|\,v_{j'}(\vec{q},\omega)\,|\,0\rangle \times[V_{nm}(\vec{q};j,j')-V_{nm}^{xc}(j,j')]\ . \qquad (48)$$

The condition for the collective modes follows as

$$\langle n\,|\,v_j(\vec{q},\omega)\,|\,0\rangle=\sum_{m,j'}\chi_{m0}^{(j')}(\vec{q},\omega)\langle m\,|\,v_{j'}(\vec{q},\omega)\,|\,0\rangle \times[V_{nm}(\vec{q};j,j')-V_{nm}^{xc}(j,j')]\ . \qquad (49)$$

We can use Eq. (45b) to bring the exchange-correlation perturbation matrix elements to the left-hand side of (49). The sum over $j'$ on the right-hand side can then be split into two parts: one over even $j'$ and one over odd $j'$. In the former, we set $\chi^{(j')}=\chi^{(e)}$, the electron polarizability, and in the latter we set $\chi^{(j')}=\chi^{(h)}$, the hole polarizability. Since $V_{nm}$ in Eq. (49) is not diagonal in the layer indices (this being due to the electrostatic interaction of the electron- and hole-density perturbation in adjacent layers), we have to write Eq. (49) as two equations, one for even $j$ and one for odd $j$. We can then use the ansatz

$$\langle n\,|\,v_j(\vec{q},\omega)\,|\,n'\rangle=\begin{cases}e^{ik_zja}\langle n\,|\,v_0(\vec{q},\omega)\,|\,n'\rangle & \text{for } j \text{ even,}\\ e^{ik_z(j-1)a}\langle n\,|\,v_1(\vec{q},\omega)\,|\,n'\rangle & \text{for } j \text{ odd,}\end{cases} \qquad (50)$$

and make the usual assumption that wave functions of carriers in adjacent layers do not overlap, to arrive at

$$\sum_m\{\delta_{nm}-\chi_{m0}^{(e)}(\vec{q},\omega)[V_{nm}^{(e\text{-}e)}(q)-V_{nm}^{xc}(e)+S_-\tilde{V}_{nm}^{(e\text{-}e)}(q)+S_+\tilde{V}_{mn}^{(e\text{-}e)}(q)]\}\langle m\,|\,v_0\,|\,0\rangle = \sum_m\chi_{m0}^{(h)}(\vec{q},\omega)[\tilde{S}_+\tilde{V}_{mn}^{(h\text{-}e)}(q)+S_-\tilde{V}_{nm}^{(e\text{-}h)}(q)]\langle m\,|\,v_1\,|\,0\rangle \qquad (51a)$$

and

$$\sum_m\{\delta_{nm}-\chi_{m0}^{(h)}(\vec{q},\omega)[V_{nm}^{(h\text{-}h)}(q)-V_{nm}^{xc}(h)+S_-\tilde{V}_{mn}^{(h\text{-}h)}(q)+S_+\tilde{V}_{mn}^{(h\text{-}h)}(q)]\}\langle m\,|\,v_1\,|\,0\rangle = \sum_m\chi_{m0}^{(e)}(\vec{q},\omega)[\tilde{S}_-\tilde{V}_{nm}^{(h\text{-}e)}(q)+\tilde{S}_+\tilde{V}_{mn}^{(e\text{-}h)}(q)]\langle m\,|\,v_0\,|\,0\rangle\ . \qquad (51b)$$

Here we have introduced the symbols $S_\pm$ and $\tilde{S}_\pm$ defined by

$$S_\pm=(1-e^{\pm 2ik_z a}e^{-2qa})^{-1}-1$$

and

$$\tilde{S}_\pm=e^{\pm ik_z a-qa}(1-e^{\pm 2ik_z a-2qa})^{-1} .$$

It is useful to define

$$S=1+S_+ +S_- =\frac{\sinh(2qa)}{\cosh(2qa)-\cos(2k_z a)} , \quad (52a)$$

and

$$S'=\tilde{S}_+ +\tilde{S}_- =\frac{2\cos(k_z a)\sinh(qa)}{\cosh(2qa)-\cos(2k_z a)} . \quad (52b)$$

Note that the structure factor in (52a) is the same as the one considered before, except that $a$ is replaced by $2a$. This is because the superlattice parameter is doubled; note the analogy with a linear diatomic chain. In Eq. (51), we have also defined

$$V_{nm}^{(e\text{-}e)}(q)=\frac{2\pi e^2}{\epsilon_s q}\int dz\, dz'\xi_n(z)\xi_0(z) \times e^{-q|z-z'|}\xi_m(z')\xi_0(z') , \quad (53)$$

$$\tilde{V}_{nm}^{(e\text{-}h)}(q)=\frac{2\pi e^2}{\epsilon_s q}\int dz\, dz'\xi_n(z)\xi_0(z) \times e^{-q(z-z')}\eta_m(z')\eta_0(z) , \quad (54)$$

and

$$V_{nm}^{\rm xc}(e)=-\int dz\, \xi_n(z)\xi_0(z)\frac{\delta v_{\rm xc}[n_0(e)]}{\delta n}\xi_m(z)\xi_0(z) . \quad (55)$$

The quantity $\tilde{V}_{nm}^{(e\text{-}e)}$ is defined as in (53), but with $|z-z'|$ replaced by $(z-z')$. The quantities $V_{nm}^{(h\text{-}h)}$ and $\tilde{V}_{nm}^{(h\text{-}h)}$ are the same as $V^{(e\text{-}e)}$ and $\tilde{V}^{(e\text{-}e)}$, except that the electron functions $\xi_n$ are replaced by the hole functions $\eta_n$; similarly, $V_{nm}^{\rm xc}(h)$ is obtained from (55) by replacing $\xi_n$ with $\eta_n$ and the electron density $n_0(e)$ by the hole density $n_0(h)$. Finally, $\tilde{V}^{(h\text{-}e)}$ can be obtained from (54) by interchanging the roles played by the electron and hole functions.

