Long-range correlated random magnetic fields in the nonlinear σ model

Mau-chung Chang and Elihu Abrahams
Serin Physics Laboratory, Rutgers University, Piscataway, New Jersey 08854
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The effect of the long-range correlated random magnetic fields which behave like $\langle h(x)h(y)\rangle \sim 1/|x-y|^{d-\sigma}$ on the critical phenomena is discussed with the use of the nonlinear σ model. The crossover between the two critical behaviors dominated by the long-range and short-range disorder fixed points is shown to occur at $(m-2)\sigma-d+4=0$ (m is the spin dimensionality). The critical exponents calculated at the long-range disorder fixed point around the lower critical dimensionality $4+\sigma$ are not the same as those in $d-2$ expansions in the pure system.

I. INTRODUCTION

The effect of quenched random magnetic fields on the critical phenomena of spin systems has been discussed extensively.[1–10] It has been shown that a random magnetic field shifts the critical dimensionality by 2. However, for Ising systems, the lower critical dimensionality is still being argued.[1,11–14] For systems with $m>2$ (m is the number of spin components) it seems to be generally accepted that the lower critical dimensionality is 4 for short-range exchange. Both Pelcovits's[9] calculation on the random-axis model and our calculation[10] on the random-field model using the replica method (hereafter, referred to as I) show that the critical exponents in dimensionality $d=4+\epsilon$ are exactly the same as those in $\epsilon=d-2$ expansion in pure systems, at least to first order in ϵ. However, these discussions are only valid for uncorrelated random magnetic field. If the random magnetic fields are correlated over long distances such that $\langle h(x)h(y)\rangle \sim 1/|x-y|^{d-\sigma}$, the critical properties can be changed. This possibility was first briefly considered by Aharony, Imry, and Ma.[5]

Very recently Kardar, McClain, and Taylor have addressed this problem.[15] Following Parisi and Sourlas[7] they used supersymmetry arguments to discuss the equivalence between the d-dimensional system in correlated random fields and the pure system in $d-2-\sigma$ dimensions. They carried out the superspace renormalization-group calculation to second-order in ϵ, $\epsilon=6+\sigma-d$, around the upper critical dimensionality $6+\sigma$. To order ϵ the critical exponents are the same as those of the pure system in $4-\epsilon$ dimensions. However, there are deviations from the simple rule $d\rightarrow d-2-\sigma$ at order ϵ^2. In this paper we discuss this problem around the lower critical dimensionality $4+\sigma$ using low-temperature renormalization-group methods (nonlinear σ model).

The paper is organized as follows. In Sec. II we use the replica trick to find the effective Hamiltonian; differential recursion relations are then calculated. In Sec. III we discuss the stability for each set of fixed points and some of the implications. We offer some concluding remarks in Sec. IV. In the Appendix we discuss a slightly generalized model for the long-range correlation function of random magnetic fields.

II. RECURSION RELATIONS

We start with the following Hamiltonian:

$$H=-\sum_{\langle ij\rangle} J_{ij}\vec{S}_i\cdot\vec{S}_j-\sum_i \vec{h}_i\cdot\vec{S}_i\ . \tag{1}$$

Here $\vec{S}_i$ is an m-component vector spin of unit length, and $\vec{h}_i$ is the random field. In I we assumed the $\vec{h}_i$ to be independent and Gaussian-distributed with $\langle h_i\rangle_{av}=0$ and $\langle h_i^2\rangle_{av}=\Delta$. In this paper we consider a long-range correlation for h_i and h_j such that $\langle h_i\rangle_{av}=0$ and $\langle h_ih_j\rangle_{av}=G(|i-j|)\sim 1/|i-j|^{d-\sigma}$ for large $|i-j|$.

