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Magnetostatic waves and spin waves in layered ferrite structures

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Collective magnetic excitations are supported by multiple ferrite layers. The character of magnetostatic waves is calculated for such systems and is illustrated for three different layers variously spaced (accurate control of the delay characteristics is shown to be possible) as well as for very many similar layers equally spaced. For volume waves on many layers, collective spin-wave-like modes of long wavelength form a continuum that evolves smoothly into narrow bands characteristic of the layers at short wavelength. For surface waves, an excitation similar to the surface wave on a continuous medium appears when the spacing is less than the layer thickness, but a dense continuum of other excitations (not analogous to those of a uniform medium) always persists even for small spacings. This unexpected spectrum is related to a novel system of modes localized on the gaps.

I. INTRODUCTION

There has recently been interest in the propagation of magnetostatic waves in two[1–6] or more[7] ferrite layers; much of this interest is stimulated by a desire to achieve technically useful delay characteristics.

We here describe a convenient way to find the dispersion relation of magnetostatic waves in an arbitrary sequence of magnetic layers interspersed with vacuum or dielectric; illustrative results are given for volume and surface modes on two or three layers. Exchange effects will be ignored.

For many similar films equally separated the modes become bands. For surface waves we find, when the separation is small, an unexpected continuum due to modes localized on the gaps; the surface-wave spectrum is not analogous to the excitations of a continuous medium. In the case of volume waves, by contrast, a collective spin-wave spectrum of long wavelength evolves continuously into magnetostatic modes of shorter wavelength; these modes are close to the magnetostatic modes of single layers.

Recently, Grünberg and Mika[8] have published a treatment of surface waves on N identical layers. Their work differs from ours both in technique and in interpretation, and each finds some results that the other does not.

The system is a set of N ferrite layers stacked along the y axis, as sketched in Fig. 1. The nth layer has thickness d_n and is at a distance s_n from layer $n+1$; the waves propagate along the x axis.

Surface permeability. At each boundary, the tangential field H_x and the normal flux B_y are continuous. These conditions are assured by requiring continuity of the ratio B_y/H_x; the authors[4,9] treat such cases through introducing a relative surface permeability μ^s defined as

$$\mu^s = -iB_y/H_x \ . \tag{1}$$

The value of μ^s at any interface can be calculated from the value at any neighboring interface.

The vector potential A_z obeys Laplace's equation in the vacuum regions above and below the stack of ferrite films, and must become small at great distances. For wave number k, therefore,

$$A_z \sim e^{ikx}e^{-|k|y}, \text{ above },$$
$$A_z \sim e^{ikx}e^{|k|y}, \text{ below }.$$

From Eq. (1), therefore, the upper and lower boundary conditions are

$$\begin{aligned} \mu^s &= \pm 1 \quad (k=\pm), \text{ above },\\ \mu^s &= \mp 1 \quad (k=\pm), \text{ below }. \end{aligned} \tag{2}$$

In many cases of technical interest the ferrite layers are next to ground planes; this topic is discussed in part 3 of the Appendix.

Differentiation of Eq. (1) yields

$$\frac{\partial \mu^s}{\partial y} = -\frac{i}{H_x}\frac{\partial B_y}{\partial y} + i\frac{B_y}{H_x^2}\frac{\partial H_x}{\partial y} = -k\frac{B_x}{H_x} - ik\frac{H_y}{H_x}\mu^s \ ,$$

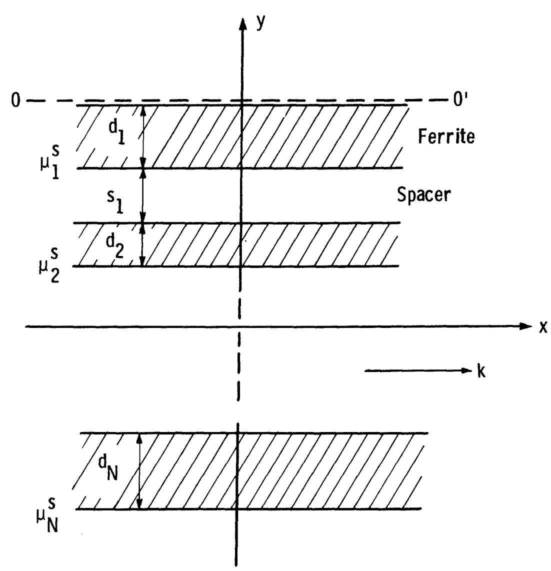

FIG. 1. Ferrite layers and the coordinates used in calculation.

the conditions $\mathrm{div}\underline{B}=\mathrm{curl}\underline{H}=0$ having been used; the second of these is the magnetostatic approximation.

We now use the constitutive relations,

$$B_x=\mu_{11}H_x+i\mu_{12}H_y \ ,$$
$$B_y=-i\mu_{12}H_x+\mu_{22}H_y \ , \tag{3}$$

which, together with the definition of μ^s, allow us to eliminate the ratios of fields from $\partial\mu^s/\partial y$. The result is

$$\frac{\partial\mu^s}{\partial y}=k[(\mu^s+\mu_{12})^2-\mu_{11}\mu_{22}]/\mu_{22} \ .$$

Therefore, if $\mu^s(0)$ is known,

$$\mu^s(y)=\frac{\mu_{22}\mu^s(0)-[\mu_{11}\mu_{22}-\mu_{12}^2-\mu_{12}\mu^s(0)]\beta^{-1}\tanh(\beta k y)}{\mu_{22}-[\mu_{12}+\mu_s(0)]\beta^{-1}\tanh(\beta k y)} \ , \tag{4}$$

where

$$\beta^2=\mu_{11}/\mu_{22} \ . \tag{5}$$

This result can also be obtained by expressing $\underline{H}$ as the gradient of a magnetic potential.[9]

Equation (4) for μ^s reduces the N-layer problem to one of iteration from interface to interface, using the known values of μ^s at the upper and lower boundaries—see Eq. (2). For positive k, for example, the value of k must be such that the value $\mu^s=1$ at the top yields $\mu^s=-1$ at the bottom.

