

## Thermoelectric effect in a weakly disordered inversion layer subject to a quantizing magnetic field

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We demonstrate that the usual Kubo formula for thermal response functions is invalid if a magnetic field is present. There exists a fundamental correction due to a lack of time-reversal symmetry. We show in particular that, in addition to being of general importance in the theory of transport, this leads to a novel thermoelectric effect in a weakly disordered two-dimensional electron gas subject to a quantizing magnetic field. The thermopower tensor is calculated within the self-consistent Born approximation using a generalized Mott formula, which is derived.

### I. INTRODUCTION

In a two-dimensional system such as an electron inversion layer the particles are free to move in a plane while they are localized and the eigenstates are quantized (discrete) in the third direction. In a strong perpendicular magnetic field the motion of the electrons in the plane is restricted to Landau orbits. Only certain orbits are allowed and the corresponding energy eigenstates are quantized; hence we have a completely quantized system. The theoretical picture of transport in such quantum systems has been revolutionized by the discovery of the quantum Hall effect<sup>1</sup> which manifests itself as extremely well-defined plateaus in the Hall resistance measured as a function of electron density or magnetic field. This discovery has generated a great deal of interest in transport properties of quantum systems in general. A particularly interesting transport property is the thermoelectric effect, which traditionally has been used as a sensitive probe of transport mechanisms in metals and semiconductors. Recently, we presented a calculation of the thermopower of an ideal two-dimensional electron gas in a quantizing magnetic field.<sup>2</sup> The thermopower of that system was shown to be a universal function of a reduced temperature ( $k_B T / \hbar \omega_c$ ) and it displayed a novel dependence on the chemical potential: The thermopower is thermally activated for values of the chemical potential between Landau levels. The analysis in this earlier work relied on some general properties of the eigenfunctions and eigenenergies of a particular system: the two-dimensional free-electron gas bounded in one direction, extended in the others. Our analysis was simple, but had the benefit of being explicit and of demonstrating the importance of edge currents.

In the present work we use the more general and powerful Kubo formalism to calculate the transport coefficients that determine the thermoelectric effect. We demonstrate that the usual Kubo formula for the thermal response function is invalid if a magnetic field is present. There

exists a fundamental correction due to the lack of time-reversal symmetry in a magnetic field. In addition to being of general importance in the theory of transport we demonstrate in particular that this leads to the existence of the novel thermoelectric effect in the two-dimensional electron gas that was found for a special case in our earlier work. Our present analysis allows us to investigate also the effect of disorder on the thermoelectric effect. We find that in the presence of static disorder treated in the so-called self-consistent Born approximation,<sup>3</sup> the thermopower tensor develops off-diagonal as well as diagonal components. The thermopower tensor varies in a rather complex manner for values of the chemical potential inside Landau levels (nonzero density of single-particle states). For values of the chemical potential between Landau levels (zero density of single-particle states) only the diagonal components are nonzero and retain the activated behavior found for the case of no disorder.

The paper is organized as follows. In Sec. II we derive formal expressions for the response functions and illustrate the results by calculating the thermopower of an inversion layer in the free-electron approximation. Section III contains a derivation of a generalized Mott formula for the thermopower and in Sec. IV we present a model calculation, the result of which is discussed and favorably compared with recent experimental information in Sec. V.

### II. TRANSPORT COEFFICIENTS IN THE KUBO FORMALISM

The thermoelectric effect is due to the interdependence of potential gradients and temperature gradients in a system where no particle current flows. Hence we need to be able to calculate the nonequilibrium particle-current density in a system influenced by temperature ( $T$ ) and potential ( $V$ ) gradients. The fundamental phenomenological expression for this current is<sup>4,5</sup>

$$J_i = L_{ij}^{11}(e/T)\partial_j V + L_{ij}^{12}\partial_j(1/T), \quad (1)$$

where  $-e$  is the electronic charge. In the presence of a magnetic field the response functions are tensor quantities and so the thermopower, which is defined by the zero-current condition,

$$\partial_j V = -S_{ij} \partial_j T, \quad (2)$$

satisfies

$$S_{ij} = (-1/eT)(L^{11})_{ik}^{-1} L_{kj}^{12}. \quad (3)$$

The coefficient  $L_{ij}^{11}$  is related to the conductivity via  $L_{ij}^{11} = (T/e^2)\sigma_{ij}$  and so it can be calculated using the standard Kubo formula involving the current-current correlation function.<sup>4,5</sup> Luttinger<sup>4</sup> first provided a so-called "mechanical" derivation of a Kubo formula for  $L^{12}$  by noting that the response to a temperature gradient  $\partial_j(1/T)$  can be shown to be equal to the response to  $(-1/T)\partial_j\psi$ , where  $\psi$  is a pseudogravitational field which couples to the Hamiltonian via

$$H = H_0 + \frac{1}{2}(\psi H_0 + H_0 \psi), \quad (4)$$

where  $H_0$  is the unperturbed Hamiltonian. Luttinger showed that *in the absence of a magnetic field* the response to  $(-1/T)\partial_j\psi$  is given by the usual Kubo formula<sup>4,5</sup> involving the particle-current-heat-current correlation function with the heat-current operator defined by

$$\vec{J}_Q = (H_0 \vec{J} + \vec{J} H_0)/2 - \mu \vec{J}, \quad (5)$$

where  $\mu$  is the chemical potential. In the absence of a magnetic field all the response functions can thus be obtained from the expression<sup>4,5</sup>

$$\bar{L}_{ij}^{\eta\zeta} = \text{Re} \lim_{\omega \rightarrow 0} \bar{L}_{ij}^{\eta\zeta}(\omega + i\delta), \quad (6)$$

where  $\delta$  is a positive infinitesimal and  $L(\omega + i\delta)$  is the analytical continuation of

$$\bar{L}_{ij}^{\eta\zeta}(i\omega) = \frac{-iT}{\hbar(i\omega)\nu} \int_0^{\hbar\beta} d\tau e^{i\omega\tau} \langle J_i^\eta(\tau) J_j^\zeta(0) \rangle. \quad (7)$$