As usual we use the replica trick. By replicating the Hamiltonian in Eq. (1) and averaging the free energy, we immediately obtain

$$-\beta\bar{F}=\frac{1}{n}\ln\sum_{\{\vec{S}\}} e^{-\beta\mathscr{H}}\Big|_{n=0}\ , \tag{2}$$

where

$$\mathscr{H}=-\sum_\alpha\sum_{\langle ij\rangle} J_{ij}\vec{S}_i^\alpha\cdot\vec{S}_j^\alpha-\frac{\beta}{2}\sum_{\alpha,\beta}\sum_{i,j} G(|i-j|)\vec{S}_i^\alpha\cdot\vec{S}_j^\beta\ , \tag{3}$$

with $G(|i-j|)\sim 1/|i-j|^{d-\sigma}$ for large $|i-j|$. The Fourier transform $\bar{G}(k)$ is constant as $k\rightarrow 0$ if $\sigma<0$. This is the short-range case. If, however, $\sigma>0$, $\bar{G}(k)\sim\Delta_1+\Delta_2 k^{-\sigma}$ for small k. Strictly speaking there are terms such as $\Delta' k^{-\sigma'}$ with $0<\sigma'<\sigma$. However, it will be shown in the Appendix that these terms are not important as far as the critical behavior is concerned.

Using the standard technique,[16,17] we write $\vec{S}_i^\alpha=(\vec{\sigma}_i^\alpha,\vec{\pi}_i^\alpha)$, where $\vec{\pi}_i^\alpha$ has $m-1$ components and $\vec{\sigma}_i^\alpha$ is in the direction of spontaneous magnetization. We take the Fourier transform $\vec{S}_i\rightarrow S(k)$ and obtain the following functional integral by expanding the constraint $\sigma_i=(1=\pi_i^2)^{1/2}$:

$$-\beta\bar{F}=\lim_{n\to 0}\frac{1}{n}\ln\int d\{\vec{\pi}_i^{\alpha}\}e^{-\beta\mathscr{H}}\,,$$

$$\begin{aligned}\beta\mathscr{H}=\sum_{\mu}\sum_{\alpha,\beta}\int\frac{d^dk}{(2\pi)^d}\Bigg[&\frac{k^2}{2T}\pi_\mu^\alpha(\vec{k})\pi_\mu^\alpha(-\vec{k})+\frac{k^2}{8T}[\pi^\alpha(\vec{k})]^2[\pi^\alpha(-\vec{k})]^2+\cdots\\&-\frac{\Delta_1}{2T^2}\{\pi_\mu^\alpha(\vec{k})\pi_\mu^\beta(-\vec{k})+\tfrac{1}{4}[\pi^\alpha(\vec{k})]^2[\pi^\beta(-\vec{k})]^2+\cdots\}\\&-\frac{\Delta_2}{2T^2}k^{-\sigma}\{\pi_\mu^\alpha(\vec{k})\pi_\mu^\beta(-\vec{k})+\tfrac{1}{4}[\pi^\alpha(\vec{k})]^2[\pi^\beta(-\vec{k})]^2+\cdots\}+\rho T[\pi_\mu^\alpha(\vec{k})\pi_\mu^\alpha(-\vec{k})+\cdots]\Bigg]\,,\end{aligned}\tag{4}$$

where μ labels the $m-1$ (transverse) components of $\vec{\pi}$, α,β are replica indices, and $\pi^{\alpha^2}(\vec{k}),\rho$ have been defined in Eqs. (2.8), (2.9), respectively, of I.

The next step is to use the Wilson-Kogut recursion method[18] to find the differential recursion relations for $1/T$, Δ_1, and Δ_2. Then the fixed points can be identified, the eigenvalues associated with them calculated, and the stability for each set of fixed points determined. The calculational details have already been given in I. They are briefly mentioned here.

The vertices due to the Δ_2 term in Eq. (4) are shown in Fig. 1. The other vertices relevant to the perturbation expansions are exactly the same as those in Fig. 1 of I. The Feynman diagrams contributing to the recursion relation for $1/T$ are shown in Fig. 2 and Fig. 1 of I. We have

$$\frac{1}{T'}=\zeta^2b^{-d-2}\left[\frac{1}{T}+K_d\ln b+\frac{\Delta_1+\Delta_2}{T}K_d\ln b\right]\,,\tag{5}$$

where $K_d=2^{-d+1}\pi^{-d/2}/\Gamma(d/2)$. We should note that the power of the denominator of the Feynman integral in Fig. 2 is different from that of Figs. 2(b) and 2(c) of I. Now they all contribute $K_d\ln b$. Here, $\ln^n b$ $(n>1)$ terms are not needed in calculating the differential recursion relations.