II. THEORY

A. The secular equation

Whether k is positive or negative, a member of the pair μ^s+1, μ^s-1 is zero at both upper and lower surfaces. We therefore define new variables u_n,v_n

$$\underline{w}_n=(u_n,v_n)=(\mu_n^s+1,\mu_n^s-1) \ , \tag{6}$$

where μ_n^s is the value of μ^s at the *bottom* of layer n. This unsymmetrical definition, from the bottoms of the layers, will require that we treat the upper vacuum layer as the upper "magnetic" layer of the system, layer $n=0$, terminating at the dotted line 00′ in Fig. 1.

To make use of the zeros in u and v we must rewrite Eq. (4) in "homogeneous" form. This is done in part 1 of the Appendix, and the result is

$$\underline{w}_n=\mathscr{C}\,\underline{M}_n\underline{w}_{n-1} \ , \tag{7}$$

where $\underline{M}_n$ is a 2×2 matrix,

$$\underline{M}_n=\begin{bmatrix} D_n(k) & E_n e^{-2ks_{n-1}} \\ F_n & D_n(-k)e^{-2ks_{n-1}} \end{bmatrix} \ . \tag{8}$$

The elements of $\underline{M}_n$ are given by

$$D_n=2\mu_{22}+(1+\mu_{11}\mu_{22}-\mu_{12}^2)\beta^{-1}\tanh(\beta k d_n) \ ,$$
$$E_n=(1-\mu_{11}\mu_{22}+\mu_{12}^2-2\mu_{12})\beta^{-1}\tanh(\beta k d_n) \ , \tag{9}$$
$$F_n=(-1+\mu_{11}\mu_{22}-\mu_{12}^2-2\mu_{12})\beta^{-1}\tanh(\beta k d_n) \ .$$

The relative permeabilities μ_{ij} can differ in the different films. It should be noted that Eq. (7) is not truly homogeneous; the "factor" $\mathscr{C}$ contains some surface permeabilities, but our object is to make use of the zeros in u and v, so that the magnitude of the factor does not enter.

If we consider one film in isolation, then for positive k we have $\mu^s=1$ above the film and $\mu^s=-1$ below the film; therefore $v_{n-1}=u_n=0$. From Eqs. (7) and (8), therefore,

$$D_n(k)=0 \ . \tag{10}$$

Similarly, for negative k, we find $D_n(-k)=0$. The diagonal elements of $\underline{M}$ therefore contain the characteristic relations for the individual films.

For a sequence of N films, Eq. (7) gives

$$\underline{w}_N=\mathscr{C}\,\underline{K}\,\underline{w}_0 \ , \tag{11}$$

where the factor is a product of prior factors, and

$$\underline{K}=\underline{M}_N\,\underline{M}_{N-1}\cdots\underline{M}_1 \ . \tag{12}$$

The quantity $\underline{w}_0$ refers to the value of $\underline{w}$ at the top of the upper film, all other $\underline{w}$'s being defined at the bottoms of the layers, as was noted after Eq. (6).

The boundary conditions in Eq. (2) reduce to $u_N=v_0=0$ for positive k and to $v_N=u_0=0$ for negative k. From Eq. (11), therefore, the secular equations are

$$K_{11}(k)=0,\quad k>0$$
$$K_{22}(k)=0,\quad k<0 \ . \tag{13}$$

Since $\underline{K}$ is merely the N-fold product of 2×2 matrices, the dispersion relation for any reasonable number of layers can be found easily.

B. Special cases

1. Single film

The result has been given already, in Eq. (10). The solutions to this equation have been extensively discussed in the literature. The most important cases are surface waves,[10,11] with $H||z$, forward volume waves,[12] with $H||y$, and backward volume waves,[11] with $H||x$.

2. Two films, $k>0$

We find

$$D_1D_2+F_1E_2e^{-2ks_1}=0 \ . \tag{14}$$

Various forms of this equation have been extensively investigated[1–6] for volume waves; we shall consider only surface waves. Since $D_i=0$ is the secular equation for film i, it is evident that terms involving E_i and F_i are interaction terms; the exponential factor ensures that interactions are small when the separation is large.

All modes are composite. Adkins and Glass[6] err in stating that the modes of the individual films, $D_i = 0$, are modes of the coupled system.

3. *Three films, $k > 0$*

The secular equation is

$$D_1 D_2 D_3 + F_1 E_2 D_3 e^{-2ks_1} + D_1 F_2 E_3 e^{-2ks_2}$$
$$+ F_1 D_2(-k) E_3 e^{-2k(s_1+s_2)} = 0 . \tag{15}$$

This equation is less formidable than it appears because—especially for volume waves—the functions D, E, and F are reasonably simple functions of frequency and wave number [see Eq. (9)]. Some solutions are given in Sec. III.