To solve Eqs. (51) as they stand is very difficult, and so we make the approximation that only the $n=m$ terms contribute. This yields two linear equations relating $\langle n|v_0|0\rangle$ and $\langle n|v_1|0\rangle$. Requiring that these matrix elements be nonvanishing gives

$$\{1-\chi_{n0}^{(e)}(\vec{q},\omega)[V_{nn}^{(e\text{-}e)}(q)-V_{nn}^{\rm xc}(e)+(S_+ +S_-)\tilde{V}_{nn}^{(e\text{-}e)}(q)]\}\{1-\chi_{n0}^{(h)}(\vec{q},\omega)[V_{nn}^{(h\text{-}h)}(q)-V_{nn}^{\rm xc}(h)+(S_+ +S_-)\tilde{V}_{nn}^{(h\text{-}h)}(q)]\}$$
$$=\chi_{n0}^{(e)}(\vec{q},\omega)\chi_{n0}^{(h)}(\vec{q},\omega)[\tilde{S}_+\tilde{V}_{nn}^{(h\text{-}e)}(q)+\tilde{S}_-\tilde{V}_{nn}^{(e\text{-}h)}(q)][\tilde{S}_-\tilde{V}_{nn}^{(h\text{-}e)}(q)+\tilde{S}_+\tilde{V}_{nn}^{(e\text{-}h)}(q)] . \quad (56)$$

### 1. *Intrasubband modes*

To obtain the intrasubband modes, we set $n=0$ in Eq. (56). Then, $V_{00}^{(e\text{-}e)}=V_{e\text{-}e}^{(h\text{-}h)}=2\pi e^2/\epsilon_s q$ and $V_{00}^{(h\text{-}e)}=V_{00}^{(e\text{-}h)}=2\pi e^2/\epsilon_s q$. We assume the long-wavelength limit, $q\ll 2k_F$, thus $\chi_{00}^{(e)}\simeq n_e q^2/m_e\omega^2$ and $\chi_{00}^{(h)}\simeq n_h q^2/m_h\omega^2$. By omitting the vertex corrections for simplicity, Eq. (56) yields

$$\left[1-\frac{2\pi e^2}{\epsilon_s q}\Pi^{(e)}(q,\omega)S\right]\left[1-\frac{2\pi e^2}{\epsilon_s q}\Pi^{(h)}(q,\omega)S\right] =\left[\frac{2\pi e^2}{\epsilon_s q}\right]^2\Pi^{(e)}\Pi^{(h)}(S')^2 \quad (57a)$$

or

$$[\omega^2-\omega_{pe}^2(q)S][\omega^2-\omega_{ph}^2(q)S]=\omega_{pe}^2(q)\omega_{ph}^2(q)(S')^2 , \quad (57b)$$

where

$$\omega_{pe}^2(q)=\frac{2\pi n_e e^2}{m_e\epsilon_s}q$$

and

$$\omega_{ph}^2(q)=\frac{2\pi n_h e^2}{m_h\epsilon_s}q$$

are the squares of the 2D electron- and hole-plasmon frequencies, respectively. Equations (57) can be solved as

$$\omega_\pm^2=\tfrac{1}{2}(\omega_{pe}^2+\omega_{ph}^2)S \pm\tfrac{1}{2}[(\omega_{pe}^2-\omega_{ph}^2)S^2+4\omega_{pe}^2\omega_{ph}^2(S')^2]^{1/2} . \quad (58)$$

In the weak-coupling limit, $qa\gg 1$, so $S=1$ and $S'=0$; Eq. (58) then yields the two solutions

$$\omega=\omega_{pe}(q),\quad \omega=\omega_{ph}(q) ,$$

and thus each layer supports its own 2D electron or hole plasmon.

In the strong-coupling limit, $qa\ll 1$. In this limit we have three cases to consider. First, we consider $k_z=0$. From Eqs. (52), we then have $S=S'=1/qa$. Equation (58) then yields the two modes

$$\omega_+^2=\Omega_{pe}^2+\Omega_{ph}^2,\quad \omega_-^2=[\Omega_{pe}^2\Omega_{ph}^2/(\Omega_{pe}^2+\Omega_{ph}^2)](qa)^2 ,$$

where $\Omega_{pe}^2=4\pi n_e e^2/m_e a\epsilon_s$ and $\Omega_{ph}^2=4\pi n_h e^2/m_h a\epsilon_s$.

The first mode is that of two independent simultaneously excited 3D electron and hole plasmons, in which the oscillating charge densities are in phase from one supercell to the next, and are out of phase within the supercell. This would be analogous to an optical phonon in a linear diatomic chain.[33] In the second mode, the oscillating change densities are in phase within the supercell and also between supercells, so that in effect they cancel each other, the restoring force disappears, and the frequency of the mode is zero.

Next, we consider the strong-coupling limit with $k_z\neq 0$. In this case, Eqs. (52) become

$$S\simeq\frac{2qa}{1-\cos(2k_z a)}$$

and

$$S'=\frac{2qa\cos(k_z a)}{1-\cos(2k_z a)} ,$$

and thus Eq. (58) yields

$$\omega_\pm^2=\frac{2\pi e^2}{\epsilon_s}\frac{aq^2}{1-\cos(2k_za)}\times\left\{\frac{n_e}{m_e}+\frac{n_h}{m_h}\pm\left[\left[\frac{n_e}{m_e}-\frac{n_h}{m_h}\right]^2+\frac{4n_en_h}{m_em_h}\cos^2(k_za)\right]^{1/2}\right\}. \quad (59)$$

Note that both branches are acoustic modes, with $\omega\propto q$. Equation (59) was first derived by Bloss.[25] The modes correspond to in-phase and out-of-phase motion between the electrons and holes within the unit supercell, with a phase difference between oscillations in adjacent cells given by $k_z$.