To determine the spin-rescaling factor ζ, we follow Nelson and Pelcovits[17] and add a uniform magnetic field term $(h/T)\int d^dx\,\sigma^\alpha(x)$, as in I. The recursion relation for h'/T' is

$$-\frac{h'}{2T'}=\zeta^2b^{-d}\left[-\frac{h}{2T}-\frac{h}{4T}\frac{(m-1)[\Delta_1+\Delta_2+T(1+h)]K_d\ln b}{(1+h)^2}\right]\,.\tag{6}$$

The Feynman diagrams are those in Fig. 3 (and Fig. 3 of I). Using $h'/T'=\zeta h/T$, we have

$$\zeta=b^d[1-\tfrac{1}{2}(m-1)(T+\Delta_1+\Delta_2)K_d\ln b]\,.\tag{7}$$

The recursion relations for $\Delta_1/2T^2$ and $\Delta_2/2T^2$ are

$$\frac{\Delta_1'}{2T'^2}=\zeta^2b^{-d}\left[\frac{\Delta_1}{2T^2}+\frac{(\Delta_1+\Delta_2)^2}{2T^2}K_d\ln b\right]\,,\tag{8}$$

$$\frac{\Delta_2'}{2T'^2}=\zeta^2b^{-d+\sigma}\frac{\Delta_2}{2T^2}\,.\tag{9}$$

The Feynman diagrams of Eq. (8) are shown in Fig. 4 (and Fig. 4 of I). Note that the one loop term does not contribute to the recursion relation for $\Delta_2'/2T'^2$, so Eq. (9) contains only one term. As mentioned in I we still cannot find a general proof to show that the infinite number of terms with the coefficient $\Delta_1/2T^2$ or $\Delta_2/2T^2$ in Eq. (4) can be renormalized consistently.

The differential recursion relations are obtained from Eqs. (5) and (7)—(9). They are

$$\frac{dT(l)}{dl}=T(2-d)+TK_d(m-2)(T+\Delta_1+\Delta_2)\,,\tag{10}$$

$$\frac{d\Delta_1(l)}{dl}=\Delta_1(4-d)+\Delta_1K_d\left[(m-3)(\Delta_1+\Delta_2+T)+\frac{(\Delta_1+\Delta_2)^2}{\Delta_1}\right]\,,\tag{11}$$

$$\frac{d\Delta_2(l)}{dl}=\Delta_2(4-d+\sigma)+\Delta_2K_d(m-3)(T+\Delta_1+\Delta_2)\,.\tag{12}$$

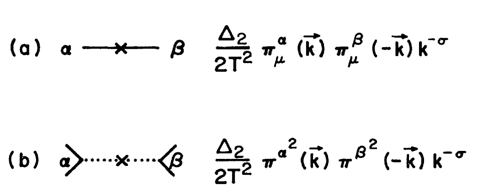

FIG. 1. Vertices relevant to the perturbation expansions in Eq. (4).

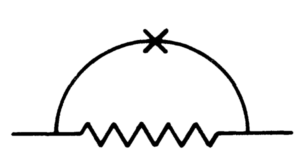

FIG. 2. Feynman diagram contributing to the recursion relation for the two-point function $(1/2T)\pi_\mu^\alpha(\vec{k})\pi_\mu^\alpha(-\vec{k})k^\sigma$.

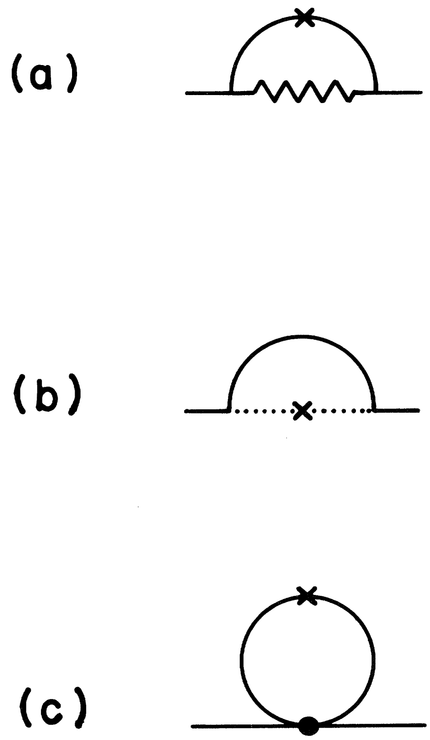

FIG. 3. Feynman diagrams contributing to the recursion relation for $(h/T)(\vec{\pi}^{\alpha})^2$.

