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Failure of bulk-correlation-length scaling for the superfluid density of confined ^{4}He

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We present new measurements of the superfluid density of ^{4}He confined between sheets of Mylar at 4600 Å average separation. These data show a full range of behavior, from bulk, to finite size, to two dimensional. We analyze these data, as well as those from three other experiments, to test predictions of finite-size scaling theory. We use a scaling function from the Ginzburg-Pitaevskii-Mamaladze theory, as well as a function we suggest in analogy to the work of Josephson. We find that only this latter function is consistent with the data. Furthermore, we find that the deviations from bulk behavior do not scale with the bulk-correlation-length exponent. We find this to be in quantitative agreement with earlier results for the specific heat of confined helium.

I. INTRODUCTION

Near a second-order phase transition the behavior of a system is characterized by the divergence of the correlation length ξ at the critical point T_c. The finite extent of any laboratory system typically presents no limitations in measuring bulk properties, since the correlation length achieves macroscopic dimensions in a region too close to the transition to be experimentally accessible. On the other hand, it is not too difficult to arrange a situation whereby one is dealing with, say, a film whose thickness becomes comparable to ξ at some temperature close to T_c. Such a system will have a rather more complicated behavior, showing the effects of its finite extent as a rounding of sharp or divergent thermodynamic properties, with eventual crossover into a lower dimensionality, i.e., for a film, from three dimensions (3D) to two.

A different situation can also be realized which would not show crossover behavior. This is the case where the thickness of the film is so small, less than about two atomic layers, that it is smaller than ξ everywhere near the transition. Its behavior will thus be nearly 2D at all temperatures. Both these limits can be achieved with liquid helium, which forms a film on all surfaces in contact with it. The transition of interest is the superfluid transition. One has in one limit the bulk, 3D transition at a temperature T_λ. This is characterized by a near divergence in the specific heat and a power-law vanishing of the superfluid density ρ_{sb}. In the other limit, the 2D behavior, one has a discontinuous drop in the superfluid density[1–3] and an analytic behavior of the specific heat[4–6] at the transition temperature T_c, which depends on the film thickness. The 2D limit has received much attention lately and appears to be well described by the theory of Kosterlitz and Thouless.[4–7]

The crossover from 3D to finite size, but not the strictly 2D behavior, has been examined by Chen and Gasparini in specific-heat studies.[8] Their work shows that one cannot scale the data in any reasonable way with the exponent of the bulk correlation length. This exponent is derived from the behavior of the superfluid density and hyperscaling arguments. Scaling with this exponent can be achieved only in an analysis in which very large corrections to scaling terms are introduced.[9] This difficulty in scaling the data is particularly surprising, since it is believed that for the superfluid transition the confining walls do not couple directly to the order parameter, and thus are not expected to play as important a role as they do in other transitions.[10–13]

There has been, up to now, no analogous analysis of the superfluid density ρ_s as was done for the specific heat. We present in this paper such an analysis as well as new data taken with a torsional oscillator. These data, which were taken with helium confined between sheets of Mylar at an average separation of 4600 Å, show the full range of behavior: bulk far away from T_c, finite size in an intermediate region, and 2D behavior in a region very close to the transition.[14]

II. SCALING OF ρ_s

When superfluid helium is confined in a geometry whose smallest dimension is d, one may write the superfluid density in the critical region as[15–17]

$$\rho_s=\rho_{sb}[1-f(d^\theta t)]\ , \tag{1}$$

where $t=1-T/T_\lambda$, with T_λ the bulk superfluid transition, and $f(d^\theta t)$ is a scaling function. This equation assumes that there is a single length which scales the data. The function f must have the properties such that $f(\infty)=0$ and $f(d^\theta t_c)=1$. That is, ρ_s must revert to its bulk value for infinite separation and must vanish at some temperature T_c below T_λ. Furthermore, under the assumption that the bulk-correlation-length exponent, defined via

$$\xi=\xi_0 t^{-\nu} , \tag{2}$$

governs the finite-size effects, one must have[15,16]

$$\theta=1/\nu . \tag{3}$$

Thus at $t=t_c=1-T_c/T_\lambda$, we must have

$$d^{1/\nu}t_c=\text{const} . \tag{4}$$

This equation for the shift in the transition temperature with confinement size has received a considerable amount of experimental attention both in the case of ρ_s (Refs. 18–22) and in the case of the specific heat,[9,23,24] where one may take the specific-heat maximum to denote t_c. One should note, however, as was emphasized in Ref. 9, that this equation describes but a single feature in the behavior of the confined superfluid: the point at which ρ_s vanishes. Equation (1), on the other hand, deals with all the data near the transition, and therefore gives a much more stringent test of the scaling hypothesis in general, and of the exponent relation, Eq. (3), in particular. The difficulty in testing the data this way is that the explicit functional form of $f(d^\theta t)$ is not known.

One approach in deducing this function is via the phenomenological theory of Ginzburg and Pitaevskii, as subsequently modified by Mamaladze.[25] In this theory, one expands the free energy in the order parameter, but allows the expansion coefficients to be nonanalytic functions of the temperature. The dependence on temperature is fixed by forcing agreement with the bulk superfluid density and the specific heat. In this theory, one has a differential equation for the order parameter which can be solved for the case of confined helium. With the order parameter vanishing at the confining walls one has[26–28]

$$\rho_s \cong \rho_{sb}(1-gl/d) , \tag{5}$$

where g depends on the geometry of confinement, and d, the smallest dimension, is much larger than l, a characteristic healing length. This equation can also be derived without using the concept of a healing length. This was done by Padmore and Reppy.[29] They point out that the confinement of the helium modifies the excitation spectrum, thereby leading to an excess normal mass, given by an equation identical in form to Eq. (5). The length l in this equation is assumed to have the same temperature dependence as the correlation length, i.e., $l\sim t^{-\nu}$. Thus we can write Eq. (5) as

$$\rho_s \cong \rho_{sb}(1-g_1 t^{-\nu}/d) , \tag{6}$$

which is in the scaling form of Eq. (1) and of course satisfies Eqs. (3) and (4). This equation is also implied in the work of Ref. 15. The approximate equality used in Eq. (5) denotes the fact that terms of order $(l/d)^2$ and higher have been omitted. From more rigorous solutions in the case of a cylindrical[28] and planar[26] geometry, we find this approximation to be at worst no more than a few percent off for the range of data we will use. This has a negligible effect on our analysis.

