Properties of surface electrons on a helium film: Effects of the film thickness and substrate

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We have investigated the influence of the film thickness and different substrates on the properties of two-dimensional electrons localized on the surface of liquid-helium films. These properties have been studied on the basis of a self-consistent-field approximation including the short-range correlations. The static structure factor and correlation energy are calculated for several values of the film thickness and different substrates. We have also determined the spectrum of the collective excitations of such a system. The results are compared with the calculations based on the random-phase approximation. We have determined the critical-range values of the film thickness above which the results turn out to be the same as for the bulk system.

I. INTRODUCTION

Recently there is considerable interest in the problem of surface electrons on thin helium films deposited on substrates. Monarkha[1] first analyzed the collective excitations of such a two-dimensional (2D) electron plasma on a thin film adsorbed on a metal, in the context of the random-phase approximation (RPA). Ikezi and Platzman[2] have investigated the stability of thin films embedded with high density of electrons and concluded, within the hydrodynamic approach, that the van der Waals forces stabilize such a film. This is an intriguing result, since as is well known, in the bulk helium the surface is unstable at a critical density of electrons, because the frequency of ripplons softens with increasing density.[3] The question of surface stability has been discussed by Tatarskii *et al.*,[4] which also takes into account a possible effective interaction between electrons and vortices coming from fluctuations in the film. These excitations cannot appear in the bulk helium because their energy is macroscopically large. The ground-state energy and the effective mass of a small electron dimple due to the deformation of the surface in the presence of a strong external electric field have been calculated by Hipólito *et al.*[5] They showed that the dimple energy increases considerably with a decrease in the thickness of the film. This fact enhances the possibility of detection of these dimples through cyclotron-resonance measurements, for example. For these phenomena, it is very important to investigate the collective behavior of the bound-electron—ripplon complex on the surface of helium films, since it plays a very important role. The experiments carried out by Volodin *et al.*[6] showed evidence of trapped electrons above a film. They demonstrate the existence of electron states on a helium film wetting both a glass and a metal surface. For the glass substrate, the electron mobility above the film is lower by a factor at least 10^3 than that above the surface of bulk helium.

More recently, Kajita and Sasaki[7] made the first successful experiment in setting surface electrons above the system of helium film on solid neon. In subsequent papers, Kajita[8] had reported measurements of the conductivity of the electrons as a function of the helium-film thickness and had discussed experimentally the stability of the system. It was shown that the electron conductivity increases with an increase in the film thickness and the surface electron is stable under a clamping electric field below a critical value which depends on the helium-film thickness. In these experiments, the temperature of the system is about 1.74 K, such that the scattering of electrons is mainly due to vapor atoms. So, in this case, the electron-ripplon coupling is not important to the scattering process, and we can consider the helium surface to be a planar one.

In previous works[9,10] two of us (N.S. and O.H.) have studied the static and dynamic properties of a classical electron system interacting via the bare Coulomb potential on the basis of the self-consistent-field approximation (SCFA). The short-range correlations are present through a local-field correction and are calculated in a self-consistent way by making the density-density response function of the system to be dependent upon the pair correlation function. The usual RPA is recovered by neglecting the local-field correction. The method was extended for the quasi-2D electron system by including the effect of the finite width of the electronic layer.[11] Our results for the quasi-2D and 2D electronic systems differ very little among them since, in the experimental situation, the electronic mean distance determined by the electron density is much larger than the width of the electronic layer given by the range of the wave function in the direction normal to the surface.

In this paper we investigate the properties of a 2D electron plasma localized on the surface of a liquid-helium

film adsorbed on several substrates. As with the usual 2D electron gas, this system can also be characterized by a plasma parameter $\Gamma=(\pi n e^4)^{1/2}/T$, where n is the density, T is the temperature in energy units, and e is the electronic charge.