Finally, we consider the strong-coupling limit with $k_za=(n+\frac{1}{2})\pi$. In this case, Eq. (58) yields the independent 3D acoustical plasmons

$$\omega_+=\Omega_{pe}qa,\quad \omega_-=\Omega_{ph}qa\ .$$

### 2. Intersubband modes

The intersubband modes are obtained from Eq. (56), with $n\neq0$. To simplify the dispersion relation, we shall expand all quantities to $O(q)$ only. In the long-wavelength limit, $q\ll 2k_F$, we have

$$\chi_{n0}^{(e)}(q,\omega)=\frac{2n_e}{\hbar}\frac{\Omega_{n0}}{\omega^2-\Omega_{n0}^2}$$

and

$$\chi_{n0}^{(h)}(q,\omega)=\frac{2n_h}{\hbar}\frac{\overline{\Omega}_{n0}}{\omega^2-\overline{\Omega}_{n0}^2}\ ,$$

where $\hbar\overline{\Omega}_{n0}$ is the subband separation between the ground and $n$th hole subbands. Equation (56) then becomes

$$[\omega^2-\Omega_{n0}^2(1+\alpha_{nn}^{(e)}-\beta_{nn}^{(e)}-\mu_{nn}^{(e)}Sq)][\omega^2-\overline{\Omega}_{n0}^2(1+\alpha_{nn}^{(h)}-\beta_{nn}^{(h)}-\mu_{nn}^{(h)}Sq)]=\Omega_{n0}^2\overline{\Omega}_{n0}^2(\mu_{nn}^{(e)}S'q)(\mu_{nn}^{(h)}S'q)\ . \quad (60)$$

Here, we have introduced a number of symbols which are defined below,

$$\alpha_{nn}^{(e)}=\frac{2n_e}{\hbar\Omega_{n0}}\frac{2\pi e^2}{\epsilon_s}\left[-\int dz\,dz'\xi_n(z)\xi_0(z)\,|z-z'|\,\xi_n(z')\xi_0(z')\right],$$

$$\alpha_{nn}^{(h)}=\frac{2n_h}{\hbar\overline{\Omega}_{n0}}\frac{2\pi e^2}{\epsilon_s}\left[-\int dz\,dz'\eta_n(z)\eta_0(z)\,|z-z'|\,\eta_n(z')\eta_0(z')\right],$$

$$\beta_{nn}^{(e)}=\frac{2n_e}{\hbar\Omega_{n0}}\frac{2\pi e^2}{\epsilon_s}V_{nn}^{\rm xc}(e),\quad \beta_{nn}^{(h)}=\frac{2n_h}{\hbar\overline{\Omega}_{n0}}\frac{2\pi e^2}{\epsilon_s}V_{nn}^{\rm xc}(h),\quad \mu_{nn}^{(e)}=\frac{2n_h}{\hbar\Omega_{n0}}\frac{2\pi e^2}{\epsilon_s}\,|z_{n0}^{(e)}|^2\ ,$$

and

$$\mu_{nn}^{(h)}=\frac{2n_h}{\hbar\overline{\Omega}_{n0}}\frac{2\pi e^2}{\epsilon_s}\,|z_{n0}^{(h)}|^2\ .$$

Equation (60) may also be written as

$$\omega_\pm^2=\tfrac{1}{2}[\Omega_{n0}^2(1+\alpha_{nn}^{(e)}-\beta_{nn}^{(e)}-\mu_{nn}^{(e)}Sq)+\overline{\Omega}_{n0}^2(1+\alpha_{nn}^{(h)}-\beta_{nn}^{(h)}-\mu_{nn}^{(h)}Sq)]$$
$$\pm\tfrac{1}{2}\{[\Omega_{n0}^2(1+\alpha_{nn}^{(e)}-\beta_{nn}^{(e)}-\mu_{nn}^{(e)}Sq)+\overline{\Omega}_{n0}^2(1+\alpha_{nn}^{(h)}-\beta_{nn}^{(h)}-\mu_{nn}^{(h)}Sq)]^2+4\mu_{nn}^{(e)}\mu_{nn}^{(h)}(\Omega_{n0}\overline{\Omega}_{n0}S'q)^2\}^{1/2}\ . \quad (61)$$

In the weak-coupling limit, $S=1$, $S'=0$, and Eq. (61) yields two modes,

$$\omega^2=\Omega_{n0}^2(1+\alpha_{nn}^{(e)}-\beta_{nn}^{(e)}-\mu_{nn}^{(e)}q)$$

and

$$\omega^2=\overline{\Omega}_{n0}^2(1+\alpha_{nn}^{(h)}-\beta_{nn}^{(h)}-\mu_{nn}^{(h)}q)\ .$$

As expected, each layer supports its own intersubband mode independently of the other layers.

In the strong-coupling limit, $qa\ll1$. For the case $k_z-0$, we have $S=S'=1/qa$, and Eq. (61) yields the two branches,

$$\omega_\pm^2=\tfrac{1}{2}[\Omega_{n0}^2(1+\alpha_{nn}^{(e)}-\beta_{nn}^{(e)}-\mu_{nn}^{(e)}/a)+\overline{\Omega}_{n0}^2(1+\alpha_{nn}^{(h)}-\beta_{nn}^{(h)}-\mu_{nn}^{(h)}/a)]$$
$$\pm\tfrac{1}{2}\{[\Omega_{n0}^2(1+\alpha_{nn}^{(e)}-\beta_{nn}^{(e)}-\mu_{nn}^{(e)}/a)-\overline{\Omega}_{n0}^2(1+\alpha_{nn}^{(h)}-\beta_{nn}^{(h)}-\mu_{nn}^{(h)}/a)]^2+(4\mu_{nn}^{(e)}\mu_{nn}^{(h)}/a^2)\Omega_{n0}^2\overline{\Omega}_{n0}^2\}^{1/2}$$

Thus we have two branches of coupled electron-hole–softened intersubband modes. In these modes, there is in-phase and out-of-phase motion of the electrons and holes out of their planes, and since $k_z=0$, this motion is repeated periodically in all the supercells.