In (7)  $\nu$  is the volume of the system,  $\beta = (k_B T)^{-1}$  is the inverse temperature,  $\vec{J}^1 = \vec{J}$  is the particle-current operator, and  $\vec{J}^2 = \vec{J}_Q$  is the heat-current operator.

It is known that this standard expression for the case of the thermal response tensor  $\bar{L}^{12}$  must be modified if a magnetic field is present. This is due to the appearance of diamagnetic surface currents.<sup>6-8</sup> We hence wish to make the point that the familiar expression (6) does not give the thermal response function in the presence of a magnetic field. The formal reason is that part of the linear response to the  $\psi$  field is due to a change in the Hamiltonian. It follows from (4) that the current operator itself is changed by

$$\delta \vec{J} = (\psi \vec{J} + \vec{J} \psi)/2. \quad (8)$$

This extra "diathermal" current is analogous to the diamagnetic current which appears in the presence of a vector potential. We will show that because of the lack of time-reversal symmetry, the diathermal current has a nonzero equilibrium expectation value if a magnetic field is present. We note in passing that there is also a change

in the heat-current operator which thus affects the thermal conductivity. By defining  $P$

$$\langle \delta J_i \rangle = (-1/T) P_{ij} \partial_j \psi, \quad (9)$$

it follows directly from Luttinger's expressions that the complete thermal response function is given by the modified formula

$$L_{ij}^{12} = \bar{L}_{ij}^{12} + P_{ij}. \quad (10)$$

For more detailed arguments consider a system described by

$$H_0 = (1/2m)(\vec{p} + e\vec{A}/c)^2 + V, \quad (11)$$

where the vector potential is in the Landau gauge

$$\vec{A} = xB\hat{y}. \quad (12)$$

and  $V$  contains all scalar potentials and interactions. Assuming a constant gradient such that  $\psi = \lambda x$  it follows from (8) and (9) that

$$P_{ix} = -(T/2) \langle x J_i + J_i x \rangle. \quad (13)$$

Using the definition of the current operator, one may write

$$P_{xx} = (iT/2\hbar\nu) \langle [H_0, x^2] \rangle. \quad (14)$$

If boundary contributions may be neglected in the matrix-element integral then  $P_{xx}$  vanishes. This is the case in the examples which will be discussed shortly. Using (12) we may also write

$$P_{yx} = -(T/2B) \langle \vec{J} \cdot \vec{A} + \vec{A} \cdot \vec{J} \rangle, \quad (15)$$

or equivalently in the absence of spin

$$P_{xy} = + \frac{Tc}{e\nu} \frac{\partial \Omega}{\partial B}, \quad (16)$$

where  $\Omega$  is the thermodynamic potential.

We now treat a simple example in order to illustrate the importance of this extra contribution to the thermally induced current. Consider a quasi-two-dimensional system such as an electronic inversion layer in which particles are free to move in a plane while motion is essentially frozen out in the third direction by discrete quantization. In the limit of a strong magnetic field normal to the plane the electronic energy states are completely quantized into Landau levels. For illustrative purposes we take as the Hamiltonian the expression in (11) but neglect  $V$ . The effects of disorder will be discussed later. We will find that the thermopower of this type of system exhibits very unusual behavior. It is straightforward to evaluate the Kubo formula given in (7) for this simple model to obtain

$$\bar{L}_{xx}^{11} = \bar{L}_{yy}^{11} = 0, \quad (17)$$

$$\begin{aligned} \bar{L}_{xy}^{11} &= -\bar{L}_{yx}^{11} \\ &= -\frac{T}{h} \sum_{N=0}^{\infty} \{ (N+1) [n_F(\hbar\omega_N) - n_F(\hbar\omega_{N+1})] \}, \end{aligned} \quad (18)$$

$$\bar{L}_{xx}^{12} = \bar{L}_{yy}^{12} = 0, \quad (19)$$

$$\begin{aligned} \bar{L}_{xy}^{12} &= -\bar{L}_{yx}^{12} \\ &= -\frac{T}{h} \sum_{N=0}^{\infty} \left[ (N+1)[n_F(\hbar\omega_N) - n_F(\hbar\omega_{N+1})] \right. \\ &\quad \left. \times \left[ \frac{\hbar\omega_N + \hbar\omega_{N+1}}{2} - \mu \right] \right], \end{aligned} \quad (20)$$

where  $n_F$  is the Fermi function. In order to gain some understanding of these expressions, suppose that the chemical potential lies between levels  $N$  and  $N+1$  and that the temperature is low ( $k_B T \ll \hbar\omega_c$ ); then we obtain

$$\bar{L}_{ij}^{12} = Q \bar{L}_{ij}^{11}, \quad (21)$$

where  $Q$  is the so-called heat of transport given by

$$Q = (\hbar\omega_N + \hbar\omega_{N+1})/2 - \mu. \quad (22)$$

In the absence of the diathermal current, evaluation of (3) would yield

$$S_{ij} = -(Q/eT)\delta_{ij}. \quad (23)$$

This predicts that the thermopower would be very large at low temperatures since  $Q$  is of the order of the magnetic energy. This form is reminiscent of that obtained for a semiconductor in which case  $Q$  is of the order of the band gap (although the analogy is actually not very close).