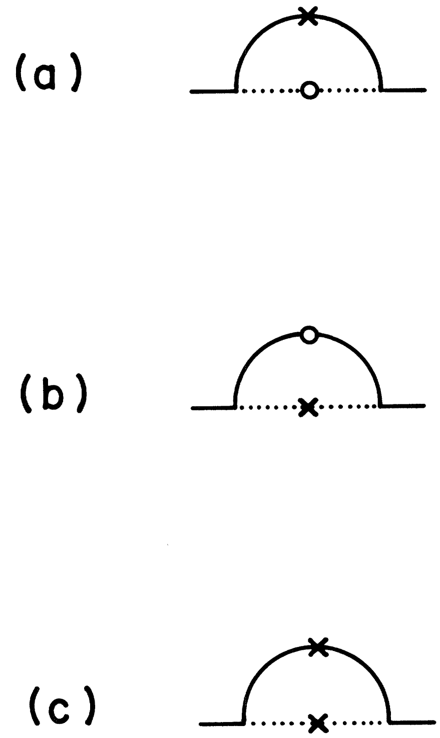

FIG. 4. Feynman diagrams contributing to the recursion relation for $(\Delta_1/2T^2)\pi_\mu^\alpha(\vec{k})\pi_\mu^\alpha(-\vec{k})$, $\alpha\neq\beta$.

III. RESULTS AND DISCUSSION

The fixed points can be determined from Eqs. (10)–(12). The three eigenvalues for each set of fixed points can be obtained by diagonalizing the 3×3 matrix from Eqs. (10)–(12). The fixed points and the eigenvalues are listed in Tables I and II, respectively.

The criterion for a fixed point to be stable is that two eigenvalues are less than 0 (irrelevant) and the third one is greater than 0 (relevant). Another condition in our case is that in the physical region Δ_2 must be ≥ 0. The reason is that $\bar{G}(k)\sim\Delta_1+\Delta_2 k^{-\sigma}$ must be greater than or equal to 0 for all momenta up to the cutoff since it is the Fourier transform of the translationally invariant correlation function G.

We limit ourselves to $d>4+\sigma$, although there is still no rigorous proof to show the absence of ferromagnetism at $d\leq 4+\sigma$. Here we discuss the stability of each fixed point.

For the Gaussian fixed point (I), we have λ_1, λ_2, and λ_3 corresponding to the eigenvalues in the T, Δ_1 and Δ_2 directions, respectively. They all are irrelevant (less than 0) and not physically interesting for the critical behavior.

In the short-range-disorder fixed point (II), λ_1, λ_2, and λ_3 are also the same as λ_T, λ_{Δ_1}, and λ_{Δ_2}, respectively. In order for II to be stable, $\lambda_3=[\sigma(m-2)-d+4]/(m-2)$ must be less than 0. This gives the limitation $\sigma<(d-4)/(m-2)$ for the stability of this short-range fixed point.

We now turn to the long-range-disorder fixed point (III). We note that λ_1 corresponds to λ_T, while λ_2 and λ_3 are the eigenvalues for the scaling fields which are linear combinations of Δ_1 and Δ_2. In the region

TABLE I. Fixed points of the recursion relations.

Fixed points	T	Δ_1	Δ_2
I—Gaussian	0	0	0
II—short-range disorder	0	$\frac{d-4}{K_d(m-2)}$	0
III—long-range disorder	0	$\frac{(4-d+\sigma)^2}{(m-3)^2K_d\sigma}$	$\frac{(d-4-\sigma)[4-d+(m-2)\sigma]}{(m-3)^2K_d\sigma}$
IV—pure	$\frac{d-2}{(m-2)K_d}$	0	0
V—short-range disorder ($T\neq0$)	$\frac{2}{K_d}$	$\frac{d+2-2m}{(m-2)K_d}$	0

TABLE II. Eigenvalues of the fixed points of Table I.