For the case $k_z\neq0$, on the other hand, $S=2qa[1-\cos(2k_za)]^{-1}$ and $S'=S\cos k_za$, and thus $Sq$ and $S'q$ are of $O(q^2)$. Equation (61) then yields the interesting result,

$$\omega^2=\Omega_{n0}^2(1+\alpha_{nn}^{(e)}-\beta_{nn}^{(e)})+O(q^2)\ ,$$

$$\omega^2=\overline{\Omega}^2_{n0}(1+\alpha^{(h)}_{nn}-\beta^{(h)}_{nn})+O(q^2)\ ,$$

and thus, to $O(q)$, we obtain separate electron and hole intersubband modes. Actually, the coupling occurs in the coefficient of $q^2$, but the fact that $k_z\neq 0$ screens zero-order and first-order (in $q$) coupling.

We have plotted Eqs. (58) and (61) in Fig. 4 for an InAs/GaSb system in which the carriers are confined to their respective layers by infinite square-well potentials. The qualitative features are in Fig. 2 for type-I systems, except that the intra- and intersubband plasmon bands are split due to the fact that for these two-component systems, the two species can move in phase or out of phase; this is analogous to the phonon modes in a periodic 1D chain.[33] (We have neglected the coupling between the intersubband and intrasubband modes.)

### 3. *Effects of dc magnetic field*

The effects of a uniform magnetic field (assumed to point parallel to the superlattice axis) are taken into account by using the solutions to the Schrödinger equation in the Landau gauge; these are

$$|\,n,l,\vec{k},j\,\rangle=e^{iky}u_{lj}(x+l_H^2k)\psi_{nj}(z-ja)\ ,$$

where $n$ is the subband index, $\vec{k}$ is the momentum in the $y$ direction, $l$ is the Landau-level index, and $j$ is the layer index.

The density response is given by

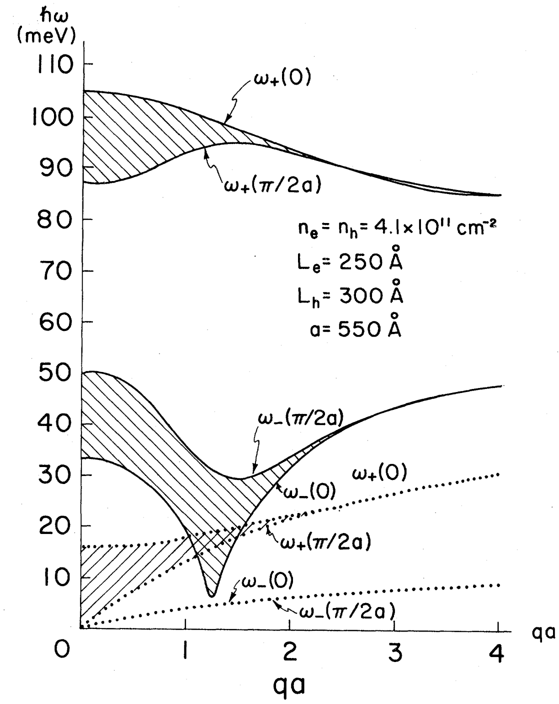


FIG. 4. Plot of $\omega$ given by Eqs. (58) and (61) as a function of $qa$, showing the band structure of the intrasubband and intersubband plasmon modes for a two-component system. Note the splitting between the modes due to the extra degree of freedom. Compare with Fig. 2. (Here, $L_e$ and $L_h$ are, respectively, the widths of the electron and hole square wells. The arguments of $\omega_\pm$ are the values of $k_za$.)

$$\delta n(\vec{q},\omega,z)=\sum_{\substack{n,n'\\ j}}\Pi^{(j)(H)}_{nn'}(\vec{q},\omega)\langle n\,|\,v_j(\vec{q},\omega)\,|\,n'\rangle\psi_{n'j}(z-ja)\psi_{nj}(z-ja)\ ,$$

where the polarizability is given by

$$\Pi^{(j)(H)}_{nn'}(q,\omega)=\frac{1}{\pi l_H^2}\sum_{l,l'}\left[\frac{f(\epsilon_{n'l'j})-f(\epsilon_{nlj})}{\epsilon_{n'l'j}-\epsilon_{nlj}-\hbar\omega}\right]|\,J^{(j)}_{l'l}(q)\,|^2\ .$$

We assume the electric quantum limit, and go through the same steps as before to obtain the condition for the collective modes. It is

$$\{1-\chi^{(e)(H)}_{n0}(q,\omega)[V^{(e\text{-}e)}_{nn}(q)-V^{\rm xc}_{nn}(e)+(S_++S_-)\widetilde{V}^{(e\text{-}e)}_{nn}(q)]\}\{1-\chi^{(h)(H)}_{n0}(q,\omega)[V^{(h\text{-}h)}_{nn}(q)-V^{\rm xc}_{nn}(h)+(S_++S_-)\widetilde{V}^{(h\text{-}h)}_{nn}(q)]\}$$

$$=\chi^{(e)(H)}_{n0}(q,\omega)\chi^{(h)(H)}_{n0}(q,\omega)[\widetilde{S}_+\widetilde{V}^{(h\text{-}e)}_{nn}(q)+\widetilde{S}_-\widetilde{V}^{(e\text{-}h)}_{nn}(q)][\widetilde{S}_-\widetilde{V}^{(h\text{-}e)}_{nn}(q)+\widetilde{S}_+\widetilde{V}^{(e\text{-}h)}_{nn}(q)]\ . \quad (62)$$