We note that since near T_λ

$$\rho_{sb}/\rho=kt^{\zeta} , \tag{7}$$

where ρ is the total density and k is a constant, and with $\zeta=\nu$ according to scaling,[30] then Eq. (6) states that the difference between the confined and unconfined superfluid density is independent of t and depends inversely on the smallest dimension. The total density ρ is introduced so that one can deal more conveniently with dimensionless quantities.

There is another approach which one may follow to determine $f(d^\theta t)$. Josephson[30] pointed out that the relationship between the order parameter ψ, which is not directly measurable, and the superfluid density can be obtained by identifying ρ_s in the kinetic energy term of a free-energy density expansion. Thus he obtained

$$\rho_{sb}=A_\perp \left[\frac{m}{\hbar}\right]^2 |\psi|^2 \sim t^{2\beta-\eta\nu'} \equiv t^{\zeta} , \tag{8}$$

where $\eta\nu'$ is the exponent associated with $A_\perp$ and β is the exponent of the order parameter. Via scaling relations and the hyperscaling relation $3\nu=2-\alpha$, one then obtains $\zeta=\nu$. In the case of a bounded superfluid, say n layers in a film geometry, we may introduce a surface order parameter per unit volume $(2/n)\psi_s$ with which, in an analogous way for ψ, we may define a surface superfluid density,

$$\rho_{ss}=A_{s\perp}\left[\frac{m}{\hbar}\right]^2 \frac{4}{n^2}|\psi_s|^2 \sim \frac{1}{n^2}t^{\zeta_s} . \tag{9}$$

The total superfluid density is then given by

$$\rho_s=\rho_{sb}+\rho_{ss} . \tag{10}$$

From this, we then obtain

$$\rho_s/\rho=(\rho_{sb}/\rho)(1-g_2 t^{(\zeta_s-\zeta)}/d^2) , \tag{11}$$

where we have introduced the smallest dimension d, and have absorbed all constants in g_2. We have also introduced explicitly a negative sign, since ρ_{ss} must be subtracted from ρ_{sb}.

The exponent ζ_s is unknown, and one should consider it as such in testing the data. If, however, one must have agreement with the shift equation and the scaling condition, Eq. (3), then clearly one must have

$$\zeta-\zeta_s=2\nu . \tag{12}$$

In this way, both Eqs. (6) and (11) predict the same relation between T_c and the confining size. In Eq. (6), however, only the bulk exponent ν appears, while Eq. (11) has the new exponent ζ_s. The latter equation emphasizes that for ξ to govern the finite-size scaling the bulk-surface relation, Eq. (12), must hold. There are other major differences in these two equations away from the point where ρ_s vanishes. One scales the data as $1/d$, the other as $1/d^2$. One predicts a temperature-independent difference between ρ_s and ρ_{sb}, while the other a difference behaving as t^{ζ_s}.

The $1/d^2$ dependence in Eq. (11) might at first be surprising. We point out, however, that in the case of surface scaling for a finite magnetic system,[31] one obtains the fact that the order parameter, the total magnetization, is the sum of a bulk plus a surface magnetization divided by the smallest dimension. Since in the case of helium ρ_s depends on the square of the order parameter, one might expect that the smallest dimension would come in as a square.

We also note that Eq. (11), with $\zeta-\zeta_s=2\nu$, may be viewed as the quadratic term in a series expansion of a scaling function $\phi(\xi/d)$ (see later discussion). More generally, it might be viewed as the quadratic term of $\phi(\xi_s/d)$, where ξ_s is a new critical length which allows for the possibility that $\zeta-\zeta_s\neq 2\nu$.

III. EXPERIMENTAL PROCEDURE

We determined the superfluid density of confined helium by measuring the mass loading of a torsional pendulum. This technique has been described before.[3] Our particular cell consists of a Mylar[32] roll formed on a magnesium toroidal shell. The Mylar substrate has dimensions 2.5 μm$\times$1 cm$\times$100 m, which gives a geometrical area of 2.0 m^2.

The cell was intended to be used for helium films and thus was designed to maximize the ratio of surface to volume. Two trial cells were prepared and wound with different tension on the Mylar ribbon. The area determination, using nitrogen absorption isotherms, showed that one can easily overwind a cell, causing the net surface area to be less than the geometric area because of the contacts between the Mylar ribbon. For the cell of our experiment, we used 0.3-μm alumina powder as an intended spacer. We estimate that the alumina contributed only 1.5% increase in the total surface area. From the geometry of the cell and the amount of Mylar used, we deduce that the average separation between surfaces is about 4600 Å.

Upon cooling to helium temperatures, the torsional pendulum with an empty cell had a Q of 2.3×10^5 at a resonant frequency of 1.4 kHz. The period could be measured with a precision of 3 psec. For thermometry, we used an Allen Bradley resistor calibrated against the vapor pressure of ^{4}He in the range 2.5—1.2 K. The temperature data, 32 points, were fitted with a five-term polynomial with a scatter typically less than 0.5 mK.

For the experiment we report here, the cell was filled with helium slightly below the λ point. Complete filling was indicated by monitoring the change in the period of the oscillator. The cell was then warmed above T_λ and allowed to cool slowly. Data of the period and amplitude of the oscillator and the thermometer resistance were taken simultaneously as the cell cooled.

IV. DATA AND ANALYSIS

In this section we discuss and analyze the results of several experiments in addition to our own, from which one can obtain the superfluid density of confined helium. We start with our own data, then proceed to the data of Henkel and Reppy,[33] which are the measurements of the angular momentum of a superfluid current. We then deal with the data of Smith and Reppy,[34] which involve the critical flow of superfluid through a slit, and lastly, we discuss the measurements of Brooks, Sabo, Schubert, and Zimmermann (BSSZ).[20] These involve the determination of the resonant frequency of a Helmholtz oscillator which is governed almost entirely by the superfluid density of the helium confined in a superleak.