Let us now consider how the existence of the diathermal current affects this result. Equation (16) is readily evaluated yielding

$$\begin{aligned} P_{xy} &= \frac{T}{h} \sum_N \left[ (2\hbar\omega_N - \mu)n_F(\hbar\omega_N) \right. \\ &\quad \left. + \int_{\hbar\omega_N}^{\infty} d\epsilon(\epsilon - \mu)n_F(\epsilon) \right]. \end{aligned} \quad (24)$$

The first term in this expression exactly cancels the contribution from (20) so that (10) becomes

$$L_{xy}^{12} = \frac{T}{h} \sum_N \int_{\hbar\omega_N}^{\infty} d\epsilon(\epsilon - \mu)n_F(\epsilon). \quad (25)$$

Thus the thermal transport coefficient is drastically modified by the existence of the diathermal current.

The physical implication of (25) can best be understood by rederiving this expression from a completely different picture which is valid for this special case. Consider a long rectangular strip in which electrons are held by a confining potential. There exist large, oppositely directed currents at each edge carried by electrons in so-called skipping orbits. Taylor<sup>9</sup> and Halperin<sup>10</sup> have recently shown that the quantized Hall resistance can be derived by considering the change in edge currents when the two edges suffer a chemical-potential difference. A similar calculation for the case when the two edges have slightly different temperatures was performed in our earlier work<sup>2</sup> and the result is precisely that obtained in (25).

The thermopower calculated using (25) is for the sake of completeness again displayed in Fig. 1. There are two remarkable features worth pointing out. The first is that as a function of chemical potential the thermopower has a series of large ( $\approx 40 \mu\text{V/K}$ ) peaks near the Landau-level

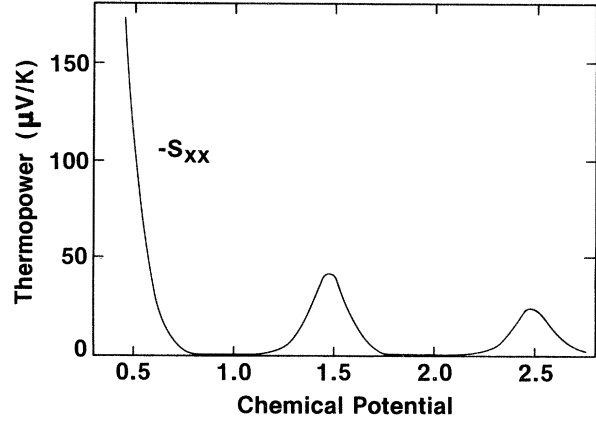


FIG. 1. Diagonal component of the thermopower vs chemical potential in units where  $\hbar\omega_c=1$  and  $k_B T=0.05$ . This corresponds to  $T=4.3$  K at  $B=12$  T in a Si(110) inversion layer. Effects of spin and valley splitting have not been included.

positions. At low temperatures the height of the peak associated with Landau level  $N$  approaches the universal value  $-\ln 2(k_B/e)/(N + \frac{1}{2})$  independent of the magnetic field, the effective mass, etc. However, this universal result is only obtained for free electrons as we shall see later. The second noteworthy feature is that for values of the chemical potential in between Landau levels the thermopower is thermally activated with an activation energy which is of the order of the magnetic energy. The source of this activation is the gap at the Fermi level. At zero temperature the system looks similar to a superconductor with zero resistance (even with impurities present as will be shown below) and zero thermopower.

Up to this point we have only dealt with free electrons. We shall now examine how the thermopower is affected by (weak) disorder.

### III. MOTT FORMULA FOR THE THERMOPOWER

In the example discussed above we saw that in the free-electron case an important cancellation of terms in the expression (10) for the thermal response function  $L^{12}$  took place. In the presence of disorder this exact cancellation *does not* arise unless the effects of the boundary conditions on the electron wave functions are accurately included in the calculation of all terms. To avoid this difficulty we shall relate the transport coefficients  $L_{ij}^{12}$  to the coefficients  $L_{ij}^{11}$  and hence obtain a Mott formula for the thermopower.<sup>11</sup> When calculating the  $L^{11}$  transport coefficients the boundary conditions do not enter in such a crucial way. To derive the desired relation we shall use a method that amounts to taking advantage of the cancellation of terms in the formal expression for  $L_{ij}^{12}$ .

The thermal response function  $L^{12}$  is a sum of the correlation function term  $\bar{L}^{12}$  and the correction term  $P$ . Both these terms can be expressed in terms of matrix elements of the particle-current operator  $\vec{J}$  and the heat-current operator  $\vec{J}_Q$  using exact single-particle eigenfunctions  $\{\psi_n\}$  and in terms of the corresponding exact energy eigenvalues  $\{E_n\}$ .