Fixed points	λ_1	λ_2	λ_3
I—Gaussian	$2-d$	$4-d$	$4-d+\sigma$
II—short-range disorder	-2	$d-4$	$\dfrac{\sigma(m-2)-d+4}{m-2}$
III—long-range disorder	$\dfrac{d-\sigma(m-2)-4+2(3-m)}{m-3}$	$A+B^{a}$	$A-B^{a}$
IV—pure	$d-2$	$\dfrac{4m-10+d(3-m)}{m-2}$	$4-d+\sigma+\dfrac{(d-2)(m-3)}{m-2}$
V—short-range disorder ($T\neq 0$)	$\dfrac{d-2+(d^2-12d-12+16m)^{1/2}}{2}$	$\dfrac{d-2-(d^2-12d-12+16m)^{1/2}}{2}$	$4-d+\sigma+\dfrac{(d-2)(m-3)}{m-2}$

[a] $A=(d-4-\sigma)(m-1)/2(m-3)$.

$$B=\frac{1}{2}\left[(d-4-\sigma)^2\frac{(m-1)^2}{(m-3)^2}+4(d-4-\sigma)(d-4+\sigma(m-2))/(m-3)\right]^{1/2}.$$

$\sigma(m-2)-d+4<0$ where the short-range-disorder fixed point (II) is stable, we have Δ_2^* (long-range-disorder fixed point) equal to

$$\frac{(d-4-\sigma)[4-d+(m-2)\sigma]}{(m-3)^2K_d\sigma}<0\ .$$

It is in the unphysical region as mentioned earlier. Therefore in this region of σ the critical behavior is dominated by the short-range-disorder fixed point.

When $4-d+(m-2)\sigma=0$, fixed points II and III coincide. The normal crossover effect occurs. When $4-d+(m-2)\sigma>0$, fixed point II becomes unstable. For fixed point III,

$$\lambda_1=\frac{d-\sigma(m-2)-4+2(3-m)}{m-3}$$

is always less than 0 ($m>3$). Also we find $\lambda_2>0$ and $\lambda_3<0$. Therefore in this region the long-range-disorder fixed point III is stable and determines the critical behavior. In Fig. 5 we show the regions of stability for fixed points II and III. We want to check whether the conjecture that the critical exponents in powers of $d-4-\sigma$ associated with the long-range fixed point (III) in $d-4-\sigma$ expansion are the same as those in $d-2$ expansion for pure systems is still valid or not.[7] For example, in the pure case, $\nu=1/(d-2)$. In fixed point III we see from that $\nu=1/\lambda_2\neq 1/(d-4-\sigma)$. Thus the conjecture fails.

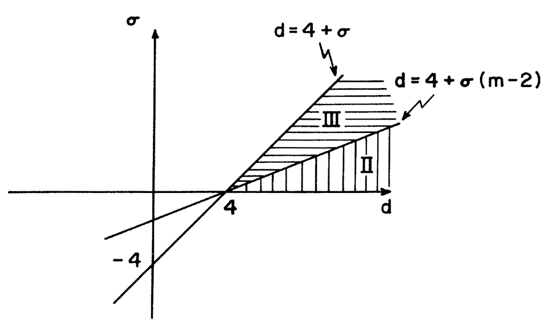

FIG. 5. Regions of stability of the short-range- (II) and long-range- (III) disorder fixed points.

So far, the discussion for fixed point III has been limited to $m>3$ (because of $m-3$ denominators). If $m=3$ we can see that the differential recursion relation in Eq. (10) becomes

$$\frac{d\Delta_2}{dl}=\Delta_2(4-d+\sigma)\ .$$

This leads to $\Delta_2^*=0$. Therefore there is no long-range-disorder fixed point if $m=3$. This can also be understood from Fig. 5. When $m=3$, the dividing line $\sigma(m-2)-d+4=0$ coincides with $\sigma-d+4=0$. That means the region dominated by the long-range fixed point shrinks to zero. Certainly this is only true to the lowest order.

It is interesting to note that in Refs. 19—21 the case $m=1$ is special in a similar way due to $m-1$ denominators in their short-range-disorder fixed point. Apparently, this feature is a purely mathematical result. There seems to be no physically intuitive argument to explain it. It is well known that in $m=1$ case, when higher-order terms are considered, the fixed points and the critical exponents are of order $\epsilon^{1/2}$ ($\epsilon=4-d$). This can be simply understood from the degeneracy in Eqs. (4.6b) and (4.6c) to second order in u and v in Ref. 19. Our $m=3$ case is different because at $m=3$ there is no degeneracy in Eqs. (11) and (12). Up to this order we have $\Delta_2^*=0$ so there is no long-range fixed point. If two-loop calculations were available, there might be a long-range-disorder fixed point with Δ_2^* being of order $(d-4-\sigma)^2$ instead of $(d-4-\sigma)^{1/2}$, provided the factor $m-3$ does not appear in the term at two-loop order of Eq. (12).