A. Torsional oscillator, present work

This method of determining the superfluid density was first used by Andronikashvili.[35] It relies on the fact that when helium is confined between surfaces whose separation is much smaller than the viscous penetration depth, then for $T>T_\lambda$, the helium just follows the motion of the surfaces. For $T<T_\lambda$, however, only the normal component remains locked, and the frequency variation of the oscillator with temperature is a measure of the amount of superfluid. With this technique one does not obtain an absolute value of ρ_s, but instead one is forced to normalize the data with results obtained by other methods. One of the advantages of this technique is that it can be used to look at the behavior of helium through the region of the superfluid transition without loss of signal, as is the case when one uses resonant sound modes.

Using this technique, we have taken data from slightly above the superfluid transition to about 1.19 K. These data are shown in Fig. 1 as ρ_s/ρ vs t in the range $2\times10^{-5}<t<10^{-2}$. The primary measurement of the shift in the period of oscillation has been reduced to ρ_s/ρ by normalizing the data at low temperature. We will discuss this procedure shortly. The obvious feature of the data is that far away from T_λ, the temperature dependence is a pure power law as in bulk helium, while as one gets closer to T_λ, there is a marked deviation. A closer inspection of this deviation already indicates that it is not a

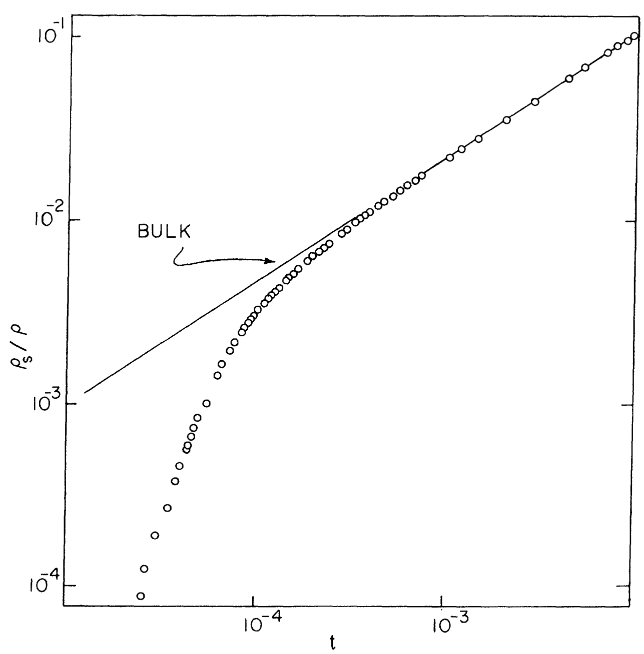

FIG. 1. Superfluid density of helium confined between sheets of Mylar at 4600 Å average separation. Data show bulk behavior for $t>10^{-3}$. This is indicated by the solid line of slope ζ. A sharp deviation from bulk behavior appears as one approaches T_λ.

constant, as would be suggested by Eq. (6), with $\rho_{sb} \sim t^{\nu}$ and $l \sim t^{-\nu}$, but rather that it does depend on temperature.

We show the data on a linear scale in the neighborhood of the transition in Fig. 2. We have plotted here the fractional period shift, the expected bulk behavior ρ_{sb}, scaled to the period shift, and the reciprocal of the amplitude of oscillation. This, in the case of our experiment, is a measure of the inverse Q, i.e., the dissipative loss in the helium flow. We note that in the case of helium films, a dramatic peak in the dissipation is observed near the transition.[3] This has been shown to be associated with the dynamics of the motion of vortices and is a characteristic of 2D behavior.[36] For thin films, the excess dissipation is typically manifest over a region of a few millikelvins near the transition. In our case, we are dealing with a situation where the film is several hundred times thicker than has been studied previously, and the region of enhanced dissipation is visible over a much narrower temperature range. As one can see from this graph, this region is less than 100 μK wide. We take this dissipation as a mark that our system has become 2D in character. The dissipation peak is superimposed on a smooth background which becomes a constant as one goes above the transition. The period shift through the region of the transition decreases with the same curvature as ρ_{sb} until one gets close to T_λ. Then, at a temperature very near the point where the dissipation increases, the curvature changes sign and the data approach zero in a more gentle manner. An important feature of the data which we have indicated by arrows is the expected universal jump in ρ_s for a 2D film with $T_c = 2.172$ K.[1] This point occurs at nearly the temperature where the dissipation has a maximum. This is in agreement with similar measurements on films having T_c's as low as 0.05 K. We do not measure a discontinuous jump at this temperature, very likely because of the finite frequency of the experiment and because within this narrow temperature region there is bound to be rounding due to nonuniformity in the confining dimension. We have further marked on Fig. 2 a width of about 60 μK below T_λ as the region in which the 3D correlation length ξ, were it to grow unimpeded, would exceed the average separation between the Mylar sheets.[37]

We have chosen for this graph the bulk transition temperature T_λ as 20 μK beyond the point where the period and amplitude stop changing. This is our estimate of the expected shift in the transition temperature for the confinement of our experiment. We have no independent measurement of T_λ. This uncertainty in T_λ introduces a possible systematic error in our subsequent analysis which we will discuss shortly.