Recall that the heat-current operator can be related to the Hamiltonian and the particle-current operator as

$$\vec{J}_Q = (H\vec{J} + \vec{J}H)/2 - \mu\vec{J}, \quad (26)$$

by using (26) and the relation between the particle current and the position operator given by

$$J_i = \mathcal{L}^{-1} \dot{r}_i = \frac{i}{\hbar\mathcal{L}} [H, r_i], \quad (27)$$

where  $\mathcal{L}$  is the length of the sample. This implies the following relation between the corresponding matrix elements:

$$(J_i)_{nm} = \frac{i}{\hbar\mathcal{L}} (E_n - E_m)(r_i)_{nm}, \quad (28)$$

and one finds that the entire correction term  $P$  is canceled by part of the correlation-function term  $\bar{L}^{12}$ . The result for  $L^{12}$  is (see the Appendix)

$$L_{ij}^{12} = \frac{T}{\mathcal{L}} \sum_{m,n} \text{Re}[(r_i)_{mn}(J_j)_{nm}] \left[ \frac{-\partial n_F(E_m)}{\partial E_m} \right] (E_m - \mu) \delta_{E_m, E_n} + \hbar T \sum_{m,n} \text{Re}[(J_i)_{mn}(J_j)_{nm}] \left[ \frac{-\partial n_F(E_m)}{\partial E_m} \right] (E_m - \mu) \pi \delta(E_m - E_n), \quad (29)$$

and for  $L^{11}$  it is (see the Appendix)

$$L_{ij}^{11} = \frac{T}{\mathcal{L}} \sum_{m,n} \text{Re}[(r_i)_{mn}(J_j)_{nm}] \left[ \frac{-\partial n_F(E_m)}{\partial E_m} \right] \delta_{E_m, E_n} + \hbar T \sum_{m,n} \text{Re}[(J_i)_{mn}(J_j)_{nm}] \left[ \frac{-\partial n_F(E_m)}{\partial E_m} \right] \pi \delta(E_m - E_n). \quad (30)$$

It is interesting to note that these results are written in a form where the states  $\psi_n$  and  $\psi_m$  have the same energy, which would seem to make sense for the present case which only involves static scattering. Furthermore, only states around the Fermi level contribute to the conductivities. The *diagonal* part of the conductivity is normally written in this form, but the *off-diagonal* conductivity (the Hall conductivity) is usually written as an integral over all states up to the Fermi level emphasizing the dissipationless nature of the drift of electrons in a magnetic field.

In passing, we note that the standard textbook expression for  $L^{11}$  in Chap. 30 of Ref. 5 has no counterpart to the first term of (30). This term is important here, since it vanishes only in zero magnetic field.

For the free-electron case it is not difficult to recover our earlier result (25) using (29) and (30). It is, however, essential to realize that the boundary conditions in the  $x$  direction cause the energy eigenvalues to differ from the energy eigenvalues of a system which is periodically continued (infinite) in both the  $x$  and  $y$  directions. On the other hand, as shown in our earlier work,<sup>2</sup> one does not need to know the *explicit* form of the eigenvalues to be able to get the correct result.

With (29) we have shown that an essential cancellation of terms takes place in the expression for the thermal response function also in the presence of disorder. In this case it is, however, not easy to do numerical work starting from (29) as we now *would* have to know the explicit form of the energy eigenvalues and matrix elements. In evaluating  $L^{11}$  we do not have to worry about finding canceling terms and we may use a formulation where boundary effects are not important. Hence we would like to find a connection between the thermal response function  $L^{12}$  and the conductivity  $L^{11}$ , and then evaluate the latter in the most convenient way. To find this connection we first observe that (29) and (30) have the form

$$L^{12} = \sum_{m,n} F_{mn} \left[ \frac{-\partial n_F(E_m)}{\partial E_m} \right] (E_m - \mu), \quad (31)$$

where  $F_{mn}$  is the common factor and

$$L^{11} = \sum_{m,n} F_{mn} \left[ \frac{-\partial n_F(E_m)}{\partial E_m} \right]. \quad (32)$$

Equation (31) can be written as

$$L^{12} = \int_{-\infty}^{\infty} d\epsilon \epsilon \left[ \frac{-\partial n_F(\epsilon)}{\partial \epsilon} \right] \sum_{m,n} F_{mn} \delta(\epsilon + \mu - E_m). \quad (33)$$

Now recalling that, at  $T=0$ ,

$$\frac{-\partial n_F(E_m)}{\partial E_m} = \delta(\mu - E_m), \quad (34)$$

one recognizes immediately that

$$\sum_{m,n} F_{mn} \delta(\epsilon + \mu - E_m) = L^{11}(T=0, \epsilon + \mu), \quad (35)$$

and hence that

$$L_{ij}^{12}(T, \mu) = \int_{-\infty}^{\infty} d\epsilon \epsilon (-\partial n_F(\epsilon) / \partial \epsilon) \times L_{ij}^{11}(T=0, \epsilon + \mu). \quad (36)$$

This is the desired relation between the transport coefficients. In the zero-temperature limit (which cannot be used in the inversion-layer case as the temperature is typically not small compared to the level broadening) we obtain a generalized Mott formula<sup>11,12</sup> for the thermopower

$$S_{ij} = -(\pi^2/3)(k_B/e)(k_B T) [\sigma^{-1}]_{ik} [\partial \sigma / \partial \mu]_{kj}, \quad (37)$$

where the conductivity tensor  $\sigma$  is directly proportional to  $L^{11}$ . We shall now turn to calculating  $L_{ij}^{11}$  and hence  $S_{ij}$  in a particular model for static disorder.