II. THEORETICAL CONSIDERATIONS

In the SCFA the density-density response function is written as

$$\chi(\vec{q},\omega)=\frac{\chi_0(\vec{q},\omega)}{1-\psi(\vec{q})\chi_0(\vec{q},\omega)} , \tag{1}$$

where $\chi_0(\vec{q},\omega)$ is the density-density response function of the ideal classical gas and $\psi(\vec{q})$ is the effective potential related to the structure factor $S(\vec{q})$ through

$$\psi(\vec{q})=\phi(\vec{q})+\frac{1}{n}\int \frac{\vec{q}\cdot\vec{k}}{k^2}\phi(\vec{k})[S(\vec{q}-\vec{k})-1]\frac{d\vec{k}}{(2\pi)^2} , \tag{2}$$

where $\phi(\vec{q})$ is the bare potential between particles.

The well-known dissipation-fluctuation theorem

$$S(\vec{q})=-\frac{\hbar}{n}\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\mathrm{Im}\chi(\vec{q},\omega)\coth(\hbar\omega/2T) \tag{3}$$

completes the self-consistent scheme. In the classical limit ($\hbar\omega \ll T$), the Kramers-Krönig relation allows us to rewrite Eq. (3) as

$$\chi(\vec{q},0)=-\frac{n}{T}S(\vec{q})$$

such that the structure factor turns out to be simply

$$S(\vec{q})=\frac{1}{1+(n/T)\psi(\vec{q})} . \tag{4}$$

Equations (2) and (4) must be solved in a self-consistent way. The density-density response function $\chi_0(\vec{q},\omega)$ of the 2D classical electron gas with a Maxwellian distribution of momenta is given by

$$\chi_0(\vec{q},\omega)=-\frac{n}{T}W\left[\frac{\omega}{q}\left[\frac{m}{T}\right]^{1/2}\right] , \tag{5}$$

where $W(z)$ is the plasma dispersion function.

The correlations of the system are described by the pair correlation function $g(r)$ related to the static structure factor through the Fourier transform as

$$g(r)=1+\frac{1}{2\pi n}\int_0^\infty dk\, J_0(kr)k[S(k)-1] . \tag{6}$$

The RPA is a trivial special case of the SCFA and corresponds to neglect the short-range correlation effects by setting $\psi(\vec{q})=\phi(\vec{q})$ in Eq. (2).

The bare potential between the electrons over the helium surface is found from the solution of Poisson's equation with appropriate boundary conditions and depends strongly on the film thickness d and the dielectric constants of the helium film ϵ and the substrate ϵ_s. Its Fourier transform can be written as[1]

$$\phi(q)=\frac{4\pi e^2}{q}F(qd) , \tag{7}$$

where

$$F(qd)=\frac{1-e^{-2qd\delta}}{(1+\epsilon)-(1-\epsilon)e^{-2qd\delta}} , \tag{8}$$

with $\delta=(\epsilon_s-\epsilon)/(\epsilon_s+\epsilon)$. We can analyze now several special limits. In the helium bulk case ($d\to\infty$), we obtain the usual potential between electrons confined in a plane,

$$\phi(q)=2\pi(e^*)^2/q , \tag{9}$$

with the effects of the helium substrate included into the renormalization electron charge $e^*=(2/1+\epsilon)^{1/2}e$, as discussed in Refs. 9 and 10. The case of the helium film adsorbed on a metal substrate corresponds to take $\delta=1$ ($\epsilon_s=\infty$) in Eq. (8).