To test the behavior of ρ_s/ρ, both Eqs. (6) and (11) require that we subtract this quantity from ρ_{sb}/ρ at the same temperature. Since the difference between these two quantities can be very small, it is important that a careful normalization of the data be made. We proceeded as follows. We have divided the shift in period from its value at the transition by 3.85 μsec, which is the total shift in going from T_λ to $T=0$. This number involves the measured period shift at 1.186 K, 3.75 μsec, plus an additional 0.10 μsec, which is our estimate of the change in ρ_{sb}/ρ down to 0 K. In this way, the fractional period shift ranges from 0 to 1, as does ρ_{sb}/ρ. More important, however, than the above normalization is the prefactor k of Eq. (7), which our data would obey in the critical region if the helium were not confined. To establish this, we have taken away the leading temperature dependence by multiplying the data near the transition by $t^{-\zeta}$.[38] Clearly in this way, if there is a portion of the critical region where ρ_s/ρ and ρ_{sb}/ρ become indistinguishable within our experimental resolution, then our data, when multiplied by this factor, would be temperature independent. We show this in Fig. 3. We see that over a limited region the data are temperature independent. Near $t=10^{-3}$, there are deviations which signal the effect of confinement. The deviations near $t=0.1$ indicate that one is exceeding the asymptotic power-law region where Eq. (7) applies. This range of validity is in agreement with measurements on bulk helium.[39–42] The systematic s-shaped deviation near $t=0.01$ is very likely the result of a fourth sound-resonant

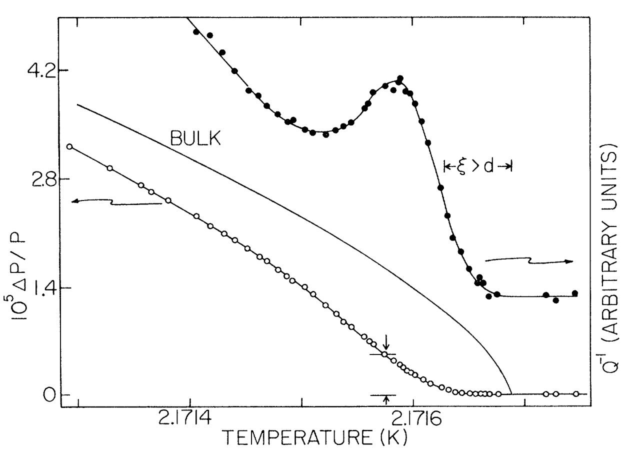

FIG. 2. Fractional period shift and Q^{-1} of the torsional oscillator near the transition. Expected bulk-helium behavior is indicated with a 20-μK shift in T_c (see text). We have also indicated by two arrows the expected jump in ρ_s for a film with $T_c = 2.172$ K, as well as the region where the 3D correlation length would tend to exceed 4600 Å if it were to grow unimpeded.

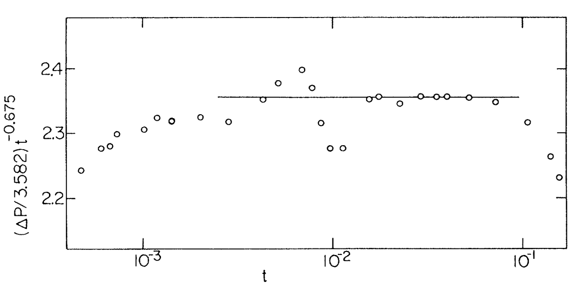

FIG. 3. Plot of the period shift in such a way as to extract the constant k of Eq. (7). Data would lie on a horizontal line if in the asymptotic region they had the same temperature dependence as in the bulk. Data for small values of t show the effect of confinement; the data for larger t show deviations because the asymptotic range of Eq. (7) is exceeded. Horizontal line yields $k=2.355$. Systematic deviation near 0.01 is a characteristic deviation due to a fourth-sound resonance mode.

TABLE I. Amplitude for the behavior of the superfluid density, $\rho_{sb}/\rho = kt^{\zeta}$. For the confined helium we have evaluated the expected value of k in a way discussed in the text.

Experiments on bulk helium	k
Persistent currents (Ref. 39)	2.41
Torsional oscillator (Ref. 40)	2.40
Fourth sound (Ref. 41)	2.38
Second sound (Ref. 42)	2.534
Experiments on confined helium	k
This work	2.355
Angular momentum of superfluid current (Ref. 33)	2.36
Flow through a slit (Ref. 34)	2.465
Helmholtz oscillator (1000, 500, 300 Å) (Ref. 20)	2.50
Helmholtz oscillator (800 Å) (Ref. 20)	2.52

resonant mode. A similar response of the oscillator is observed in the case of films when one encounters a third sound mode.[43]

The horizontal line through the data gives us the amplitude of the power-law dependence which our experiment would yield for bulk helium. This number, 2.355, is close to what is obtained with other techniques. We summarize this in Table I. This table also shows why one cannot take this amplitude from other experiments. It is a number which, even though it can be reproduced to within a few percent, is not sufficiently precise for the subtraction analysis. A self-consistent determination from the data is not only desirable but essential.

We proceed now to test Eqs. (6) and (11). To do this, we plot in Fig. 4 $k-(\rho_s/\rho)t^{-\zeta}$ vs t. This, apart from the power-law factor, is the difference between the solid line and the data as shown in Fig. 1. From Fig. 4, we see that this difference does obey a power law over a region slightly less than two decades in temperature. Nearer the transition, for $t<5\times10^{-5}$, we obtain a sharp deviation from this behavior as one crosses over into the 2D regime. The arrows on the linear plot of Fig. 2, which indicated the magnitude of the universal jump in ρ_s, are at the temperature at which this deviation becomes visible. Thus, when plotted as in Fig. 4, the data show that the onset of 2D behavior is relatively sharp.