#### IV. THERMOPOWER IN THE PRESENCE OF DISORDER

For an explicit model calculation of the thermopower of a weakly disordered inversion layer in a strong magnetic field we consider a two-dimensional system described by the Hamiltonian (11) with a random impurity potential,  $V$ , obeying

$$\langle V(\vec{r}_i)V(\vec{r}_j) \rangle_{av} = 2\pi l^2 \hbar^2 \sigma^2 \delta^2(\vec{r}_i - \vec{r}_j), \quad (38)$$

where  $\sigma$  is a measure of the disorder and for dimensional convenience we have included two powers of the magnetic length  $l$ .

The effect of the random potential (38) on the broadening of the Landau levels and possible localization of states in the tails are poorly understood at the moment. For illustrative purposes we therefore treat the disorder in the simplest possible manner using the self-consistent Born approximation (SCBA).<sup>3</sup> We will work in a representation where  $H_0$  is diagonal and hence we need the eigenfunctions of the kinetic energy operator. With the use of the Landau gauge (12), these have the form

$$\psi_{kN}(x,y) = \mathcal{L}^{-1/2} e^{iky} F_N(x + kl^2), \quad (39)$$

where  $\mathcal{L}$  is the length of the normalizing box in the  $y$  direction and  $F_N$  is the  $N$ th harmonic-oscillator wave function. The eigenvalue associated with  $\psi_{kN}$  is

$$\hbar\omega_N = (N + \frac{1}{2})\hbar\omega_c, \quad (40)$$

where  $\omega_c$  is the classical cyclotron frequency. The levels are highly degenerate since the eigenvalue is independent of  $k$ .

In the SCBA the dressed Green's function for propagation in the  $N$ th Landau level satisfies

$$G_N(z) = 1/[z - \omega_N - \Sigma(z)], \quad (41)$$

where the self-energy is independent of  $k$  and  $N$  and obeys

$$\Sigma(z) = \sigma^2 \sum_{M=0}^{\infty} \frac{1}{z - \omega_M - \Sigma(z)}. \quad (42)$$

Making the usual high-field approximation ( $\sigma \ll \omega_c$ ) yields

$$\Sigma(\epsilon + i\delta) = \frac{\epsilon - \omega_L}{2} - i\sigma \left[ 1 - \left( \frac{\epsilon - \omega_L}{2\sigma} \right)^2 \right]^{1/2}, \quad (43)$$

$$L_{xx}^{11} = \frac{T\omega_c^2}{16\pi^2\hbar} \int_{-\infty}^{\infty} d\epsilon \left( \frac{-\partial n_F(\epsilon)}{\partial \epsilon} \right) \sum_N [(N+1)A_N(\epsilon)A_{N+1}(\epsilon)] \quad (50)$$

and

$$L_{xy}^{11} = \frac{T\omega_c^2}{8\pi^2\hbar} \int_{-\infty}^{\infty} d\epsilon n_F(\epsilon) \sum_N \left[ (N+1) \left[ A_N(\epsilon) \frac{\partial \text{Re}G_{N+1}(\epsilon)}{\partial \epsilon} - A_{N+1}(\epsilon) \frac{\partial \text{Re}G_N(\epsilon)}{\partial \epsilon} \right] \right], \quad (51)$$

where  $A(\epsilon) = -2\text{Im}G(\epsilon + i\delta)$  is the spectral density of the Green's function  $G$ .

The results (50) and (51) for  $L^{11}$  can then be used in (36) which gives  $L^{12}$  and hence the thermopower tensor from

where  $\omega_L$  is the center of the nearest Landau level to  $\epsilon$ . Together with (41) this leads to a semielliptical density of states for each Landau level,

$$\rho = (2\pi^2 l^2 \hbar \sigma)^{-1} [1 - (\omega/2\sigma)^2]^{1/2}. \quad (44)$$

Here  $\omega$  is the deviation of the energy from the center of the nearest level.

To calculate the conductivity we start from the fundamental expression (7) for  $L^{11}$ , and use Eqs. (A3)–(A8) to express it in terms of Green's functions and current matrix elements. As all vertex corrections vanish in the SCBA in the limit of a short-range potential the response function is simply given by

$$L_{ij}^{11}(i\omega) = \frac{iT}{\hbar(i\omega)\nu} \sum_{m,n} (J_i)_{mn} (J_j)_{nm} \times G_n(ip + i\omega) G_m(ip). \quad (45)$$

It is straightforward to establish that the only nonzero matrix elements of the particle-current operator are

$$\langle k,N | J_x | k,N+1 \rangle = \frac{-i\hbar}{ml\mathcal{L}} \left( \frac{N+1}{2} \right)^{1/2}, \quad (46)$$

$$\langle k,N | J_y | k,N+1 \rangle = \frac{-\hbar}{ml\mathcal{L}} \left( \frac{N+1}{2} \right)^{1/2}, \quad (47)$$

where  $|k,N\rangle$  are the eigenstates of the kinetic energy operator. As there are  $\mathcal{L}^2/2\pi l^2$  allowed  $k$  values<sup>13</sup> one finds for the tensor  $L^{11}$

$$L^{11}(i\omega) = \frac{iT}{\hbar(i\omega)\nu} \frac{\omega_c^2}{4\pi} \sum_{N=0}^{\infty} \left[ (N+1) \begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix} Q_N(i\omega) \right], \quad (48)$$

where

$$Q_N(i\omega) = \hbar\beta^{-1} \sum_p G_N(ip) G_{N+1}(ip + i\omega). \quad (49)$$

The real and imaginary part of  $Q_N(\omega + i\delta)$  is readily evaluated and one finds that

(3). If the temperature is small compared to the width of the Landau levels the simpler Mott formula (37) for the thermopower can be used. Under typical experimental conditions this is, however, not the case.