4. *Many equal films*

When the matrices $\underline{M}$ are all equal,

$$\underline{K} = \underline{M}^N .$$

If $\underline{S}$ diagonalizes $\underline{M}$,

$$\underline{S}^{-1} \underline{M}\, \underline{S} = \underline{\Lambda} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} ,$$

then

$$\underline{K} = \underline{S}(\underline{S}^{-1}\underline{M}\,\underline{S})(\underline{S}^{-1}\underline{M}\,\underline{S}) \cdots \underline{S}^{-1}$$
$$= \underline{S}\,\underline{\Lambda}^N \underline{S}^{-1} .$$

The symmetry of the system ensures that positive and negative wave numbers are the same at the same frequency. The secular equation, $K_{11} = 0$, reduces to

$$(\lambda_2 - M_{11})\lambda_1^N - (\lambda_1 - M_{11})\lambda_2^N = 0 ,$$

or

$$\left[\frac{\lambda_1}{\lambda_2}\right]^N = \frac{\lambda_1 - M_{11}}{\lambda_2 - M_{11}} . \tag{16}$$

The eigenvalues of $\underline{M}$ are

$$\lambda_{1,2} = \tfrac{1}{2}\{(M_{11} + M_{22}) \pm [(M_{11} - M_{22})^2 + 4M_{12}M_{21}]^{1/2}\} . \tag{17}$$

When N is very large, it is obvious that nearly all solutions of Eq. (16) must arise when λ_1 and λ_2 have the same magnitude, i.e., when the square root is imaginary, because otherwise the left-hand side of the equation becomes very large (or small). This condition is met when

$$(M_{11} - M_{22})^2 + 4M_{12}M_{21} \le 0 . \tag{18}$$

It is shown in part 2 of the Appendix that there are no other solutions, apart from a singular surface-wave solution, and that the number of modes in the band defined by Eq. (18) is equal to the number of films. These results, Eqs. (16)–(18), are formally the same as those given by Grünberg and Mika[8] in the case of surface waves.

III. RESULTS

A. Types of systems

Numerical results have been obtained for three dissimilar ferrite layers and for many equal layers, for both surface waves and for forward volume waves. The principal diagonal and off-diagonal permeabilities, μ and κ, are

$$\mu = 1 - \omega_H \omega_M / (\omega^2 - \omega_H^2) ,$$
$$\kappa = \omega\, \omega_M / (\omega^2 - \omega_H^2) , \tag{19}$$

where ω is the frequency, and

$$\omega_H = \gamma H_0, \quad \omega_M = 4\pi\gamma M .$$

Here H_0 is the internal bias field, M is the magnetization, and γ is the gyromagnetic ratio.

For surface waves, the bias field is along the z axis, and

$$\mu_{11} = \mu_{22} = \mu, \mu_{12} = \kappa . \tag{20a}$$

For forward volume waves, the bias field is normal to the plane of the films, and therefore

$$\mu_{11} = \mu, \quad \mu_{22} = 1, \quad \mu_{12} = 0 . \tag{20b}$$

In this case all solutions lie within the spin-wave region, $\omega_H^2 < \omega^2 < \omega_H^2 + \omega_H \omega_M$, and μ_{11} is negative. In Eqs. (5) and (9) we must set

$$\alpha^2 = -\mu_{11}/\mu_{22} ,$$
$$\beta^{-1}\tanh(\beta kd) = \alpha^{-1}\tan(\alpha kd) . \tag{21}$$

We summarize the principal simple excitations briefly, for later comparison.

1. *Magnetostatic modes of a layer*

For forward volume waves on a single film, Eqs. (9), (10), (20b), and (21) give

$$\tan(\alpha kd) = 2\alpha/(\alpha^2 - 1) . \tag{22}$$

Solutions span all real values of α (negative μ); the frequency range is

$$\omega_H < \omega < (\omega_H^2 + \omega_H \omega_M)^{1/2} .$$

For each principal solution k_1, further modes $k = k_1 + n\pi/\alpha d$ exist, where n is a positive integer. The higher modes should be regarded as spin waves within the layer, and exchange effects must often be taken into account in describing them.

For surface waves, Eqs. (9), (10), and (20a) give

$$\tanh(kd) = 2\mu/(\kappa^2 - \mu^2 - 1) . \tag{23}$$

This is equivalent to the form given by Damon and Eshbach[11] and by Bongianni,[10] and solutions exist only above the spin-wave spectrum, within the range

$$(\omega_H^2 + \omega_H \omega_M)^{1/2} < \omega < \omega_H + \tfrac{1}{2}\omega_M .$$

These magnetostatic modes are shown in Fig. 2, for yttrium iron garnet (YIG) with a magnetization $4\pi M = 1780$

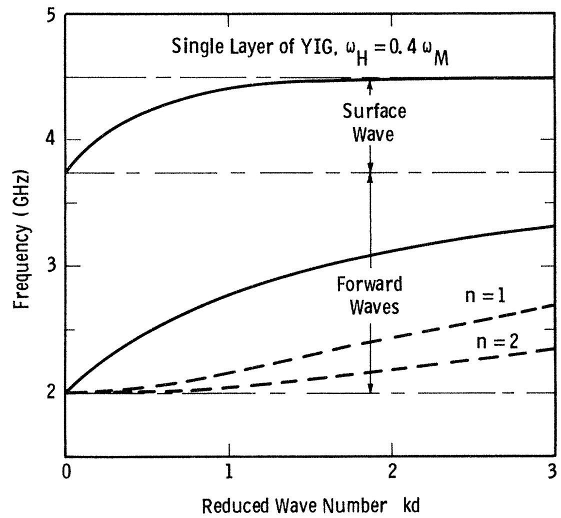

FIG. 2. Simple magnetostatic excitations of a single layer of YIG, $4\pi M = 1780$ G. The bias field H_0 is $0.4\times 4\pi M$.