The exponent determined from a least-squares fit of the straight-line portion of the data in Fig. 4 is 1.206 ± 0.029. Had we taken T_λ as the point where the period and amplitude become constants rather than 20 μK higher, the slope of the line in Fig. 4 would have been 1.18 ± 0.03. We might, in light of this, take the exponent as 1.18 ± 0.06. We postpone a discussion of this exponent until we

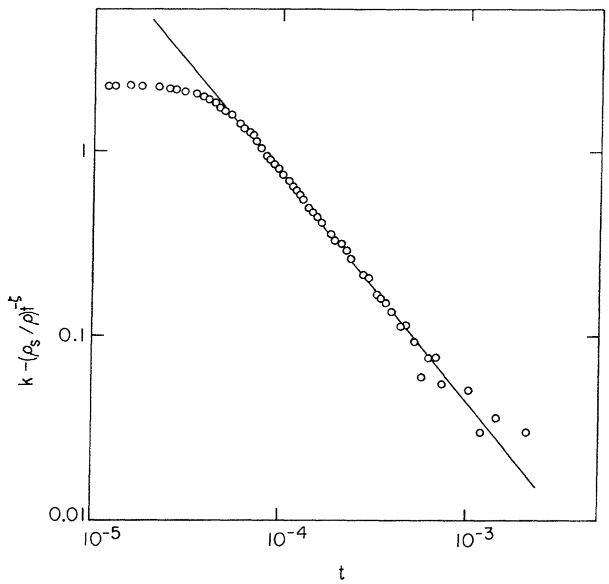

FIG. 4. Difference in superfluid density between bulk and confined helium plotted in such a way as to test Eqs. (6) and (11). Straight-line portion is the power law predicted by these equations. Sharp deviation near $t=5\times10^{-5}$ is the onset of 2D behavior where these equations are no longer applicable.

present the results from other experiments. These results will be summarized in Table II.

B. Data of Henkel and Reppy

Henkel and Reppy[33] obtain ρ_s/ρ from a measurement of the angular momentum associated with a superfluid current. This was done by generating the current in a gyroscope which contained the confining medium. The magnitude of the angular momentum, which is directly proportional to ρ_s, was picked up as a deflection of the gyroscope and converted into the resonant frequency of a tunnel diode oscillator. The change in frequency of the oscillator Δf, from its value at T_λ, is directly proportional to ρ_s. Although various confining geometries were used in their work, we found that only the data for the 2000-Å GA-Metricel filters covered a sufficient range of temperature to be useful in our analysis. These filters are of a cellulose triacetate material and do not provide as uniform a confining geometry as, for instance, the Nuclepore filters used in the work of BSSZ or, for that matter, the planar geometry of our own experiment. However, if we just want to look at the deviation from bulk behavior, and as long as we do not look too close to the transition, these data are still useful.

TABLE II. Results of the analysis to extract the finite-size scaling exponent from various experiments. According to bulk-surface scaling this exponent should be $\zeta-\zeta_s=2\nu=1.350\pm0.002$. Average of the exponents below is 1.0 ± 0.1.

Experiment	Confinement	Scaling exponent
This work	4600-Å planar separation	1.18 ± 0.06
Ref. 33	2000-Å filters	0.97 ± 0.14
Ref. 34	3870-Å slit	0.95 ± 0.05
Ref. 20	1000-, 800-, 500-, 300-Å pores	0.82 ± 0.02

To analyze the data in the same manner as our own, we first must extract the value of k. We chose to work directly with the frequency shift Δf and have plotted first $\Delta f t^{-\zeta}$, which should be constant sufficiently far from T_λ but still in the critical region where a power law $\Delta f = f_0 t^{\zeta}$ is expected to apply. We find that the data which extend only as far as $t = 2.7\times10^{-2}$ never quite reach a constant. However, they approach a value of $f_0 = 213\pm1$ kHz. This, when converted to an amplitude, yields a value of $k = 2.36$ in quite reasonable agreement with other data. This can be seen in Table I.

In Fig. 5, we show the data plotted in a way to test Eqs. (6) and (11), i.e., $f_0 - \Delta f\, t^{-\zeta}$ vs t. We find, as in the case of our own data, that a power law fits these data but over a rather limited range of t, with deviations closer to the transition. The exponent for the line in Fig. 5 is 0.97 ± 0.14. The error here is not a statistical error, but reflects the uncertainty in f_0.

C. Data of Smith and Reppy

In the work of Smith and Reppy,[34] superfluid helium was allowed to flow at the critical velocity through a slit with nominal width of 3850 Å. The total volume flow was measured in this experiment, thus yielding the product of the superfluid velocity and the superfluid density. One then must use a model to extract the value of ρ_s/ρ within the slit. This is done by obtaining a relationship between ρ_s/ρ and v_s from the low-temperature region where $\rho_s \cong \rho_{sb}$.[26] We have taken their tabulated values of ρ_s/ρ and followed the same procedure as in the anaylsis of the previous data to determine k. These data, unlike those of Henkel and Reppy, extend sufficiently far from T_λ that k can be established rather unambiguously. We find $k = 2.465\pm0.005$. With this value of k, we have plotted in Fig. 5 $k - (\rho_s/\rho)t^{-\zeta}$ vs t. We note that for this experiment the bulk transition temperature is established independently of the behavior of ρ_s; hence there is no ambiguity about the value of T_λ. In Fig. 5, we observe again that far from T_λ the data fit a power law, and close to T_λ there are deviations similar to what we have already observed. This presumably signals the onset of the 2D region. We note, however, that this is much sooner than would be expected when one compares the confining dimension between their experiment and that of our own. This suggests that the slit might have had regions where the separation was smaller than the quoted values. This possibility does not affect the present analysis, since we are not attempting to scale the data with size. The exponent associated with the line in Fig. 5 is 0.95 ± 0.05 and is listed in Table II. The error quoted with this exponent is not statistical but corresponds to the variation in k.

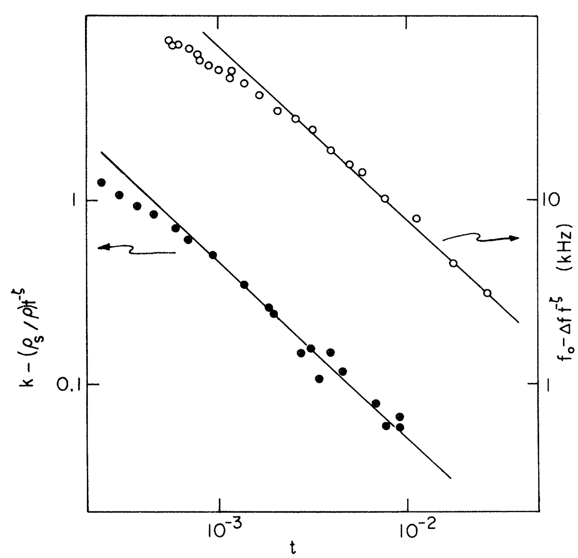

FIG. 5. Data of Refs. 33 (○) and 34 (●) plotted in such a way as to test Eqs. (6) and (11). Straight-line portions yield exponents given in Table II.

