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Logarithmic dynamic scaling in spin-glasses

K. Binder
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106 and Institut für Festköperforschung, Kernforschungsanlage, D-5170 Jülich, West Germany

A. P. Young
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106 and Department of Mathematics, Imperial College, London SW7 2BZ, England

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Evidence is presented that "freezing" in spin-glasses is a consequence of anomalous critical slowing down associated with a zero-temperature transition: temperature and field dependences of the relaxation time are compatible with $\ln\tau \propto T^{-\nu z} f(HT^{-\Delta})$ both for nearest-neighbor $\pm J$-Ising models and $Eu_{0.4}Sr_{0.6}S$. The exponents ν, z, and Δ are estimated. Also, the nonlinear susceptibility of *Cu*Mu 1% is shown to be very similar to that of the $d=3$, $\pm J$ model, compatible with our description.

The nature of the freezing transition in real spin-glasses has remained controversial; some data (e.g., Refs. 1 and 2) seem to support the idea of a thermal-equilibrium phase transition, similar to that occurring in the infinite-range Edwards-Anderson model[3,4]—only the critical exponents being different from the mean-field ones. Other data show that the spectrum of relaxation times starts to broaden dramatically far above the freezing temperature T_f, and at T_f it ranges from microscopic times τ_0 (e.g., $\tau_0 \approx 10^{-12}$ sec) to the time scale of observation, t (e.g., Refs. 5 and 6). However, a simple picture of thermal activation over constant energy barriers (e.g., Ref. 7) does not work: the typical relaxation time τ increases faster than predicted by an Arrhenius law as T is lowered,[6] and the resulting frequency dependence of T_f is also weaker than the Arrhenius behavior $T_f^{-1}(\omega) \propto \ln(1/\omega\tau_0)$ (e.g., Refs. 8–10). Thus, if one accepts the theoretical prediction that the lower critical dimension for an Edwards-Anderson transition is $d_l = 4$, so that no nonzero transition temperature is possible at $d=3$,[11–14] one has to find *another cooperative mechanism* which is responsible for the freezing.

In this Rapid Communication we wish to present evidence that one can understand freezing as a consequence of the phase transition occurring at $T=0$ for $d<d_l$. The Edwards-Anderson susceptibility χ_{EA} and correlation length ξ_{EA}, defined for Ising spins by

$$[\langle S_i S_j \rangle_T^2]_{av} \propto r_{ij}^{-(d+2+\eta)} \exp(-r_{ij}/\xi_{EA}) \;, \qquad \chi_{EA} = \sum_j [\langle S_i S_j \rangle_T^2]_{av} \tag{1}$$

are assumed to diverge as $T \to 0$, i.e.,

$$\chi_{EA} \propto T^{-\gamma}, \quad \xi_{EA} \propto T^{-\nu} \;, \tag{2}$$

the brackets $[\cdots]_{av}$ denoting a configurational average over the quenched disorder. At a finite-temperature transition the various locally ordered states, in which a correlated region of size ξ_{EA}^d can exist, are mutually accessible by small-amplitude fluctuations of a coarse-grained local order parameter, and hence the relaxation time exhibits "ordinary" critical slowing down, $\tau \propto \xi_{EA}^z$, z being a dynamic exponent.[15] Near a transition at $T=0$, on the other hand, the local spin alignment is strong and fluctuations relating the various locally ordered states in anisotropic systems are "walls";[16] nucleating these walls requires thermal activation and hence it is natural to expect[17,18] that

$$\ln(\tau/\tau_0) \propto \Delta F(\xi_{EA})/T \propto \tilde{J} T^{-1} \xi_{EA}^{z-1/\nu} \propto T^{-z\nu} \;, \tag{3}$$

where $\tilde{J}$ is a typical interaction energy in the system and one assumes that ξ_{EA} also controls the heights of typical free-energy barriers. Static scaling at the $T=0$ transition implies, for the field dependence of M and ξ_{EA},[18–20]

$$1 - TM/H = T^{\beta} \tilde{M}(HT^{-\Delta}) \;, \tag{4a}$$

$$\xi_{EA}(T,H) = T^{-\nu} \tilde{\xi}(HT^{-\Delta}) \;, \tag{4b}$$

where[21] $\Delta = 1 + (\gamma+\beta)/2$. Equations (3) and (4) then immediately yield the field dependence of the relaxation time

$$\ln[\tau(T,H)/\tau_0] \propto T^{-z\nu} f(HT^{-\Delta}) \;. \tag{5}$$

Ample evidence is already available[19,22,23] to show that a zero-temperature transition occurs in the $d=2$ Ising model, with exponents

$$\nu \approx 2, \quad \nu z \approx 2, \quad \gamma \approx 4 \quad (d=2) \tag{6}$$

for a continuous distribution and probably a slightly different value for γ with a $\pm J$ distribution because $\eta \approx 0.4$ in this case.[21,22] In this paper we argue that the evidence in $d=3$ is also consistent with a $T=0$ transition.