In the limit of thin helium films ($qd \ll 1$) the function $F(qd)$ given by Eq. (8) assumes the following form:

$$F(qd)=\frac{1}{1+\epsilon_s}+\frac{\epsilon_s^2-\epsilon^2}{\epsilon(1+\epsilon_s)^2}qd , \tag{10}$$

such that, for a metal substrate, we have a constant potential

$$\phi(q)=\frac{4\pi e^2}{\epsilon}d . \tag{11}$$

This implies that, for very thin films, the electron gas interacts through a dipolar potential. For a substrate with a large dielectric constant, as, for example, a semimetal, we obtain

$$F(qd)=\frac{\epsilon+\epsilon_s qd}{\epsilon\epsilon_s} . \tag{12}$$

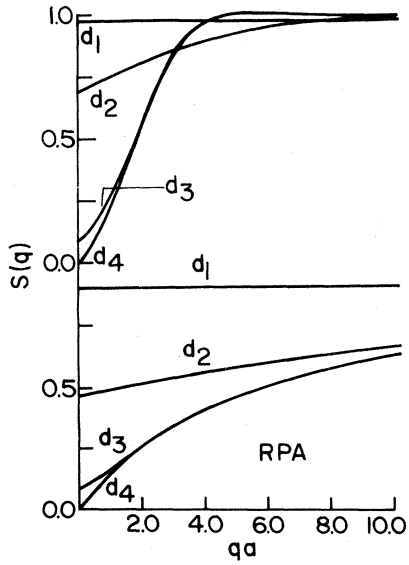

FIG. 1. Structure factor $S(q)$ in both SCFA and RPA for several values of the film thickness ($d_1=0.01$, $d_2=0.1$, $d_3=1.0$, and $d_4=100$), the plasma parameter $\Gamma=3$, all for a metal substrate.

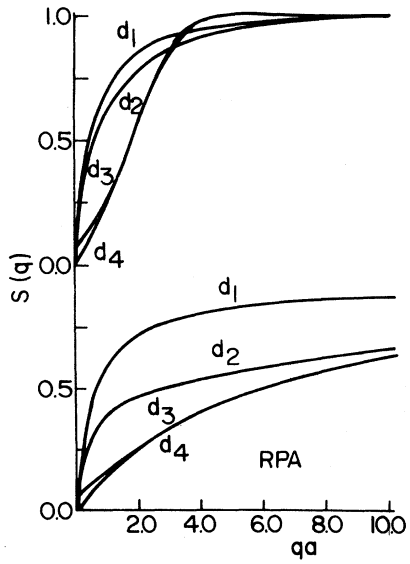

FIG. 2. Structure factor $S(q)$ for a semimetal substrate. Parameters are the same as those employed in Fig. 1.

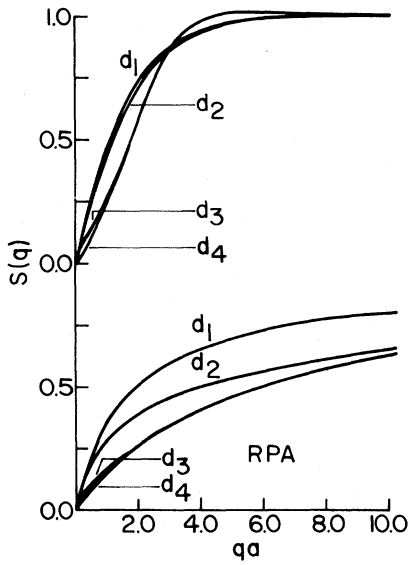

FIG. 3. Structure factor $S(q)$ for a glass substrate. Parameters are the same as those employed in Fig. 1.

III. RESULTS AND DISCUSSIONS

The self-consistent solution is obtained by the standard method of iteration. With a reasonable input $S(q)$ in Eq. (2), the effective potential $\psi(q)$ is calculated using numerical integration and a new $S(q)$ is then obtained. The whole procedure is repeated until self-consistency in $S(q)$ is achieved. The results of these calculations for the structure factor as a function of wave number for the plasma parameter $\Gamma=3$ and for several values of the film thickness are shown in Figs. 1–3. The film thickness d is in units of the core radius $a=(\pi n)^{-1/2}$. In this situation and for a temperature of 2 K the characteristic value $d=10^{-2}$, for example, corresponds to a real helium layer of 280 Å. In Fig. 1 we show the results for a metal substrate, in Fig. 2, for a semimetal-like material ($\epsilon_s=20$), and in Fig. 3, for a glass substrate ($\epsilon_s=5.6$). For comparison, we also present the results within the framework of RPA.