We point out that an analysis to check the applicability of Eq. (6) was also done by Smith.[26] He generated a plot very similar to Fig. 5, and by drawing a line to fit the data *closest* to the transition, found consistency with this equation. Apart from our use of the exponent $\zeta = 0.675$ rather than $\frac{2}{3}$, and of a slightly different choice of k, our analysis basically differs from his by biasing a straight line through the data farther away from the transition. Certainly, if one is concerned with inhomogeneities, this is the more reliable region to use, and it is in terms of these inhomogeneities that deviations closer to the transition can be understood. If the data closer to the transition are assumed to be representative of Eq. (6), then it is hard to understand why farther from the transition, but still in a region where this equation should apply, one obtains systematic deviations from a straight line.

D. Data of BSSZ

The measurements of BSSZ involve the determination of the resonant frequency of a Helmholtz oscillator where the frequency-determining element is the helium confined in the pores of a Nuclepore filter.[44] Several filters were used in this experiment with pores of 1000, 800, 500, and 300 Å. These filters are as uniform a confining cylindrical geometry as one can presently achieve.[45] Hence the data are useful not only for the determination of deviations from bulk behavior as we have done so far, but also in scaling the data with size. Data of the specific heat of helium confined in these filters were taken by Chen and Gasparini and analyzed in detail for finite-size scaling in Ref. 9.

In the measurements with a Helmholtz oscillator, one is not able to follow the behavior of ρ_s too close to the transition, since the signal amplitude becomes vanishingly small as ξ approaches about one-quarter of the pore diameter. Thus one of the difficulties in analyzing these data is that the region where ρ_s has the strongest deviation from ρ_{sb} is not available. Since, especially for the larger confinement, one is relegated to look at rather small differences, it becomes even more crucial for these data to establish an accurate value of k. We have followed the same procedure as in the earlier analysis. We have looked at the region sufficiently far from T_λ where one expects the smallest deviation from ρ_{sb}. For the case of the 1000- and 800-Å size filters k could be extracted relatively easily.

For the smaller confinements there were deviations from bulk behavior even in the range of $t=10^{-2}$–10^{-1}. For these, we have taken the same amplitude as for the 1000-Å filters. This is reasonable, and not too crucial of a choice, since for the 500- and 300-Å filters the deviation from bulk behavior are much larger. The values of k for these data are given in Table I.

The data are plotted in Fig. 6 as $(kt^{\zeta}-\rho_s/\rho)d$, where for the first time we are attempting to scale the data with size. Clearly one can see from this figure that, while one could fit a straight line through any one set of data, these data do not collapse on a universal curve. This indicates that the size scaling is not correct. The data plotted according to Eq. (11) with the d^2 factor are shown in Fig. 7. The data now do collapse on a single line. We believe this is the first instance in which the superfluid density for confined helium has been scaled in this manner. The exponent obtained from a least-squares fit of these data is 0.82 ± 0.02. This is tabulated in Table II along with the results from the analysis of the other data.

BSSZ have taken a rather different approach in the analysis of these data, but their results support our own. While they point out the relationship between ρ_s and ρ_{sb} given by Eq. (5), they fit the data to a relationship where a prefactor of ρ_{sb} is treated as a variational parameter. We note that while with this approach one is able to fit the data well, the remaining term of Eq. (5) does not scale with size as this equation would predict. This is in keeping with our own observations. They also fit the data with a pure power law by using the bulk exponent ζ and a shifted, reduced transition temperature t_{01}. Specifically,

$$\rho_s/\rho=c(t-t_{01})^{\zeta} . \quad (13)$$

This equation, with $\zeta=\nu$ and Eq. (4), $d^{1/\nu}t_{01}=a$, can be written as

$$\rho_s/\rho=ct^{\nu}[1-a(td^{1/\nu})^{-1}]^{\zeta}=ct^{\nu}Y(td^{1/\nu}) . \quad (14)$$

Thus Eq. (13) is of the proper scaling form. For the results to support scaling, however, one must have $c=k$, the prefactor of ρ_{sb}/ρ; t_{01} must scale as $d^{-1/\nu}$. From the analysis of BSSZ, one finds that $c\cong k$, but their values of t_{01} scale not with $1/0.675$ but rather with $1/(0.53\pm0.08)$. Thus, although not emphasized quite in the way we have done, the results of the analysis of BSSZ show that while one may successfully use functions which agree in form with the finite-size scaling ansatz, the exponents do not come out quite correct.

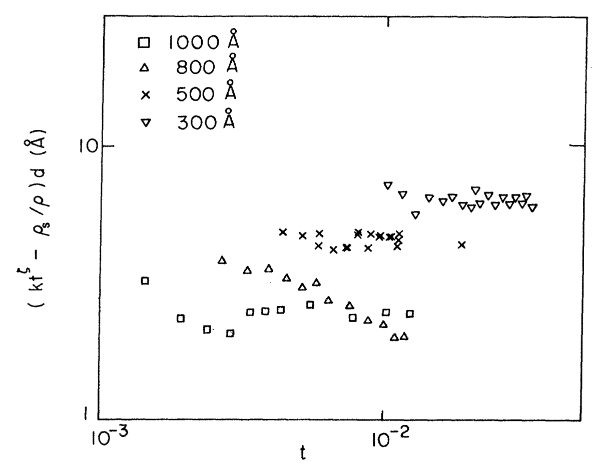

FIG. 6. Data of Ref. 20 for helium confined in cylindrical pores plotted according to Eq. (6). If the data scaled with size as this equation predicts, they should collapse on a universal curve.