Results for $\ln\tau$ from simulations of the $d=3$, $\pm J$ nearest-neighbor Ising model[24] are shown in Fig. 1(a) and are consistent with Eq. (3), but now

$$z\nu \approx 4 \quad (d=3) \;. \tag{7}$$

Since $T_f(\omega)$ can be defined from $\omega\tau[T_f(\omega), H=0]=1$, this result implies $[T_f(\omega)]^{-4} \propto \ln(1/\omega\tau_0)$. It is encouraging that data available for $Eu_{0.4}Sr_{0.6}S$ over a broad frequency range[8] are compatible with this result, Fig. 1(b). Equation (3) also leads to

$$\frac{1}{T_f} \frac{\Delta T_f}{\Delta \log_{10}\omega} = \frac{1}{z\nu \log_{10}(1/\omega\tau_0)} \;. \tag{8}$$

For Ruderman-Kittel-Kasuya-Yosida (RKKY) systems this quantity seems to be rather nonuniversal, varying from about $\frac{1}{50}$ in *Ni*Mn to around $\frac{1}{2000}$ in *Cu*Mn, *Ag*Mn, and *Au*Mn. With reasonable values of $\omega\tau_0$, e.g., $\omega\tau_0 \sim 10^{-10}$,

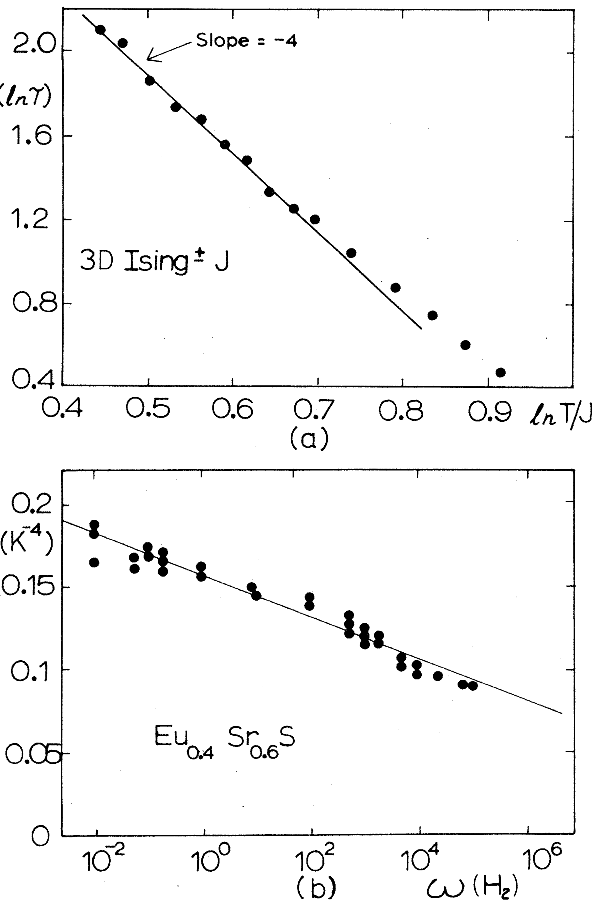

FIG. 1. (a) Log-log plot of $\ln\tau(T,H=0)$ vs T/J for the $d=3$, $\pm J$-Ising spin-glass (Ref. 24). (b) Experimental data (Ref. 8) for $T_f(\omega)$ in $Eu_{0.4}Sr_{0.6}S$ replotted as $[T_f(\omega)]^{-4}$ vs $\ln\omega$.