## V. RESULTS AND DISCUSSION

The diagonal and off-diagonal components,  $S_{xx}$  and  $S_{xy}$ , of the thermopower tensor are displayed in Figs. 1–4 for a fixed temperature and varying amount of disorder. Figure 1 shows the free-electron result already discussed above. Only  $S_{xx}$  is nonzero so the thermopower tensor is diagonal in the absence of disorder and has large ( $\approx 40 \mu\text{V/K}$ ) peaks for values of the chemical potential,  $\mu$ , near the Landau-level positions. At low temperatures the height of the peak associated with Landau level  $N$  approaches the universal value  $-(\ln 2)(k_B/e)(N + \frac{1}{2})^{-1}$  independent of the magnetic field, the effective mass, etc. However, this universal result is only obtained for free electrons. With disorder the thermopower tensor develops off-diagonal components increasing in magnitude with increasing disorder. Compared to the disorder-free case the diagonal component becomes somewhat broadened and develops structure as the disorder increases and  $\mu$  is inside a Landau level, especially for higher Landau levels where the thermally activated contribution is relatively less prominent. The structure of the results in Figs. 1–4 can be understood from an analysis of the relevant conductivity and thermal-response-function tensor components in the low-temperature small-disorder limit, but we shall not carry out this analysis. When the chemical potential lies outside the Landau levels the thermopower is thermally activated with an activation energy which is of the order of the magnetic energy. Even in the presence of disorder the system looks similar to a superconductor with zero resistance and zero thermopower. This has important consequences for the high-accuracy determinations of the fine structure constant using the quantum Hall effect as errors due to thermal voltages will be exponentially small at the center of a Hall step at low temperature.

For some time only limited preliminary experimental results for the thermopower of a two-dimensional system in a quantizing magnetic field were available.<sup>14</sup> However, very recently Obloh *et al.* have presented measurements of the thermopower tensor of GaAs heterostructures.<sup>15</sup> The shape and magnitude of the diagonal as well as the off-

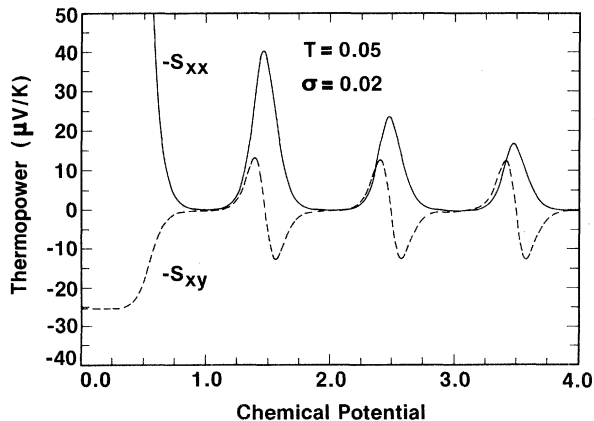


FIG. 2. Diagonal and off-diagonal components of the thermopower tensor vs chemical potential in units where  $\hbar\omega_c = 1$ ,  $k_B T = 0.05$ , and  $\hbar\sigma = 0.02$ . If  $B = 12$  T this corresponds to  $T = 4.3$  K and a Landau-level width ( $4\hbar\sigma$ ) of 0.6 meV (6.9 K).

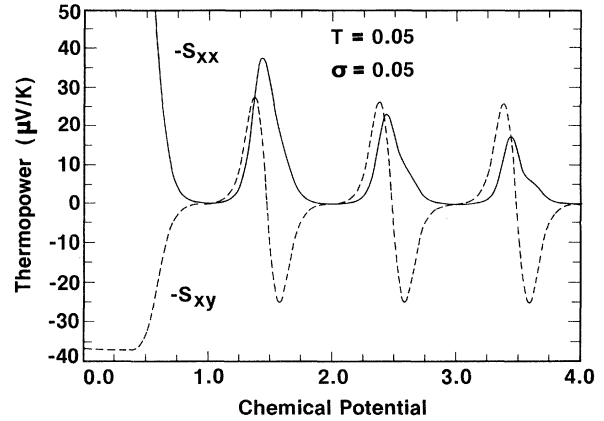


FIG. 3. Diagonal and off-diagonal components of the thermopower tensor vs chemical potential in the same units as in Figs. 1 and 2 except that the disorder has been increased to correspond to  $\hbar\sigma = 0.05$  or 1.5 meV (17 K) if  $B = 12$  T.

diagonal components of the measured thermopower tensor<sup>15</sup> seem to agree quite well with the results of the present work. The smaller maximum value of  $-S_{xx}$  for a low-mobility sample compared to a high-mobility sample is consistent with our results as is the reported dependence of  $S_{xx}$  on temperature and temperature gradient. A detailed quantitative comparison between the present theory and these recent experimental results will be presented elsewhere.<sup>16</sup> Further experimental work on systems with varying mobility would be a valuable source of information about transport properties of disordered Landau-level systems.