For the intrasubband modes, we set $n=0$. The dispersion relation then becomes

$$\left[1-\frac{2\pi e^2}{\epsilon_s q}\Pi^{(e)(H)}(q,\omega)S\right]\left[1-\frac{2\pi e^2}{\epsilon_s q}\Pi^{(h)(H)}(q,\omega)S\right]=\left[\frac{2\pi e^2}{\epsilon_s q}\right]^2\Pi^{(e)(H)}(q,\omega)\Pi^{(h)(H)}(q,\omega)(S')^2 \quad (63)$$

or

$$(\omega^2-\omega_{ce}^2-\omega_{pe}^2S)(\omega^2-\omega_{ch}^2-\omega_{ph}^2S)=\omega_{pe}^2\omega_{ph}^2(S')^2\ ,$$

which can be solved to give

$$\omega^2=\tfrac{1}{2}[\omega_{ce}^2+\omega_{ch}^2+(\omega_{pe}^2+\omega_{ph}^2)S]\pm\tfrac{1}{2}\{[\omega_{ce}^2-\omega_{ch}^2+(\omega_{pe}^2-\omega_{ph}^2)S]^2+4\omega_{pe}^2\omega_{ph}^2(S')^2\}^{1/2}\ . \quad (64)$$

(We have omitted effects of exchange and correlation for simplicity.)

In the weak-coupling limit, $S=1$ and $S'=0$, and Eq. (64) yields the two modes

$$\omega^2=\omega_{ce}^2+\omega_{pe}^2$$

and

$$\omega^2=\omega_{ch}^2+\omega_{ph}^2 \ .$$

Thus in this limit, each layer supports its own electron (or hole) 2D magnetoplasmon, as expected.

In the strong-coupling limit with $k_z=0$, we have $S=S'=1/qa$, and Eq. (64) yields the two branches

$$\omega^2=\tfrac{1}{2}(\omega_{ce}^2+\omega_{ch}^2+\Omega_{pe}^2+\Omega_{ph}^2)\pm\tfrac{1}{2}[(\omega_{ce}^2-\omega_{ch}^2+\Omega_{pe}^2-\Omega_{ph}^2)^2+4\Omega_{pe}^2\Omega_{ph}^2]^{1/2} ,$$

which are the coupled 3D magnetoplasmon modes of a two-component system. For $k_z\neq 0$, on the other hand, Eq. (64) yields

$$\omega^2=\frac{1}{2}\left[\omega_{ce}^2+\omega_{ch}^2+\frac{2(\Omega_{pe}^2+\Omega_{ph}^2)(qa)^2}{1-\cos(2k_za)}\right]\pm\frac{1}{2}\left[\left[\omega_{ce}^2-\omega_{ch}^2+\frac{2(\Omega_{pe}^2-\Omega_{ph}^2)(qa)^2}{1-\cos(2k_za)}\right]^2+\frac{16\Omega_{pe}^2\Omega_{ph}^2(qa)^4\cos^2(k_za)}{[1-\cos(2k_za)]^2}\right]^{1/2} ,$$

which are coupled electron-hole acoustic magnetoplasmons. When $k_za=(n+\frac{1}{2})\pi$, these modes are decoupled by the screening, and reduce to

$$\omega^2=\omega_{ce}^2+\Omega_{pe}^2(qa)^2, \quad \omega^2=\omega_{ch}^2+\Omega_{ph}^2(qa)^2 \ .$$

These are independent electron and hole acoustic magnetoplasmons.

For the intersubband modes, the effects of the magnetic field show up only in the $q^2$ term. Since we investigate these modes only to $O(q)$, they are unaffected in this order, and we obtain the same dispersion relations as above.

#### 4. *Effects of electron-phonon interaction*

The effects of electron-phonon coupling may be taken into account by replacing the background dielectric constant with a frequency-dependent dielectric function,

$$\epsilon_s(\omega)=\frac{\omega^2-\omega_{\rm L}^2}{\omega^2-\omega_{\rm T}^2}\epsilon_\infty ,$$

as before.

Consider first the intrasubband modes. Then,

$$\frac{2\pi e^2}{\epsilon_s(\omega)q}\Pi^{(e,h)}(q,\omega)=\frac{\omega^2-\omega_{\rm T}^2}{\omega^2-\omega_{\rm L}^2}\frac{\omega_{pe,h}^2}{\omega^2} ,$$

Setting this into Eq. (57a) yields

$$\left[1-\frac{\omega^2-\omega_{\rm T}^2}{\omega^2-\omega_{\rm L}^2}\frac{\omega_{pe}^2S}{\omega^2}\right]\left[1-\frac{\omega^2-\omega_{\rm T}^2}{\omega^2-\omega_{\rm L}^2}\frac{\omega_{ph}^2S}{\omega^2}\right]=\left[\frac{\omega^2-\omega_{\rm T}^2}{\omega^2-\omega_{\rm L}^2}\frac{\omega_{pe}^2S'}{\omega^2}\right]\left[\frac{\omega^2-\omega_{\rm T}^2}{\omega^2-\omega_{\rm L}^2}\frac{\omega_{ph}^2S'}{\omega^2}\right].$$

This can be solved for $\omega^2$ to give

$$\omega^2=\frac{(1+\gamma_\pm\omega_{\rm L}^2)\pm[(1+\gamma_\pm\omega_{\rm L}^2)^2-4\gamma_\pm\omega_{\rm T}^2]^{1/2}}{2\gamma_\pm} , \qquad (65)$$

where

$$\gamma_\pm=\frac{(\omega_{pe}^2+\omega_{ph}^2)S\pm[(\omega_{pe}^2-\omega_{ph}^2)^2S^2+4(\omega_{pe}\omega_{ph}S')^2]^{1/2}}{2(\omega_{pe}\omega_{ph})^2(S^2-S'^2)} . \qquad (66)$$

Note that Eq. (65) yields four branches, since the $(\pm)$ sign in front of the radical is uncorrelated with the $(\pm)$ subscripts on $\gamma$.