G, in an internal bias field $H_0 = 0.4\times 4\pi M$. Only the first three of the volume-wave modes are shown.

2. *Gap modes*

Localized modes can exist on a gap between two semi-infinite ferrites when the bias field is along the z axis. For a gap s, standard arguments yield

$$\tanh(ks) = 2\mu/(\kappa^2 - \mu^2 - 1) \ . \tag{24}$$

The spectrum is identical with that of surface waves on an isolated layer of the same thickness, Eq. (23). These modes are mentioned because they are important in understanding the surface wave characteristics of a layered system.

3. *Spin waves*

This term (often reserved for waves of short wavelength) will here be used for the excitations of a continuous medium. When exchange effects are small and the wave number is at an angle θ to the bias field, the frequency is

$$\omega = (\omega_H^2 + \omega_H \omega_M \sin^2\theta)^{1/2} \ .$$

The frequency range is the same as that of forward volume waves, and the frequency is independent of wave number. In particular, when H_0 is along the y axis we find

$$k_y / k_x = \cot\theta = \alpha \ , \tag{25}$$

where $\alpha = (-\mu)^{1/2}$.

The surface wave of a semi-infinite medium, first reported by Damon and Eshbach,[11] is among the elementary excitations. When H_0 is along the z axis, only one frequency exists,

$$\omega = \omega_H + \tfrac{1}{2}\omega_M \ , \tag{26}$$

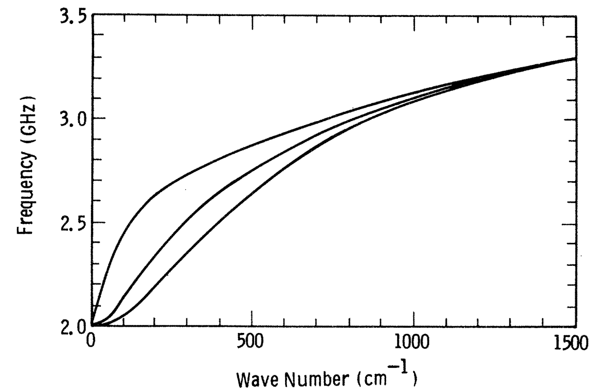

FIG. 3. Three equal layers: splitting of the fundamental forward volume-wave modes on YIG. The layer thicknesses are 20 μm, the separations are 30 μm, and the bias field is $0.4\times 4\pi M$.

which is at the top of the surface wave band for a finite layer. At this frequency all positive wave numbers are allowed.

B. Few ferrite layers

1. *Volume waves, three layers*

Wave numbers are computed from Eq. (15), the functions D, E, and F being given in Eq. (9); the substitution in Eq. (21) must be used when μ_{11} is negative.

The splitting of the fundamental mode into three branches is illustrated in Fig. 3, for three equal films equally separated. The layer thickness d is 20 μm, and the separation s is 30 μm; the material is YIG, and the magnetic bias field remains $0.4\times 4\pi M$, for easy comparison with Fig. 2. Higher modes, corresponding to the harmonics shown in Fig. 2, certainly exist, but the modes shown are clearly derived from the fundamental mode because they coalesce at high wave number when the interaction, proportional to e^{-2ks}, is weak.

The effect of differing magnetizations is illustrated in Fig. 4 for the same geometry as in Fig. 3, but with the upper and lower layers having magnetizations 100 G greater and less than the magnetization of YIG. For films of different magnetization one must take account of the conservation of normal flux, because for forward volume waves the bias field is normal to the film; if the bias field in film i is H_i, then

$$H_2 = H_1 + 4\pi(M_1 - M_2) \ . \tag{27}$$

The layer with the greatest magnetization is therefore subject to the lowest bias field, and the band edge for volume waves, γH, is lowest in this layer.

This effect can be seen clearly in Fig. 4, where the characteristic is discontinuous each time a new layer starts to support waves. At the lowest frequency there is only one fundamental mode; then the wave numbers fall to zero and two modes appear; in the upper range there are three modes derived from the fundamental mode. In the low-frequency region, where only one or two solutions are

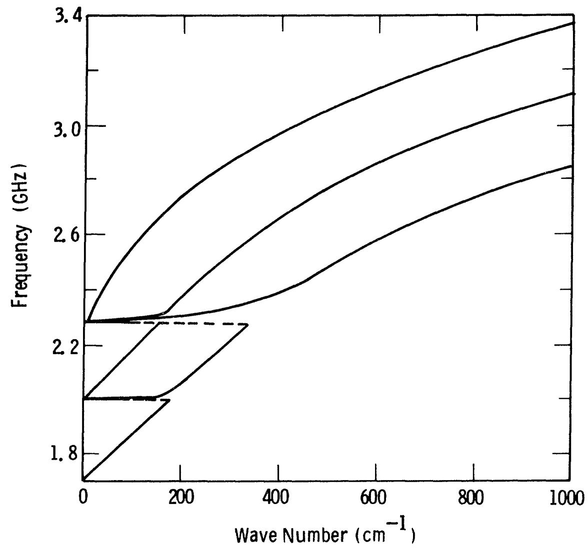

FIG. 4. Different magnetizations in three layers: structure of the fundamental forward volume waves for the same geometry as in Fig. 3. The magnetizations are $4\pi M_1 = 1680$ G, $4\pi M_2 = 1780$ G, $4\pi M_3 = 1880$ G; the bias field in the middle film is $0.4 \times 4\pi M_2$.

shown, we have excluded modes derived from the harmonics (see Fig. 2) of one of the active films. Such modes can be recognized if one finds field as a function of position, because the field crosses zero in the center of an active film (an example is shown in Fig. 6 of Ref. 4).