## APPENDIX

In this appendix we shall derive the results (29) and (30) for the transport coefficients  $L^{12}$  and  $L^{11}$ . First, we consider the thermal response coefficient  $L^{12}$  which is a sum of a correlation-function term  $\bar{L}^{12}$  and a correction term  $P$ . Let us start by expressing the correction term  $P$  given by (13) using a complete set of eigenstates  $\{\psi_m\}$ . We do

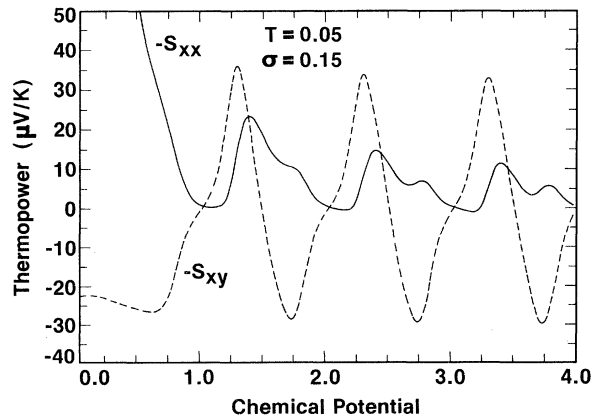


FIG. 4. Diagonal and off-diagonal components of the thermopower tensor vs chemical potential in the same units as in Figs. 1–3 except that the disorder has been increased to correspond to  $\hbar\sigma = 0.15$  or 4.5 meV (52 K) if  $B = 12$  T.

not need to know the explicit form of either the eigenstates or the corresponding set of energy eigenvalues  $\{E_m\}$  to write  $P$  as

$$P_{ij} = \frac{T}{2\mathcal{L}} \sum_{m,n} n_F(E_m) [(r_i)_{mn} (J_j)_{nm} + (J_j)_{mn} (r_i)_{nm}]$$

$$= \frac{T}{2\mathcal{L}} \sum_{m,n} (r_i)_{mn} (J_j)_{nm} [n_F(E_m) + n_F(E_n)]. \quad (\text{A1})$$

To evaluate the particle-current–heat-current correlation term  $\bar{L}^{12}$  given by (7) we first note that the heat-current operator can be written as

$$\vec{J}_Q = (H\vec{J} + \vec{J}H)/2 - \mu\vec{J}. \quad (\text{A2})$$

Hence if we write the particle-current operator as

$$\vec{J} = \sum_{m,n} \vec{J}_{mn} c_m^\dagger c_n, \quad (\text{A3})$$

using single-particle creation and destruction operators, the heat-current operator takes the form

$$\vec{J}_Q = \sum_{p,q} [(E_p + E_q)/2 - \mu] \vec{J}_{pq} c_p^\dagger c_q. \quad (\text{A4})$$

By employing the operator expressions (A3) and (A4) in the formula (7) for the particle-current–heat-current correlation function one finds an expression containing the following integral over “time”  $\tau$ :

$$\int_0^{\hbar\beta} d\tau e^{i\omega\tau} \langle T_\tau [c_m^\dagger(\tau) c_n(\tau) c_p^\dagger(0) c_q(0)] \rangle, \quad (\text{A5})$$

where  $T_\tau$  is a time-ordering operator. Having a single-particle Hamiltonian we can use Wick’s theorem.<sup>5</sup> Defining the Matsubara Green’s function and its Fourier transform<sup>5</sup> by the relations

$$G_n(\tau) = -\langle T_\tau [c_n(\tau) c_n^\dagger(0)] \rangle \quad (\text{A6})$$

and

$$G_n(\tau) = \beta^{-1} \sum_p e^{-ip\tau} G_n(ip), \quad (\text{A7})$$

we find that (A5) becomes

$$\hbar\beta G_n(0) \delta_{nm} G_q(0) \delta_{pq} - \beta^{-1} \sum_p G_n(ip + i\omega) \delta_{np} G_m(ip) \delta_{mq}. \quad (\text{A8})$$

Before we proceed to evaluate the Matsubara sum over  $ip$  in (A8) we pause to note that whereas the correction term  $P$  as given by (A1) contains one matrix element of the particle-current operator and one of the position operator, the term  $\bar{L}^{12}$  at present contains two particle-current matrix elements as we have used (A3) and (A4). With respect to combining the two terms we note that the following relation between the position operator and the particle-current operator holds:

$$J_i = \mathcal{L}^{-1} \dot{r}_i = \frac{i}{\hbar\mathcal{L}} [H, r_i]. \quad (\text{A9})$$

It follows that the relation between the corresponding matrix elements is

$$(J_i)_{nm} = \frac{i}{\hbar\mathcal{L}} (E_n - E_m) (r_i)_{nm}. \quad (\text{A10})$$