In the case of weak coupling, we obtain two sets of modes corresponding, respectively to $\gamma_-$ and $\gamma_+$. These are

$$\omega_\pm^2=\tfrac{1}{2}(\omega_{pe}^2+\omega_{\rm L}^2)\pm\tfrac{1}{2}[(\omega_{pe}^2+\omega_{\rm L}^2)^2-4\omega_{pe}^2\omega_{\rm T}^2]^{1/2} \qquad (67a)$$

and

$$\omega_\pm^2=\tfrac{1}{2}(\omega_{ph}^2+\omega_{\rm L}^2)\pm\tfrac{1}{2}[(\omega_{ph}^2+\omega_{\rm L}^2)^2-4\omega_{ph}^2\omega_{\rm T}^2]^{1/2} . \qquad (67b)$$

The modes are identified as coupled 2D-electron- (or hole-) plasmon—optical-phonon modes; they occur independently in each layer.

In the limit of strong coupling, we consider three cases: $k_z=0$, $k_z\neq 0$, and $k_za=(n+\frac{1}{2})\pi$. For $k_z=0$, $\gamma_+^{-1}=0$ and $\gamma_-^{-1}=\Omega_{pe}^2+\Omega_{ph}^2$, so the modes are

$$\omega_+^2=\omega_{\rm L}^2 ,$$

$$\omega_1^2=\tfrac{1}{2}\{(\Omega_{pe}^2+\Omega_{ph}^2+\omega_{\rm L}^2)\pm[(\Omega_{pe}^2+\Omega_{ph}^2+\omega_{\rm L}^2)^2-4(\Omega_{pe}^2+\Omega_{ph}^2)\omega_{\rm T}^2]^{1/2}\} .$$

The $\omega_+$ modes are just the longitudinal-optical phonons, while the $\omega_-$ modes are coupled optical-phonon—electron-hole plasmons.

For the case of $k_z\neq 0$, on the other hand, we obtain the four branches,

$$\omega^2=\tfrac{1}{2}(\omega_{\rm L}^2+\alpha_\pm q^2)\pm\tfrac{1}{2}[(\omega_{\rm L}^2+\alpha_\pm q^2)^2-4\omega_{\rm T}\alpha_\pm q^2]^{1/2} ,$$

where

$$\alpha_\pm q=\frac{2(\omega_{pe}\omega_{ph})^2a}{(\omega_{pe}^2+\omega_{ph}^2)\pm[(\omega_{pe}^2-\omega_{ph}^2)^2+4(\omega_{pe}\omega_{ph})^2\cos^2(k_za)]^{1/2}} .$$

Note that $\alpha_\pm$, as defined here, is independent of $q$, but depends on $k_z$. These excitations are coupled phonon—electron-hole acoustical-plasmon modes.

For the case in which $k_z a=(n+\frac{1}{2})\pi$, the oscillations are out of phase from one supercell to the next. We then obtain two decoupled sets of excitations of the form

$$\omega_\pm^2=\tfrac{1}{2}[\omega_L^2+\tfrac{1}{2}\Omega_{ph}^2(qa)^2] \pm\tfrac{1}{2}\{[\omega_L^2+\tfrac{1}{2}\Omega_{ph}^2(qa)^2]^2-2\omega_T^2\Omega_{ph}^2(qa)^2\}^{1/2}$$

and

$$\omega_\pm^2=\tfrac{1}{2}[\omega_L^2+\tfrac{1}{2}\Omega_{pe}^2(qa)^2] \pm\tfrac{1}{2}\{[\omega_L^2+\tfrac{1}{2}\Omega_{pe}^2(qa)^2]^2-2\omega_T^2\Omega_{pe}^2(qa)^2\}^{1/2},$$

which are coupled phonon-3D-electron (or -hole) acoustical plasmons.

Next, let us consider the effect of electron-phonon coupling on the intrasubband modes in a magnetic field. In this case, we have

$$\frac{2\pi e^2}{\epsilon_s(\omega)q}\Pi^{(e,h)(H)}(q,\omega)=\frac{\omega^2-\omega_T^2}{\omega^2-\omega_L^2}\frac{\omega_{pe,h}^2}{\omega^2-\omega_{ce,h}^2},$$

which may be substituted into Eq. (63) to yield

$$\left[1-\frac{\omega^2-\omega_T^2}{\omega^2-\omega_L^2}\frac{\omega_{pe}^2S}{\omega^2-\omega_{ce}^2}\right]\left[1-\frac{\omega^2-\omega_T^2}{\omega^2-\omega_L^2}\frac{\omega_{ph}^2S}{\omega^2-\omega_{ch}^2}\right] =\left[\frac{\omega^2-\omega_T^2}{\omega^2-\omega_L^2}\right]^2\frac{(\omega_{pe}\omega_{ph}S')^2}{(\omega^2-\omega_{ce}^2)(\omega^2-\omega_{ch}^2)}. \quad (68)$$

Solving this equation analytically, as it stands, for $\omega^2$ is a difficult task, since we obtain a quartic. The solution is greatly simplified, however, if we assume that the electron and hole cyclotron frequencies are the same. This essentially amounts to assuming that the effective masses of the electrons and holes are the same. This is not usually the case, but we will be able to obtain at least a qualitative idea of the structure of the modes this way. By setting $\omega_{ce}=\omega_{ch}=\omega_c$, Eq. (68) gives

$$\omega_\pm^2=\frac{1+(\omega_c^2+\omega_L^2)\gamma_\pm}{2\gamma_\pm}\pm\frac{1}{2\gamma_\pm}\{[1+(\omega_c^2+\omega_L^2)\gamma_\pm]^2 -4\gamma_\pm(\omega_T^2+\omega_L^2\omega_c^2\gamma_\pm)\}^{1/2}, \quad (69)$$

where $\gamma_\pm$ is the same as in Eq. (66).