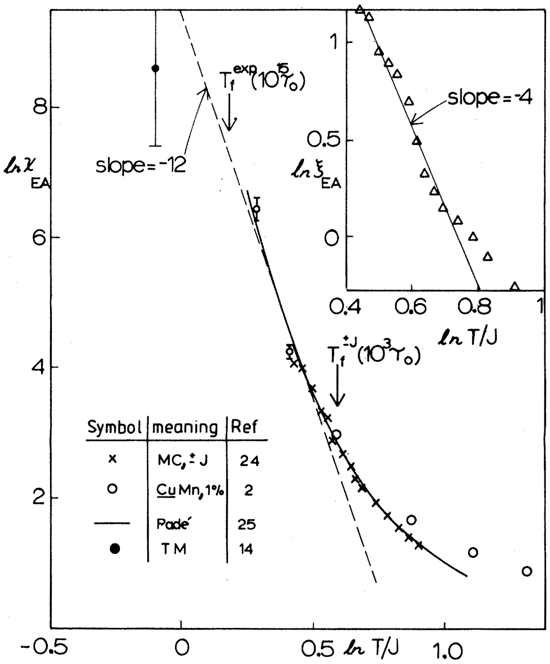

FIG. 2. Log-log plot of χ_{EA} vs T/J for the $d=3$ $\pm J$-Ising spin-glass. Crosses are Monte Carlo (MC) results (Ref. 24) and the solid curve is a Padé analysis (Ref. 25) of the high-temperature series (Ref. 11). The filled circle is a transfer matrix (TM) calculation (Ref. 14) and is obtained from $\chi_{EA} \approx 8\pi\xi_{EA}^3$ with $\xi_{EA} \approx 6 \pm 2$. Due to the smallness of the lattice $(4\times4\times10)$ we consider this estimate to be a lower bound only. An arrow marks the freezing temperature observed in earlier MC work (Ref. 12) for $\tau \approx 10^3\tau_0$. The open circles are data (Ref. 2) for 1% *Cu*Mn with the temperature axis scaled by taking $J=8.39$ K. Our χ_{EA} is $(a_3+2)/3$ in the notation of Ref. 2. The "static" transition temperature of 10.03 K, quoted by Ref. 2, is also shown. Taking "static" to mean $\omega \approx 10^{-3}$ Hz and assuming $\tau_0 \approx 10^{-12}$ sec, the relaxation time must be of order $10^{15}\tau_0$ at this temperature. The insert shows ξ_{EA} against T/J on a log-log plot for the $d=3$ Ising model, obtained by MC simulation (Ref. 24).

one would need $z\nu \simeq 20$ for *Cu*Mn, although our estimate, Eq. (7) works fairly well for *Ni*Mn. We shall return to this discrepancy at the end.

Figure 2 presents recent Monte Carlo estimates[24] of χ_{EA} and ξ_{EA} for the $d=3$, $\pm J$-Ising spin-glass, together with estimates of χ_{EA} from Padé approximants.[25] There is substantial curvature in this log-log plot, particularly for χ_{EA}, so that a phase transition at nonzero T_f is not ruled out. However, the $T_f(\tau)$ seen in Monte Carlo work[12] for $\tau \approx 10^3$ Monte Carlo steps per spin is clearly a dynamical effect because χ_{EA} and ξ_{EA} are rather small at this temperature. The data are certainly consistent with Eq. (2) and yield exponent values

$$\gamma \approx 12, \quad \nu \approx 4 \quad (d=3) \;, \tag{9}$$

which correspond to pure exponential decay of $[\langle S_i S_j\rangle^2]_{av}$ (Ref. 22) and lead to $\Delta \approx 7$. We also plot experimental results for χ_{EA} (Ref. 2) on 1% *Cu*Mn, taking a value of $J=8.39$ K to set the temperature scale. Agreement between theory and experiment is reasonable[26] and shows that a zero-temperature transition is a possible explanation of the data. *No* rescaling of the field has been carried out, which shows that RKKY spin-glasses behave in a surprisingly similar way to this very short-range Ising model in spite of Dzyloshinski-Moriya anisotropies, etc.[27] Monte Carlo simulations need larger fields to observe deviations from zero-field behavior than do experiments because freezing occurs at a higher temperature where χ_{EA} is smaller.

Interestingly, the chosen value of J also gives a ratio between experimental[2] and short-time Monte Carlo[12] freezing temperatures (shown by arrows in Fig. 2) which agrees with the prediction above that $T_f^{-4}(\omega) \propto \ln(1/\omega\tau_0)$.