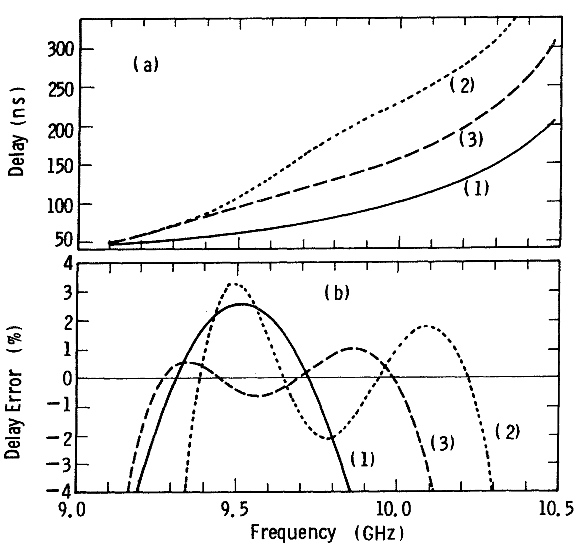

FIG. 5. Control of delay by multiple films, to obtain linear dependence of delay on frequency. (a) The group delay over 1 cm for a single film, (1) $d = 30$ μm; for two films, (2) $d_1 = d_2 = 15$ μm, $s = 30$ μm; and for three films, (3) with $d_1 = d_3 = 5$ μm, $d_2 = 20$ μm, $s_1 = s_2 = 30$ μm; the bias field is 3214 G. (b) Deviation of the delay from linearity in the above cases.

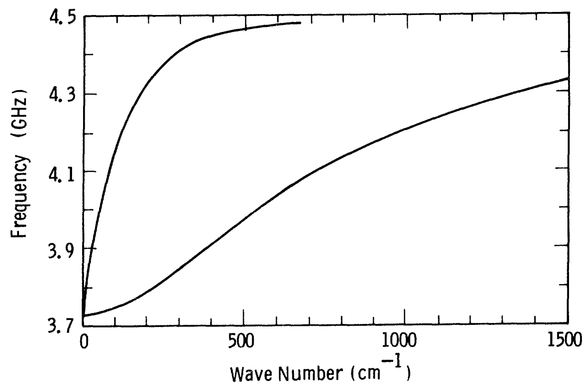

FIG. 6. Surface wave and gap wave on two equal YIG films slightly separated; $d_1 = d_2 = 20$ μm, $s = 5$ μm, bias field $0.4 \times 4\pi M$. At high wave numbers the lower curve is an excitation localized on the narrow gap.

Magnetostatic waves are used in frequency-dependent delay lines, between fixed antenna and receiver; common system requirements are that the delay either be constant or else change linearly with frequency. This is unfortunate; for magnetostatic waves on a single layer the group velocity v_g varies almost linearly with frequency, so the group delay $1/v_g$ does not.

The use of multiple layers to modify the delay is shown in Fig. 5(a), which gives the frequency dependence of the group delay of the fundamental mode for single, double, and triple films of equal magnetization. The parameters are as follows: one film, $d = 30$ μm; two films, $d_1 = d_2 = 15$ μm, $s = 30$ μm; three films, $d_1 = d_3 = 5$ μm, $d_2 = 20$ μm, $s_1 = s_2 = 30$ μm. These results are given at higher frequency (~ 10 GHz, bias field $H_0 = 3214$ G) than our other results, because volume waves have a larger bandwidth at such frequencies. The departure of the delay from linearity is shown in Fig. 5(b), which shows the improvement that can be produced by multiple films; note that the best operating bands are not coincident when the bias field is the same for all three systems.

2. *Surface waves, two layers*

Our principal concern has been to show the existence of the gap mode, Eq. (24); as far as we know it has not been demonstrated before. We therefore considered two equal layers, $d_1 = d_2 = 20$ μm, separated by a narrow gap, $s = 5$ μm. The result, calculated from Eq. (14), is shown in Fig. 6.

The first mode is very nearly a surface wave for the two films together, with an "effective" thickness $d_1 + d_2 = 40$ μm. Except at the lowest frequencies, the second mode is the gap mode for $s = 5$ μm; its wave number is almost 8 times that of the fundamental mode, in agreement with the ratio of "effective" thicknesses.

C. Many similar films

1. *Volume waves*

We shall consider only the fundamental mode—the solid volume-wave line in Fig. 2—for the same parameters