In the weak-coupling limit, Eq. (69) yields two sets of modes given by

$$\omega_\pm^2=\tfrac{1}{2}(\omega_{ph}^2+\omega_c^2+\omega_L^2) \pm\tfrac{1}{2}[(\omega_{ph}^2+\omega_c^2+\omega_L^2)^2-4(\omega_{ph}^2\omega_T^2+\omega_c^2\omega_L^2)]^{1/2} \quad (70a)$$

and

$$\omega_\pm^2=\tfrac{1}{2}(\omega_{pe}^2+\omega_c^2+\omega_L^2) \pm\tfrac{1}{2}[(\omega_{pe}^2+\omega_c^2+\omega_L^2)^2-4(\omega_{pe}^2\omega_T^2+\omega_c^2\omega_L^2)]^{1/2}. \quad (70b)$$

Equations (70) may be compared with Eqs. (67). We see that the effect of the magnetic field is to shift the 2D plasmon frequencies by the cyclotron frequency, and thus we obtain coupled phonon-electron (or -hole) magnetoplasmon modes.

In the strong-coupling limit, we have four branches for $k_z\neq0$. These are given by

$$\omega^2=\tfrac{1}{2}(\omega_{L0}^2+\omega_c^2+\alpha_\pm q^2) \pm\tfrac{1}{2}[(\omega_{L0}^2+\omega_c^2+\alpha_\pm q^2)^2-4(\omega_L^2\omega_c^2+\omega_T^2\alpha_\pm q^2)]^{1/2},$$

with $\alpha_\pm$ given by the expression given before. This result is also understandable physically: The coupled electron-hole acoustical-plasmon frequencies are shifted upward by the cyclotron frequency. Thus we have coupled phonon—electron-hole acoustical magnetoplasmons.

For $k_z a=(n+\frac{1}{2})\pi$, we obtain two decoupled sets of phonon-electron (or -hole) 3D acoustical magnetoplasmons. The dispersion relations are

$$\omega_\pm^2=\tfrac{1}{2}[\omega_{L0}^2+\omega_c^2+\tfrac{1}{2}\Omega_{pe}^2(qa)^2] \pm\tfrac{1}{2}\{[\omega_L^2+\omega_c^2+\tfrac{1}{2}\Omega_{pe}^2(qa)^2]^2 -4[\omega_{L0}^2\omega_c^2+\tfrac{1}{2}\omega_T^2\Omega_{pe}^2(qa)^2]\}^{1/2}$$

and

$$\omega_\pm^2=\tfrac{1}{2}[\omega_{L0}^2+\omega_c^2+\tfrac{1}{2}\Omega_{ph}^2(qa)^2] \pm\tfrac{1}{2}\{[\omega_{L0}^2+\omega_c^2+\tfrac{1}{2}\Omega_{ph}^2(qa)^2]^2 -4[\omega_{L0}^2\omega_c^2+\tfrac{1}{2}\omega_{T0}^2\Omega_{ph}^2(qa)^2]\}^{1/2}.$$

For $k_z=0$, on the other hand, Eq. (68) yields the optical-phonon mode

$$\omega_\pm^2=\omega_{L0}^2,$$

and the cyclotron mode

$$\omega_-^2=\omega_c^2.$$

Finally, we consider the effect of electron-phonon coupling on the intersubband modes. A straightforward generalization of the procedure given in the discussion of the effects of electron-phonon coupling on the intersubband modes in type-I superlattices yields the expression

$$\left[1-\frac{\omega^2-\omega_T^2}{\omega^2-\omega_L^2}\frac{\Omega_{n0}^2}{\omega^2-\Omega_{n0}^2}\lambda_{nn}^{(e)}(q)\right]\left[1-\frac{\omega^2-\omega_T^2}{\omega^2\omega_L^2}\frac{\overline{\Omega}_{n0}^2}{\omega^2-\overline{\Omega}_{n0}^2}\lambda_{nn}^{(h)}(q)\right]=\left[\frac{\omega^2-\omega_T^2}{\omega^2-\omega_L^2}\right]^2\frac{\Omega_{n0}^2}{\omega^2-\Omega_{n0}^2}\frac{\overline{\Omega}_{n0}^2}{\omega^2-\overline{\Omega}_{n0}^2}(\mu_{nn}^{(e)}S'q)(\mu_{nn}^{(h)}S'q), \quad (71)$$

where we have defined

$$\lambda_{nn}^{(e)}(q)=\alpha_{nn}^{(e)}-\beta_{nn}^{(e)}-\mu_{nn}^{(e)}Sq\ ,$$

and similarly for $\lambda_{nn}^{(h)}(q)$. As in the case of the intrasubband modes, Eq. (71) is difficult to solve as it stands [compare Eq. (68)]. We make the approximation that the electron and hole subband separations are equal, $\Omega_{n0}=\overline{\Omega}_{n0}$, to obtain a qualitative idea of the modes implied by Eq. (71). Then, Eq. (71) yields

$$\omega_{\pm}^2=\frac{1}{2}\left[\frac{1}{\gamma_\pm}+(\omega_{\rm L}^2+\Omega_{n0}^2)\right]\pm\frac{1}{2}\left[\left[\frac{1}{\gamma_\pm}+(\omega_{\rm L}^2+\Omega_{n0}^2)\right]^2-4\left[\frac{1}{\gamma_\pm}\omega_{\rm T}^2+\omega_{\rm L}^2\Omega_{n0}^2\right]\right]^{1/2}, \tag{72}$$

where now

$$\gamma_\pm=\frac{1}{\Omega_{n0}^2}\frac{\lambda_{nn}^{(e)}+\lambda_{nn}^{(h)}\pm[(\lambda_{nn}^{(e)}-\lambda_{nn}^{(h)})^2+4\mu_{nn}^{(e)}\mu_{nn}^{(h)}(S'q)^2]^{1/2}}{2[\lambda_{nn}^{(e)}\lambda_{nn}^{(h)}-\mu_{nn}^{(e)}\mu_{nn}^{(h)}(S'q)^2]}\ . \tag{73}$$