Now let us discuss the $m<3$ case. Obviously the nonlinear σ model is inapplicable at $m=1$. For $m=2$ we see that the fixed points II, IV, and V also have $m=2$ denominators. Thus we should treat this case separately. If we set $m=2$ in Eqs. (10)—(12) we find that the long-range-disorder fixed point (III) has

$$\Delta_2^*=\frac{(d-4-\sigma)(4-d)}{K_d\sigma}<0$$

(for $d>4+\sigma$), which is unphysical and the fixed points II, IV, and V do not exist. The Gaussian fixed point (I) is stable in all directions. Therefore it does not determine the critical behavior. For $m=2$, within the nonlinear σ model formalism at one-loop order, there is no phase transition.[22] In the pure case ($\Delta_1=\Delta_2=0$) we know that $T^*=[(d-2)/(m-2)K_d]$ is finite as $d\rightarrow 2$ and $m\rightarrow 2$. This is a signal for the Kosterlitz-Thouless transition. For the short-range-disorder case ($\Delta_1\neq 0$ and $\Delta_2=0$) we have $\Delta_1^*=[(d-4)/K_d(m-2)]$. So it is likely to have Kosterlitz-Thouless transition at $d=4$ and $m=2$. This has been mentioned by Cardy.[14] Furthermore, in the case of long-range disorder, Δ_1^* and Δ_2^* contain factors $4-d+\sigma$ and $m-3$ in the numerator and denominator, respectively. Therefore the mechanism for the transition at $d=4+\sigma$ and $m=3$ is probably also of Kosterlitz-Thouless type. However, at the present time we cannot say anything about it.

Finally we make comments on the fixed points IV and V. We again use the stability criterion that two eigenvalues are negative and the third is positive. Indeed in some combinations of d, m, and σ, there are stable fixed points. However, we should note that the scaling fields are not T and Δ_1 themselves, but rather linear combinations of them. Also, at low temperature ($T\rightarrow 0$) one is quite far away from T^* in fixed points IV and V. Therefore these fixed points are not physically interesting in discussing the critical behavior. When $T\rightarrow 0$ the critical behavior should be dominated by fixed points II and III in which T is an irrelevant operator and the randomness is a relevant one.

IV. CONCLUSION

In this work we have discussed the long-range correlated random magnetic fields using low-temperature renormalization-group calculations. We have found the long-range-disorder and short-range-disorder fixed points. They are stable in different regions in Fig. 5. The crossover happens at $(m-2)\sigma-d+4=0$. Also the critical exponents calculated at the long-range-disorder fixed point in powers of $d-4-\sigma$ are no longer the same as those in $d-2$ expansions in the pure systems or in $d-4$ expansions in the uncorrelated random field model.

In this theory it is ambiguous to determine the flow diagrams exactly. The reason is that in the initial Hamiltonian (unrenormalized or physical) $\bar{G}(k)$ is

$$\Delta_1+\Delta_2 k^{-\sigma}+\sum_{i=3}^{N}\Delta_i k^{-\sigma_i}\quad(\sigma_i<\sigma)\ ,$$

and these Δ_i are not supposed to be zero initially. In the Appendix we show that the fixed point $\Delta_i^*\neq 0$ ($i>2$) is always unstable. Therefore we can only be sure of the following.

(i) If (d,σ) is in region II in Fig. 5 and the starting values for T, Δ_1, Δ_2, and Δ_i ($i>2$) are sufficiently close to fixed point II, then flow is to that point which determines the critical behavior.

(ii) If (d,σ) is in region III in Fig. 5 fixed point III should be dominant for the parameters close to that point. In this case we should be careful that the relevant scaling field is not Δ_1 or Δ_2 but rather a linear combination of them.