We can now evaluate the Matsubara sum<sup>5</sup> using the explicit form of the Green’s function

$$G_n(z) = 1/(z + \mu - E_n), \quad (\text{A11})$$

and perform the analytical continuation  $i\omega \rightarrow \omega + i\delta$ . The result is

$$\bar{L}_{ij}^{12} = \lim_{\omega \rightarrow 0} \left[ \frac{-T}{\hbar^2 \omega \mathcal{L}} \sum_{m,n} \text{Re}[(r_i)_{mn} (J_j)_{nm}] (E_m - E_n) \left[ \frac{E_m + E_n}{2} - \mu \right] [n_F(E_m) - n_F(E_n)] \mathcal{P} \left[ \frac{1}{E_m - E_n + \omega} \right] \right]$$

$$+ \lim_{\omega \rightarrow 0} \left[ \frac{T}{\hbar \omega} \sum_{m,n} \text{Re}[(J_i)_{mn} (J_j)_{nm}] \left[ \frac{E_m + E_n}{2} - \mu \right] [n_F(E_m) - n_F(E_n)] \pi \delta(E_m - E_n + \omega) \right]. \quad (\text{A12})$$

The principal-part factor in (A12) can be expanded as

$$\mathcal{P} \left[ \frac{1}{E_m - E_n + \omega} \right] = \mathcal{P} \left[ \frac{1}{E_m - E_n} \right] - \omega \mathcal{P} \left[ \frac{1}{(E_m - E_n)^2} \right]. \quad (\text{A13})$$

The first term of (A13), which would lead to a divergence as  $\omega \rightarrow 0$ , vanishes as can readily be verified by exchanging dummy indices  $m$  and  $n$  and noting that

$$\text{Re}[(r_i)_{nm} (J_j)_{mn}] = \text{Re}\{[(r_i)_{mn} (J_j^\dagger)_{nm}]^*\} = \text{Re}[(r_i)_{mn} (J_j)_{nm}] \quad (\text{A14})$$

as the current operator is Hermitian. The second part of (A13) can be written as

$$\mathcal{P} \left[ \frac{1}{(E_m - E_n)^2} \right] = \frac{1}{(E_m - E_n)^2} (1 - \delta_{E_m, E_n}). \quad (\text{A15})$$

If we take the contribution of the  $\delta$ -function part of (A15) to the first term and all of the second term of (A12) we find that

$$L_{ij}^{12} = \frac{T}{\mathcal{L}} \sum_{m,n} \text{Re}[(r_i)_{mn}(J_j)_{nm}] \left[ \frac{-\partial n_F(E_m)}{\partial E_m} \right] (E_m - \mu) \delta_{E_m, E_n} \\ + \hbar T \sum_{m,n} \text{Re}[(J_i)_{mn}(J_j)_{nm}] \left[ \frac{-\partial n_F(E_m)}{\partial E_m} \right] (E_m - \mu) \pi \delta(E_m - E_n). \quad (\text{A16})$$

The remaining part of (A15) (the 1) gives together with the correction term  $P$  after a rearrangement of terms

$$\frac{T}{\mathcal{L}} \sum_{m,n} \text{Re}[(r_i)_{mn}(J_j)_{nm}] (E_m - E_n)^{-1} [(E_m - \mu)n_F(E_m) - (E_n - \mu)n_F(E_n)], \quad (\text{A17})$$

where the sum runs over all states. The expression (A17) is in fact zero. To see this we first note that it is finite. In our geometry where the system is bounded in the  $x$  direction and infinite (periodically extended) in the  $y$  direction the matrix elements entering (A17) are finite if  $i=x$  and  $j=y$ . We invoke the relation (A10) once more and change dummy indices to obtain

$$\frac{-T}{\hbar \mathcal{L}^2} \sum_{m,n} \text{Im}[(r_i)_{mn}(r_j)_{nm} - (r_i)_{nm}(r_j)_{mn}] (E_m - \mu) n_F(E_m). \quad (\text{A18})$$

With  $j=y$  some care must be taken as  $(y)_{nm}$  is not well defined. However, by introducing a damping factor  $\sqrt{\eta} \exp(-\eta |y|)$  to the wave function and interpreting  $(y)_{nm}$  as the limiting value when  $\eta \rightarrow 0$ , this difficulty can be circumvented.

Equation (A18) can be simplified by summing over states  $\psi_n$  which yields an expression containing the factor

$$(r_i r_j - r_j r_i)_{mm} n_F(E_m) (E_m - \mu). \quad (\text{A19})$$

This is zero as the position operators commute. Hence (A16) is indeed the result (29) for the thermal response function  $L_{ij}^{12}$ .

It is straightforward to derive an expression for  $L_{ij}^{11}$  that has a similar form as (A16). The starting equation is the same as (A12) except for the factor  $[(E_m + E_n)/2 - \mu]$  which appeared there because  $L^{12}$  but not  $L^{11}$  contains the heat-current operator. There is no correction term analogous to  $P$ . The steps are very similar to those detailed above for the thermal response coefficient and shall not be repeated. Again a term containing the factor (A19) (except for  $E_m - \mu$ ) vanishes and the result is

$$L_{ij}^{11} = \frac{T}{\mathcal{L}} \sum_{m,n} \text{Re}[(r_i)_{mn}(J_j)_{nm}] \left[ \frac{-\partial n_F(E_m)}{\partial E_m} \right] \delta_{E_m, E_n} + \hbar T \sum_{m,n} \text{Re}[(J_i)_{mn}(J_j)_{nm}] \left[ \frac{-\partial n_F(E_m)}{\partial E_m} \right] \pi \delta(E_m - E_n). \quad (\text{20})$$

Finally we note that the two expressions (A16) and (A20) are identical except for the extra factor of  $(E_m - \mu)$  in the expression for the thermal response function. This factor is just the amount of heat carried in the state  $\psi_m$ .

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