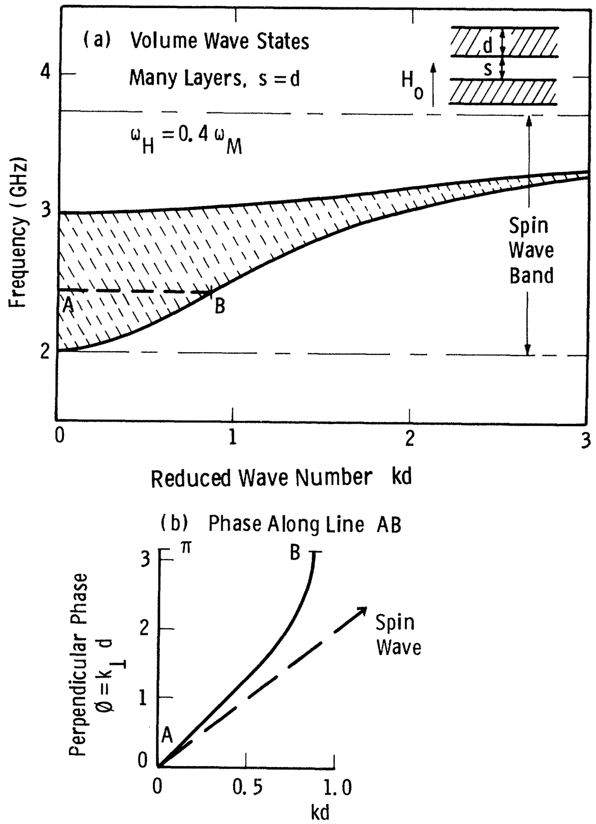

FIG. 7. Many films, $d=s$. (a) Formation of a continuum from the fundamental forward volume wave (other modes exist to the right; cf. Fig. 2). (b) Comparison with spin waves; dependence of normal wave number on tangential wave number at constant frequency.

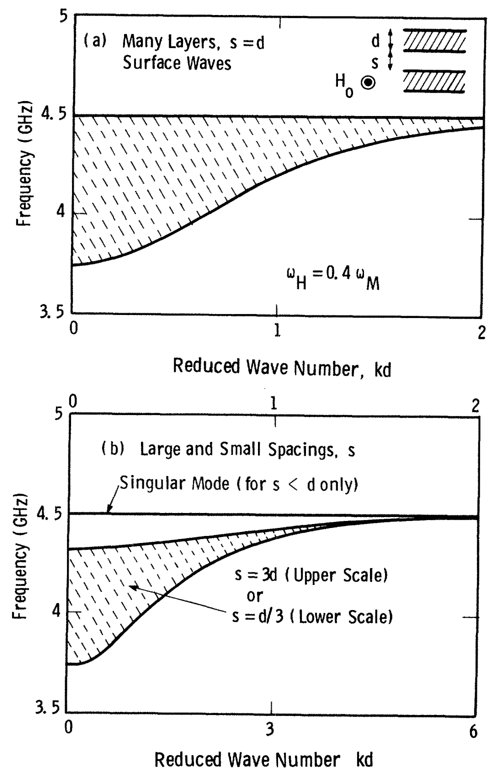

FIG. 9. Surface waves on many films. (a) Formation of a surface-wave continuum on YIG for equal thickness and separation, bias field $0.4\times 4\pi M$. (b) Narrowing of the continuum for lesser and greater spacings, and the appearance of a singular surface wave for lesser spacings.

that were used in that figure. The broadening of the fundamental mode into a continuum is shown in Fig. 7(a), for spacing equal to layer thickness. Waves may exist in all of the shaded area; the limits of the spectrum are found from Eq. (18), using the matrix elements M_{ij} defined in Eqs. (8) and (9), together with the permeabilities in Eqs. (19) and (20).

At low wave numbers the spectrum clearly belongs to collective magnetic modes of the system; i.e., each mode resembles a spin wave. This identification is made clearer by finding the dependence of "perpendicular" wave number (i.e., along the y axis) on tangential wave number, for a particular frequency. The result is shown in Fig. 7(b); at low wave numbers it is almost tangent to the characteristic of a spin wave in an isotropic medium, Eq. (25).

The greatest frequency at which $k=0$ lies in the band is given by

$$\omega_{\max}^2=\omega_H^2+\omega_H\omega_M/(1+s/d)\ . \tag{28}$$

Thus, when the spacing s is small the width of the continuum expands to cover the entire spin-wave band. This result is not in itself remarkable; it is mentioned only because surface waves do not display the same behavior.

The contraction of the continuum by increase of the spacing is illustrated in Fig. 8. It is evident that magnetostatic modes of high wave number evolve continuously from collective modes of long wavelength.

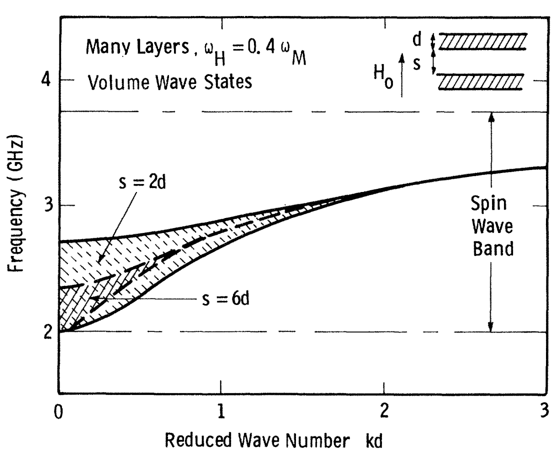

FIG. 8. Narrowing of the volume wave continuum by increased separation.

2. Surface waves

The results of Damon and Eshbach[11] [see Eq. (26)] lead one to expect a rather narrow continuum for surface waves, since many coupled layers are more like a continuous solid than a single layer is. This expectation is not realized. Figure 9(a) shows the broadening of the surface-wave mode into a rather wide continuum for spacing equal to layer thickness. A remarkable feature of this result is that low wave numbers are allowed at all frequencies up to the top of the band.

It can be shown that, for surface waves, Eq. (18) reduces to

$$T(\omega)^2(t_s^2+t_d^2-t_s^2t_d^2)-2T(\omega)t_dt_s+t_d^2t_s^2<0\ , \tag{29}$$

in which

$$t_d=\tanh(kd),\quad t_s=\tanh(ks)\ ,$$

and $T(\omega)$ is the right-hand side of Eq. (23); $T(\omega)=\tanh(k_0d)$ for an isolated layer.