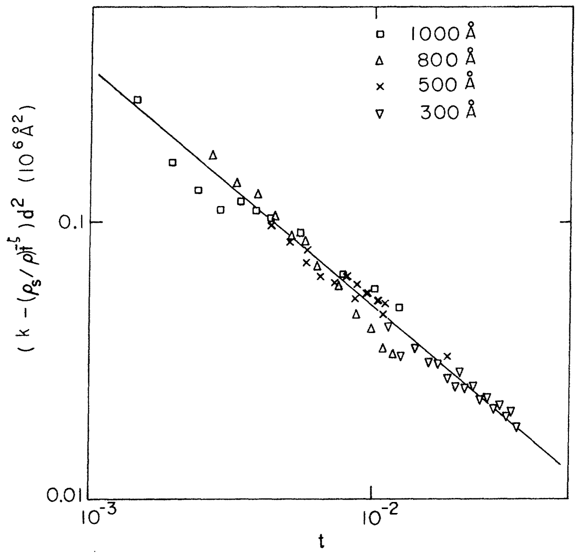

FIG. 7. Same data as in Fig. 6 plotted with the d^2 size scaling of Eq. (11). Factor of $t^{-\zeta}$ is introduced in this figure to make it analogous to Figs. 4 and 5.

V. DISCUSSION

A. Results from the superfluid density

We have seen in the last part of the preceding section that the data of BSSZ do not vary with d as suggested by Eq. (6). This equation comes from a mean-field theory and involves the variation of the order parameter near a wall over a distance dictated by the bulk correlation length. This picture does not seem to be adequate to describe the experimental results. Even if one ignores the size dependence of Eq. (6), one would still have to contend with the fact that Eq. (6), when plotted as in Figs. 4, 5, and 7, predicts the exponent of the t dependence to be $\zeta=0.675$. The results from all the experiments, as can be seen in Table II, clearly do not support this prediction. Regarding Eq. (11), it seems to have the correct size scaling and, of course, the power-law dependence in t which is obeyed by all the data. The scaling exponent, $\zeta-\zeta_s$, varies substantially from experiment to experiment; however, the common feature is that this exponent is always less than $\zeta-\zeta_s=2\nu=1.350$. This signals a failure of the bulk-surface scaling relation, Eq. (12). If we simply average the results shown in Table II, we would conclude that $\zeta-\zeta_s=1.0\pm0.1$.

One might ascribe the lack of agreement with scaling to a possible lack of homogeneity in confinement. This, one would argue, would be a serious problem in the interpretation of these data. This is not really the case if one looks only at the temperature dependence in a region not too close to the transition. This is due to the fact that the deviation from bulk behavior is predominantly a surface effect; thus the details of the geometry are not really important. The geometry is, of course, very important if one looks close to the transition where one crosses over to a lower dimensionality, or, at any temperature if one scales the deviation from bulk behavior with confining size. What is also convincing about our analysis is that we have looked at data from a variety of experiments with quite different confining media, using quite different measuring techniques.

In the case of our own data, they are the only ones which follow the behavior of ρ_s into a lower dimensionality. This latter behavior seems to be at least qualitatively consistent with measurements of thin films and theoretical expectations for 2D behavior. One would expect that if gross confinement inhomogeneities were present, they would tend to smear the character of the 2D transition, especially the dissipation peak, over a wide temperature region. This is not observed. In the case of helium confined to Nuclepore filters, one has probably the most homogeneous confinement geometry. In addition, the data do not extend very close to T_λ so as to be strongly affected by inhomogeneities. In the case of the data of Refs. 33 and 34, we have also limited ourselves to a region far from the transition where one is least susceptible to inhomogeneities. We would conclude after these considerations that the lack of agreement with the scaling prediction is a serious discrepancy. This is further reinforced by the comparison in the next section.

B. Comparison with results from the specific heat

In the work of Chen and Gasparini on the specific heat of helium confined to Nuclepore filters, it was also found that the data disagreed with finite-size scaling.[8] It was found in addition[9] that agreement with scaling could be forced if one assumed corrections-to-scaling terms which are very large, larger than the leading amplitudes and a factor of 1000 larger than what is encountered in the case of the bulk specific heat.[46,47] A description of these data both above and below T_λ was achieved in terms of the surface specific heat with exponent α_s. These data yield a value for α_s which is the same above and below T_λ, thus in *agreement with surface scaling*. The magnitude of α_s, however, $\alpha_s = 0.44 \pm 0.01$, is in strong *disagreement with bulk-surface scaling* where one expects $\alpha_s = \alpha + \nu = 0.655 \pm 0.004$. In the case of ρ_s, we have not attempted an analysis with corrections-to-scaling terms. This does not seem warranted for these data. In addition for bulk helium at saturated pressure ρ_{sb} does not require corrections-to-scaling terms in the temperature region we have used.

We can now make a more quantitative connection between our results on the superfluid density and the results from the specific heat. The exponent $\alpha_s - \alpha$ plays the role of the shift exponent for the specific-heat maximum. In particular, from Ref. 9, we can write their Eq. (15) in the case of no corrections-to-scaling as

$$D = (-12.2 A_s / A) t_m^{-(\alpha_s - \alpha)} , \qquad (15)$$

where we have written the above in such a way that D is the diameter of the pores in Å rather than a film thickness as in Ref. 9. A_s and A are the amplitudes of the surface and bulk specific heats, respectively, and t_m is the reduced temperature shift for the specific-heat maximum. From our equation (11), we obtain

$$D = (g_2)^{1/2} t_v^{-(\zeta - \zeta_s)/2} , \qquad (16)$$

where t_v is the point where ρ_s vanishes. Thus the exponents $\alpha_s - \alpha$ and $(\zeta - \zeta_s)/2$ play the same role, and we expect them to be equal. This can be best illustrated as follows by looking at all the data rather than the points t_m and t_v. From the scaling of the specific heat we note (see Ref. 9) that the difference between the specific heat of confined and unconfined helium ΔC can be written as

$$(-\Delta C\, d / R)^2 \sim t^{-2\alpha_s} , \qquad (17)$$

where R is the gas constant. For ρ_s we have

$$[k - (\rho_s/\rho) t^{-\zeta}] d^2 \sim t^{\zeta_s - \zeta} . \qquad (18)$$

Thus both the specific heat and the superfluid density, when plotted according to Eqs. (17) and (18), should collapse on curves which will be straight lines on a log-log plot. In light of the suggestion that $2(\alpha_s - \alpha) = \zeta - \zeta_s$, these lines should be parallel to the extent that $\alpha = -0.02 \cong 0$.[46,47] This, as can be seen in Fig. 8, works quite well. The exponents from the specific heat are[9]