In the weak-coupling limit, Eqs. (72) and (73) yield two sets of modes, corresponding, respectively, to $\gamma_-$ and $\gamma_+$. These are given by

$$\omega_\pm^2=\tfrac{1}{2}[\Omega_{n0}^2(1+\alpha_{nn}^{(e)}-\beta_{nn}^{(e)}-\mu_{nn}^{(e)}q)+\omega_{\rm L}^2]\pm\tfrac{1}{2}\{[\Omega_{n0}^2(1+\alpha_{nn}^{(e)}-\beta_{nn}^{(e)}-\mu_{nn}^{(e)}q)+\omega_{\rm L}^2]^2-4[\omega_{\rm L}^2+\omega_{\rm T}^2(\alpha_{nn}^{(e)}-\beta_{nn}^{(e)}-\mu_{nn}^{(e)}q)]\Omega_{n0}^2\}^{1/2} \tag{74a}$$

and

$$\omega_\pm^2=\tfrac{1}{2}[\Omega_{n0}^2(1+\alpha_{nn}^{(h)}-\beta_{nn}^{(h)}-\mu_{nn}^{(h)}q)+\omega_{\rm L}^2]\pm\tfrac{1}{2}\{[\Omega_{n0}^2(1+\alpha_{nn}^{(h)}-\beta_{nn}^{(h)}-\mu_{nn}^{(h)}q)+\omega_{\rm L}^2]^2-4[\omega_{\rm L}^2+\omega_{\rm T}^2(\alpha_{nn}^{(h)}-\beta_{nn}^{(h)}-\mu_{nn}^{(h)}q)]\Omega_{n0}^2\}^{1/2}\ . \tag{74b}$$

These expressions show that in the weak-coupling limit each layer supports its own coupled phonon-intersubband mode.

In the strong-coupling limit, on the other hand, we have two cases to consider, $k_z=0$ and $k_z\neq 0$. For $k_z=0$, Eqs. (72) and (73) yield the four branches

$$\omega^2=\tfrac{1}{2}[\omega_{\rm L}^2+\Omega_{n0}^2(1+b_{n\pm})]\pm\tfrac{1}{2}[\omega_{\rm L}^2+\Omega_{n0}^2(1+b_{n\pm})]^2-4\Omega_{n0}^2[\omega_{\rm L}^2+\omega_{\rm T}^2b_{n\pm}]\ ,$$

where we have defined $\eta_{nn}=\alpha_{nn}-\beta_{nn}$ and

$$b_{n\pm}=\frac{2[\eta_{nn}^{(e)}\eta_{nn}^{(h)}-a^{-1}(\eta_{nn}^{(e)}\eta_{nn}^{(h)}-\eta_{nn}^{(h)}\eta_{nn}^{(e)})]}{[\eta_{nn}^{(e)}+\eta_{nn}^{(h)}-a^{-1}(\mu_{nn}^{(e)}+\mu_{nn}^{(h)})]\pm\{[\eta_{nn}^{(e)}-\eta_{nn}^{(h)}-a^{-1}(\mu_{nn}^{(e)}-\mu_{nn}^{(h)})]^2+4a^{-2}\mu_{nn}^{(e)}\mu_{nn}^{(h)}\}^{1/2}}\ .$$

In this case, the excitations have the character of combined phonon-electron-hole intersubband modes.

For the case $k_z\neq 0$, $S\simeq 2qa[1-\cos(2k_za)]^{-1}$ and $S'\simeq S\cos(k_za)$, so that $Sq$ and $S'q$ are of $O(q^2)$. Since we limit our analysis to $O(q)$ for the intersubband modes, we can, to this order, put $Sq$ and $S'q$ equal to zero. The dispersion relations then reduce to those of Eqs. (74), differing only by terms of $O(q^2)$. The reason for this is that since $k_z\neq 0$, the density oscillations change in phase from one supercell to the next and the coupling between the electron and hole intersubband modes is screened out. We have already seen this sort of behavior in our previous discussion of the intersubband modes of type-II superlattices in the absence of electron-phonon coupling.

## III. CONCLUSIONS AND DISCUSSION

In this paper we have presented a survey of the electronic collective modes in single- and multiple—quantum-well systems, including the effects of electron-phonon coupling, magnetic fields, and retardation. We have found a very large variety of such modes: quasi-2D plasmons and magnetoplasmons, acoustical plasmons, intersubband modes, coupled phonon—quasi-2D plasmon and phonon—quasi-2D magnetoplasmon modes.

We have shown that the intrasubband modes display the appropriate crossover behavior (from 2D to 3D) on going from the weak-to strong-coupling regime. The intersubband modes also display a change in behavior upon going from weak- to strong-coupling regime, but the behavior in the strong-coupling limit cannot really be called 3D, since there is no 3D analog to intersubband modes.

The electrostatic modes (i.e., those which exist in the nonretarded limit) can be detected by light scattering, inelastic-electron-scattering, and infrared-absorption measurements. The effects of dispersion in the $x$-$y$ plane are more easily seen by light scattering experiments. In particular, it should be possible to observe the large softening of the intersubband modes in type-I superlattices for $q\neq 0$, which was described above. The $q\neq 0$ intersubband modes have been seen by Olego *et al.*,[30] who also observed the acoustical intrasubband plasmon predicted in Eq. (24). The electrostatic modes in type-II superlattices are probably not so simple to observe. Part of the difficulty is that the applicability of the theoretical results is not certain; the subband approximation may not be justified in systems in which the conduction-band edge is below the valence-band edge. Clearly, more work needs to be done on this system.

In our work, we have not considered overlap of carrier wave functions in adjacent layers. For systems in which the layers are thin, such overlap is important. In particular, the peculiar band structure of these systems (in which the minibands are no longer flat) suggests the possibility of so-called saddle-point excitons, in which an electron is excited from a valence-band maximum to a conduction-band saddle point (and vice versa). We have already discussed the necessary modifications to the above theory. In addition, it would be interesting (and important) to include the effects of the spacer layers (i.e., the layers separating the electron layers from each other) on the linear response of these systems.

## ACKNOWLEDGMENTS

This work was supported in part by the National Science Foundation Grant No. DMR-81-21069.

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