Another interesting problem which still needs further investigation is the phase diagram. So far, we have discussed it in the ferromagnetic state. Just as in the random axis model, in the low-T region the phase boundary separates a spin-glass phase and a ferromagnetic phase. We conjecture it is the same in the system with correlated random magnetic fields. Certainly the phase diagram will be complicated because it has at least Δ_1, Δ_2, and T parameters. We have mentioned before that fixed point (V) is not physically interesting as far as critical behavior is concerned in the low-T region. However, this point might be related to a multicritical point.

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APPENDIX

In this Appendix we discuss the slightly generalized model

$$\bar{G}(k)\sim\Delta_1+\Delta_2 k^{-\sigma}+\sum_{i=3}^{N}\Delta_i k^{-\sigma_i},\quad \sigma_i<\sigma,\quad i>2\ .$$

We follow the same lines as in the text to derive the following differential recursion relations:

$$\frac{dT(l)}{dl}=T(2-d)+TK_d(m-2)(T+\Delta_1+\Delta_2+\Omega)\ ,\qquad\text{(A1)}$$

$$\frac{d\Delta_1(l)}{dl}=\Delta_1(4-d)+TK_d\Delta_1\left[(m-3)(T+\Delta_1+\Delta_2+\Omega)+\frac{(\Delta_1+\Delta_2+\Omega)^2}{\Delta_1}\right],\qquad\text{(A2)}$$

$$\frac{d\Delta_2(l)}{dl}=\Delta_2(4-d+\sigma)+\Delta_2K_d(m-3)(T+\Delta_1+\Delta_2+\Omega)\ ,\qquad\text{(A3)}$$

$$\frac{d\Delta_{i>2}(l)}{dl}=\Delta_i(4-d+\sigma_i)+\Delta_iK_d(m-3)(T+\Delta_1+\Delta_2+\Omega)\ .\qquad\text{(A4)}$$

Here $\Omega=\sum_{i=3}^{N}\Delta_i$.

Besides the fixed points listed in Table I we can find another set of fixed points with $T=0$, $\Delta_1^*\neq 0$, $\Delta_2^*=0$, $\Delta_j^*\neq 0$, and $\Delta_i^*=0$, $i,j>2$. Δ_1^* and Δ_j^* are the same as those in fixed point III with σ replaced by σ_j, and

$$\Delta_1^*+\Delta_j^*=\frac{d-4+\sigma_j}{K_d(m-3)}\ .$$

We can easily obtain

$$\lambda_{\Delta_i\,(j\neq 1)}=\sigma_i-\sigma_j\ .\qquad\text{(A5)}$$

As long as $\sigma_j\neq\sigma$, there is always a σ_i ($i>j$) such that $\lambda_{\Delta_i}=\sigma_i-\sigma_j>0$. Since one eigenvalue (i.e., λ_2, cf. Table

II) for Δ_1 and Δ_j is already always positive, we have at least two positive eigenvalues, thus this set of fixed points is always unstable.

To make the theory complete we still have to show that the stability of fixed points II and III is not influenced by the addition of all the terms Δ_i ($i=3,\ldots,N$). For the long-range-disorder fixed point (III) we check λ_{Δ_i} from Eq. (A4). It is

$$\lambda_{\Delta_i}=\sigma_i-\sigma<0 \; . \tag{A6}$$

Since this eigenvalue is always less than 0 it will not change the stability. Similarly for the short-range-disorder fixed point (II), we have

$$\lambda_{\Delta_i}=4-d+\sigma_i+(m-3)\frac{d-4}{m-2}=\frac{\sigma_i(m-2)-d+4}{m-2} \; . \tag{A7}$$

In region II of Fig. 5 where the fixed point II is stable, we have $\sigma(m-2)-d+4<0$. Since $\sigma_i<\sigma$, then

$$\lambda_{\Delta_i}=\frac{\sigma_i(m-2)-d+4}{m-2}<0 \; .$$

We have proved the fact that the stability of the fixed points II and III still remain the same even if the terms $\sum \Delta_i k^{-\sigma_i}$ are considered. Therefore we are convinced that the critical behavior is always dominated by the fixed points in Table I obtained from $\bar{G}(k)\sim\Delta_1+\Delta_2 k^{-\sigma}$, but the details of the flow diagrams certainly depend on the initial values of $\Delta_{i\,(i>2)}$.

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