This result is entirely symmetrical between s and d; thin layers far apart have the same spectrum as thick layers close together. We must conclude that waves propagating mostly on the thin layers in the first case are replaced with modes propagating mostly on the narrow spaces in the second case: these are the "gap modes" mentioned in Sec. III A [see Eq. (24)]. The existence of such a mode is shown in Fig. 6.

The greatest frequency at which $k=0$ lies in the continuum is given by

$$\omega_{\max}^2=\omega_H^2+\omega_H\omega_M+\omega_M^2sd/(s+d)^2\ . \tag{30}$$

This function goes through a maximum when $s=d$; at this spacing $\omega_{\max}$ for $k=0$ is $\omega_H+\omega_M/2$, the case shown in Fig. 9(a). At all other spacings the continuum is narrower, in contrast to the volume wave case, Eq. (28), where the width of the continuum always increases as the spacing falls.

The decrease in the width of the continuum for both large and small spacings is shown in Fig. 9(b). It is plain that when the gaps are small we must regard the principal excitations as gap modes weakly linked through large thicknesses of ferrite.

Figure 9(b) also shows a singular surface-wave mode that exists only when the spacing is less than the layer thickness. The existence of this mode is demonstrated in part 2 of the Appendix, and its appearance is the only discernible effect of interchanging s and d.

The progressive broadening and subsequent narrowing of the surface-wave continuum have previously been shown by Grünberg and Mika,[8] in Fig. 2 of their work; these authors have also demonstrated the appearance of the singular surface wave.

That a layered structure should support a surface wave with a frequency independent of wave number is worthy of remark. The explanation is simple. At the frequency of the Damon and Eshbach[11] surface waves, a magnetic field decreasing as $e^{-k|y|}$ in the ferrite layer can join onto a field increasing as $e^{k|y|}$ in the intervening dielectric. The field displays a sawtooth pattern as one goes down through the layers; decreases by e^{-kd} in each ferrite layer are followed by increases of e^{ks} in the next dielectric layer. For all positive k, therefore, the mode is localized near the surface provided that $s<d$.

IV. CONCLUSION

A compact way of calculating the characteristics of magnetostatic waves on multiple ferrite films has been described and demonstrated. The properties of three unequal films, separated by dielectric spacers, have been illustrated, and have been shown to lead to accurate control of delay over 800 MHz bandwidth.

For very many like films equally separated, the magnetostatic modes are split into a continuum.[8] At long wavelengths the modes are collective modes of the entire system; at short wavelengths they evolve smoothly into narrow bands centered on the magnetostatic modes of the layers (or of the gaps between the layers).

For volume waves, the long-wavelength modes of a layered medium resemble the spin waves of a continuous medium. A decrease in the spacing between the layers broadens the continuum until the entire spin-wave spectrum is covered.

The surface-wave spectrum of a very thick layered medium is less simply explained: Indeed, the only satisfactory part of the results is the abrupt appearance of the Damon and Eshbach surface wave when the spacing becomes less than the layer thickness.

In a continuous semi-infinite medium, the classical surface wave exists at only one frequency. In a layered medium, by contrast, other waves extend throughout the medium at all frequencies within the surface-wave band of a single layer; the number of modes at each frequency is equal to the number of layers. Comparison with a continuous medium is further complicated by a curious symmetry between layers and gaps, which are equally capable of supporting surface waves (the gap modes have not, as far as we know, been reported before). In contrast to the volume wave case, this symmetry leads to the conclusion that the continuum is narrow when the gaps are small, as well as when they are wide.

The density and frequency distribution of these modes have no simple parallel in a continuous medium. The narrowing of the continuum, however, implies that energy transfer from layer to layer is slow when the gaps are very small. While the modes exist, therefore, most of them may not be accessible when the structure looks "almost like" a uniform magnetic medium.

Note added in proof. The properties of many equal layers under a bias field of arbitrary in-plane direction have been described by R. E. Camley, T. S. Rahman, and D. L. Mills [Phys. Rev. B 27, 261 (1983)].

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APPENDIX

1. Connection between layers

For layer n, let the surface permeabilities μ^s at the bottom and at the top be μ_n and μ_n', respectively. Equation (4) has the form

$$\mu_n=(a\mu_n'+b)/(c+d\mu_n')$$

(the interval $y=-d_n$ must here be counted as negative).

We seek to convert this to a "homogeneous" form involving $u_n=\mu_n+1$ and $v_n=\mu_n-1$, for reasons given before Eq. (6). Simple algebra gives

$$u_n=\mu_n+1=\frac{1}{2(c+d\mu_n')}(D_nu_n'+E_nv_n')\ ,$$
$$v_n=\mu_n-1=\frac{1}{2(c+d\mu_n')}[F_nu_n'+D_n(-k)v_n']\ , \tag{A1}$$

where

$$D_n=a+b+c+d,\quad E_n=a-b-c+d\ ,$$
$$F_n=a+b-c-d,\quad D_n(-k)=a-b+c-d\ .$$

Explicit forms for these quantities, from the values of a, b, c, and d, are given in Eq. (9).