We have also calculated $S(q)$ for a solid-neon substrate and the results are almost the same as those for the helium bulk case. This is quite obvious, since the dielectric constants of neon and helium are so close that δ goes to nearly zero in Eq. (8), and we get approximately the bare potential given by Eq. (9), irrespective of the thickness d.

For thickness $d \geq 100$, the structure factor is independent of the substrate and similar to that of the bulk helium. In the region $1 < d < 100$, the differences between the several substrates are found only for small q's. As we can see from Fig. 1 the special feature is shown by electrons on the helium-metal system, since $S(q)$ is very sensitive to the film thickness according to the fact that the Fourier transform of the potential, in this case, varies from a constant value to a usual 2D electron-gas behavior. Sizable differences are found between the results based on SCFA and RPA. As is well known, the inadequacy of RPA is due to the neglect of short-range correlation effects via the effective potential $\psi(q)$. This failure is strongly manifested in 2D by the logarithmic divergence of the equation of state.

With the self-consistent values obtained for the structure factor we evaluated the correlation energy which is given by[9]

$$E_c=\frac{n}{4\pi}\int_0^\infty dk\,\phi(k)k[S(k)-1]\,. \tag{13}$$

In Table I we present the correlation-energy-density results E_c/nT for some values of the film thickness and substrates. As previously noted, the results for $d=100$ (thick film) are almost the same as the bulk helium. So this thickness is the critical value above which the properties of 2D electrons on a thick film are the same as those of the bulk case, independently of the substrate. In Fig. 4

TABLE I. Values of the correlation-energy density normalized to the kinetic energy $(-)E_c/nT$ for several thicknesses, plasma parameters, and substrates.

		Metal		Semimetal		Glass	
d	Γ	3	0.1	3	0.1	3	0.1
0.01		0.054	0.0003	0.206	0.0015	0.720	0.006
0.1		0.485	0.007	0.666	0.008	1.200	0.012
1.0		1.991	0.021	1.999	0.021	2.168	0.023
100.0		2.658	0.032	2.605	0.031	2.606	0.031

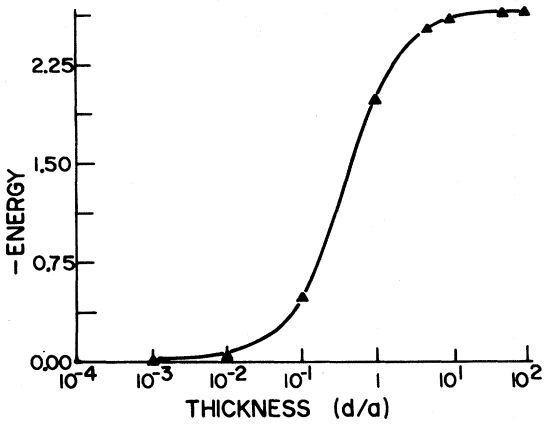

FIG. 4. Correlation-energy density E_c/nT as a function of thickness for a metal substrate ($\Gamma=3$). Line is a guide to the eye. Note the asymptotic value in the bulk limit.

we plot the correlation energy as a function of the thickness for a metal substrate. From this figure, the asymptotic value of the correlation energy becomes clear for $d \geq 100$. Once we have obtained the correlation energy, we can calculate all the thermodynamics properties of the system, such as the Helmholtz free energy and the specific heat, as shown in Ref. 9.