Finally, we consider the field dependence predicted by Eq. (5). This implies that $T_f(\omega,H)/T_f(\omega,0)$ is only a function of $H/[T_f(\omega,H)]^{\Delta}$, so that lines of freezing temperatures in the H-T plane at different frequencies should all collapse on to a single curve if plotted in this scaled form. Choosing $\Delta \approx 7$, which is obtained from Eq. (9), recent data[28] on $Eu_{0.4}Sr_{0.6}S$ are shown in Fig. 3. Considering the considerable scatter in the original data and the fact that no adjustable parameter is used, we consider the evidence for scaling satisfactory. There are not yet any $d=3$ computer simulations in a field to compare with but the the insert to Fig. 3 shows that simulation data for the $d=2$ $\pm J$ model[23] does scale in the expected manner. Again, Δ (≈ 2.8) is taken from other exponents [$\nu \approx 2$, $\eta \approx 0.4$ (Refs. 19, 21, and 22)] so it is not an adjustable parameter.

We note that, within their accuracy, the present estimates of z and ν, Eqs. (6), (7), and (9), for both $d=2$ and 3 are

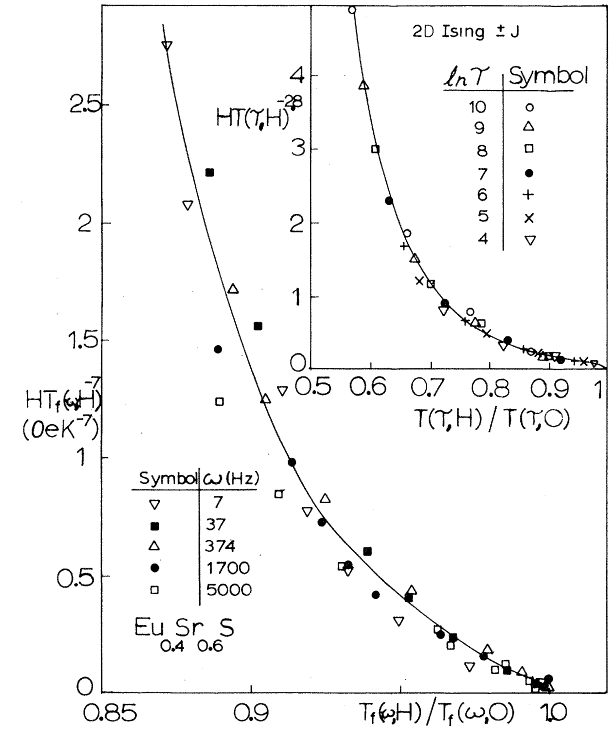

FIG. 3. Scaling plot of the H-T lines where freezing occurs at a fixed frequency for $Eu_{0.4}Sr_{0.6}S$ (from Fig. 2 of Ref. 28). The insert shows a similar scaling plot of lines of constant relaxation time for the $d=2$, $\pm J$-Ising model (from Fig. 4 of Ref. 23). $T(\tau,H)$ is the temperature where the relaxation time reaches a given value τ for the chosen field H.

consistent with the suggestion[18] that $z-1/\nu=(d-1)/2$, which results from the assumption that one needs only to reverse of order $\xi_{EA}^{d/2}$ spins to go from one ordered state of a cluster to another.

We do not claim that the given exponent values are highly accurate, partly because of errors in the data, but more importantly because they are obtained at rather high temperatures where the asymptotic critical region may not yet have been reached. In this case the effective exponents would increase further as the temperature decreases. Perhaps this could explain the nonuniversal (and generally larger) values of $z\nu$ observed in RKKY systems if, for some reason, those measurements are closer to the true critical region. Another complication is that real systems are Heisenberg but with anisotropy which induces crossover to Ising behavior at low temperatures. The form of this crossover remains to be elucidated.

In the alternative hypothesis, of a finite-temperature transition, one expects[29] that, asymptotically close to T_c, conventional critical slowing down[15] would occur where τ, rather than $\ln\tau$, scales with a power of ξ_{EA}. However, if the transition temperature is small compared with the mean-field prediction one would have a temperature range above T_c where activation of walls dominates the dynamics, followed by crossover to conventional slowing down as one approaches T_c. To our knowledge, this crossover has not been seen experimentally.

To conclude, we have proposed that spin-glasses may be described by a $T=0$ transition and given evidence in favor of this hypothesis. It would be useful to have more accurate experiments on a variety of systems to verify the specific predictions that have been made. It would be particularly helpful to check if there are really differences between insulators, such as $Eu_xSr_{1-x}S$, which seem to fit the picture very well, and RKKY systems which do not fit quite so straightforwardly.

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without order to a state with nonzero Edwards-Anderson order parameter, and hence $\beta=0$. Since $\beta=\nu(d-2+\eta)/2$ this implies $\eta=2-d$, $\gamma=(2-\eta)\nu=d\nu$, and $\Delta=1+d\nu/2$ (see Ref. 18).

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