It is convenient to iterate from one ferrite layer to another, rather than treat the intervening dielectric as a magnetic layer. From Eq. (9) we have for the vacuum layer s_{n-1}

$$[D(-k)]_{\mathrm{vac}}=e^{-2ks_{n-1}}D_{\mathrm{vac}},\quad E_{\mathrm{vac}}=F_{\mathrm{vac}}=0\ ,$$

where

$$D_{\mathrm{vac}}=2[1+\tanh(ks_{n-1})]\ .$$

Travel across the dielectric layer, characterized by μ_n' at the bottom and by μ_{n-1} at the top, therefore gives

$$u_n'=\mathscr{C}\,u_{n-1},\quad v_n'=\mathscr{C}\,v_{n-1}e^{-2ks_{n-1}}\ . \tag{A2}$$

Substitution into Eq. (A1) yields Eq. (9). The total factor is a function, not only of k, but of μ_n' and μ_{n-1}; truly homogeneous equations are not attainable.

2. Number of solutions

It will first be shown that, for large N, Eq. (16) has no roots other than the band defined by Eq. (18), apart from a singular surface-wave solution. A singular solution can arise if the first term in λ_1 is zero outside the range of wave numbers that satisfy Eq. (18), i.e., if there is a wave number such that

$$M_{11}+M_{22}=0\ , \tag{A3}$$

and Eq. (18) is not satisfied. There would then be a split-off band.

The solution to Eq. (A3) is

$$\tanh(\beta kd)\tanh(ks)=\frac{-2\beta\mu_{22}}{1+\mu_{11}\mu_{22}-\mu_{12}^2}$$
$$=\tanh(\beta k_0d)\ , \tag{A4}$$

where k_0 is the wave number on an isolated film. There are, therefore, no solutions outside the range of frequencies within which magnetostatic waves exist on an isolated film, and there are solutions at all such frequencies.

Upon substituting Eq. (A4) into condition (18) we obtain, after some algebra,

$$(M_{11}-M_{22})^2+4M_{12}M_{21}=-16\mu_{22}^2e^{-2ks}\mathrm{sech}^2(\beta kd)\ , \tag{A5}$$

which is certainly always negative. There is therefore no distinct split-off band.

The phase change between consecutive layers is

$$\phi=\mathrm{Arc}(\lambda_1)\quad\text{or}\quad\tan\phi=\mathrm{Im}\lambda_1/\mathrm{Re}\lambda_1\ .$$

If the right-hand side of Eq. (16) is $\exp(i\psi)$, then solutions appear when

$$e^{2iN\phi}=e^{i\psi}$$

or

$$\phi=\psi/2N+n\pi/N\ , \tag{A6}$$

where n is an integer. When the number of films, N, is very large, the solutions are closely spaced ($\Delta\phi=\pi/N$), and the band becomes a continuum.

The limits of the band are defined by $\mathrm{Im}\lambda_1=0$; it is easy to show that this condition is satisfied when the lower limit of the band is at $k=0$, as well as for bands that span positive values of k. From Eqs. (A4) and (A5), we find that $\mathrm{Re}\lambda_1=0$ within the band, so the values of ϕ always vary between 0 and π. The integer n in Eq. (A6) can therefore take all values from 0 to $N-1$; the total number of solutions is N.

Singular surface wave. A special solution to Eq. (16) may arise when λ_1 and λ_2 have different magnitudes, provided that the right-hand side of the equation vanishes (or is infinite). This can occur only if

$$M_{12}M_{21}=0\ .$$

Except at $k=0$, this requires that the sum of permeabilities in either E or F, defined in Eq. (9), be zero. This condition cannot be met for volume waves, but for surface waves it is satisfied at the frequency

$$\omega=\omega_H+\tfrac{1}{2}\omega_M\ . \tag{A7}$$

This is the frequency of the Damon and Eshbach[11] surface wave on an isotropic semi-infinite solid, Eq. (26).

Take the square root in Eq. (17) as $+(M_{11}-M_{22})$ when $M_{12}M_{21}$ is zero. Then $\lambda_1=M_{11}$ and $\lambda_2=M_{22}$; Eq. (16) is now

$$(M_{11}/M_{22})^N=0\ .$$

When N is very large, solutions exist provided that $M_{11}<M_{22}$. At the frequency $\omega_H+\omega_M/2$ this condition reduces to

$$e^{-2k(d-s)}<1\quad\text{or}\quad s<d\ . \tag{A8}$$

This case is not included in the continuum defined by Eq. (18).

A layered structure therefore supports a surface wave precisely like that on a uniform semi-infinite solid, so long as the spacing is less than the thickness of the layers.

3. Effect of ground planes

In many cases of technical interest a ground plane may be adjacent to the ferrite system. The boundary condition at a ground plane is

$$\mu^s = 0 \,, \tag{A9}$$

because the normal field, B_y, must vanish at a good conductor. We shall give only a formal treatment of this topic.

Let ground planes be at distances s_0 above and s_N below the ferrite system. At these planes $u=1$ and $v=-1$; at the top of the ferrite system Eq. (A2) gives

$$u_0 \sim 1, \quad v_0 \sim -e^{-2ks_0} \,.$$

The boundary conditions at the bottom plane can be rewritten as

$$u_{N+1} + v_{N+1} = 0 = u_N + e^{-2ks_N} v_N \,.$$

If we regard this as an inner product with $\underline{w}_N$, then Eq. (11) allows us to write the dispersion relation as the vanishing of a total inner product,

$$(1 \;\; e^{-2ks_N})\underline{K} \begin{bmatrix} 1 \\ -e^{-2ks_0} \end{bmatrix} = 0 \,, \tag{A10}$$

where $\underline{K}$ is the matrix product in Eq. (12).

This result is a generalization of Eq. (13), being true for both positive and negative wave numbers. It is easily seen that, if s_0 and s_N are very large, then for positive k Eq. (A10) reduces to $K_{11}=0$, and for negative k to $K_{22}=0$, which are the results in the text.

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