From the poles of the density-density response function $\chi(\vec{q},\omega)$ [Eq. (1)], we can determine the collective excitations of the system. In the long-wavelength limit and for small damping we obtain the dispersion of the plasmons in SCFA as

$$\omega_q^2=\frac{n}{m}q^2\psi(q)\left[1+\frac{3T}{n\psi(q)}\right]. \tag{14}$$

In Figs. 5–7 we show the results of the plasma dispersion relation for $\Gamma=3$, for several values of the film thickness, and for metal, semimetal, and glass substrates. For comparison, we also plot the results from RPA. As before, the situation with a metal substrate shows the most interesting result. In the limit of thin films, there is a soundlike mode ($\omega=cq$) with an acoustic electron velocity given by

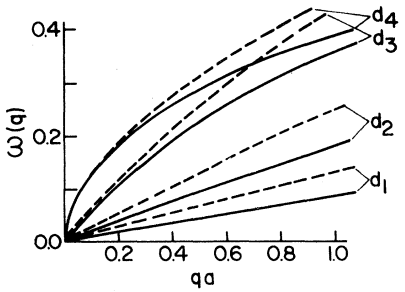

FIG. 5. Long-wavelength dispersion-relation curves in units of $\omega_0=(2\pi ne^2k_D/m)^{1/2}$ for a metal substrate. Dashed lines are the results from RPA. Parameters are the same as those of Fig. 1. Observe the transition from an acoustic mode to a usual 2D plasmon.

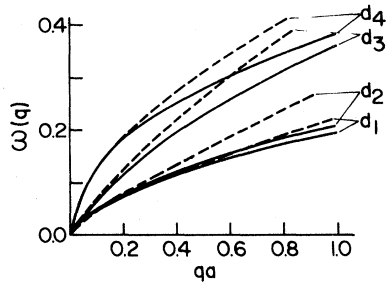

FIG. 6. Same as Fig. 5 for a semimetal substrate.

$$c=c_0\left[1+\frac{3}{2}\frac{T}{mc_0^2}\right], \tag{15}$$

where $c_0=(\alpha/2)^{1/2}v_T$. Here $\alpha=[1-S(0)]/S(0)$ is the fractional deviation of the constant structure factor (see Fig. 1) and $v_T=(2T/m)^{1/2}$ is the thermal electron velocity. In RPA we have $\alpha_{\rm RPA}=2k_Dd/\epsilon$ with $k_D=2\pi ne^2/T$ as the 2D Debye wave number.

From the dispersion-relation curves (Figs. 5–7) we observe that the short-range correlations between particles present in the system correct the overestimated screening results given by the RPA by a decrease in the acoustic-electron velocity. With an increase in the thickness, the long range of the electron interaction begins to appear until the bulk limit is reached, with the typical plasmon mode

$$\omega_q^2=\frac{2\pi ne^2}{m}q\left[1+\left[\frac{3}{k_D}-\gamma\right]q\right], \tag{16}$$

where $\gamma=(1/4\pi n)\int_0^\infty[S(k)-1]dk$ is the correction to RPA due to the short-range effects. We also have found that in the long-wavelength limit the damping of these excitations remains exponentially small.

In conclusion, we have shown that the thickness of the helium film and the substrates play an important role in

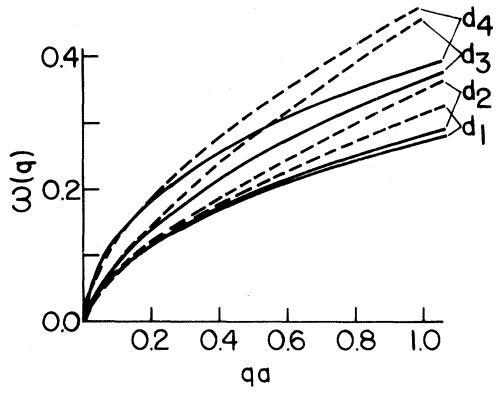

FIG. 7. Same as Fig. 5 for a glass substrate.

the correlational properties of a 2D electron plasma. Numerical results for the structure-factor functions and plasmon dispersion relation represent a definite improvement over the RPA calculations. Finally, we would like to mention that the motion of the electron perpendicular to the helium-film surface was not taken into account. Calculations in this direction are now in progress and the results will be presented later.

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