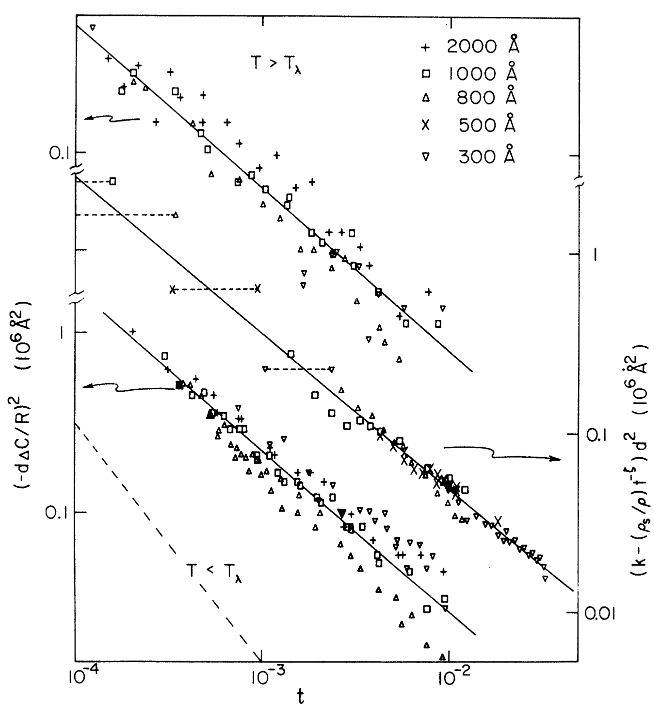

FIG. 8. Scaling of specific heat and superfluid density for confinement in cylindrical pores. If the data are scaled with the bulk correlation length, the least-squared-fitted lines through the data should be parallel to the dashed line. See comments in text about symbols linked with dashed lines and solid symbols.

$$2(\alpha_s-\alpha)=\begin{cases}0.94\pm0.03, & T<T_\lambda\\ 0.91\pm0.03, & T>T_\lambda\ .\end{cases}$$

These are in reasonable agreement with $\zeta-\zeta_s=0.82\pm0.02$, which, as we have already seen, is the result from the data of BSSZ. These exponents also agree with the results we have obtained from all the other data, which on the average yield $\zeta-\zeta_s\cong1.0\pm0.1$. These exponents should all be 1.35 according to scaling. This is the slope of the dashed line in Fig. 8.

In Fig. 8, we have also plotted as symbols linked by dashed lines the extrapolated points at which BSSZ judged ρ_s to vanish. As we discussed, two functions were used for this purpose, hence the two symbols. The scaling function we have used, Eq. (11), yields the temperature at which ρ_s vanishes to lie in between the estimates of these two other functions. It is at first surprising that Eqs. (11) and (16), derived on the basis of a bulk plus surface contribution, might be able to describe the data up to the vanishing of ρ_s. This perhaps is due to the fact that with channels, one has a crossover from 3D to 1D; hence one does not have a different transition which intervenes, as is the case for films when one crosses from 3D to 2D. We have seen already, in the case of our own data, Fig. 4, that the deviation from a finite-size description becomes marked at the start of the 2D region onset. This would preclude such an extrapolation in case of films.[48] Presumably, in 1D the transition is removed to $T=0$.

Also in Fig. 8, we note that the specific-heat data for $T<T_\lambda$ corresponding to the maximum, the solid symbols, fit quite nicely within the description of the surface plus bulk specific heat. There is, in a sense, nothing special about these points. Conventional wisdom would have suggested that this description should break down for the maximum because of the *assumption* that the bulk correlation length becomes equal to the confining dimension. What these data suggest is that the maximum is mainly the result of a modification of bulk behavior stemming from a surface effect. This, according to the exponents we have obtained, is not governed, at least exclusively, by the bulk correlation length.

The comments about crossover from 3D to 1D, as opposed to 2D, apply to the specific heat as well. In fact, it seems very likely that all features of the specific heat which have been observed so far, even in the case of 2D films, are the result of finite-size effects rather than dimensionality. In strictly two dimensions, the specific heat is expected to be analytic at the transition with a broad maximum at a higher temperature.[5] This has yet to be verified experimentally.

C. Other experiments; results from onset temperature

There are no other experiments with helium where the analysis we have described has been carried out. There are, however, a number of experiments which have obtained the shift in the transition temperature or onset temperature for superfluid behavior as a function of confinement. We show in Fig. 9 all of the data we are aware of for confinement in Nuclepore filters. Figure 9 was prepared as follows. The data of specific-heat maximum for both films[49] and filled pores were plotted first. The solid line represents a least-squares fit of these data only. The remaining data all represent the determination of superfluid onset. In all cases, except for the specific heat, the data represent the measurement done on a single filter of given nominal size. For the specific heat, the data represent confinement in ~500–700 filters of the same nominal size. Thus if one is concerned with a particular filter having the specified nominal pore size, the specific-heat data are probably the most reliable because of the effective averaging among a large number of filters.

The $\triangle$ are the data of Ihas and Pobell.[19] These authors determined the temperature at which a Nuclepore membrane is no longer effective in generating second sound. This is then taken as the point at which ρ_s vanishes in the pores. The identification of this temperature is somewhat ambiguous due to the fact that one must choose a cutoff signal level. The two linked symbols for these data represent choosing either 0.1 μV as the cutoff level, or a somewhat higher level for the smaller pore filters in order to compensate for the filter's efficiency in generating second sound. The $\blacktriangledown$ are the data of Thomlinson *et al.*[50] and are obtained by using the same technique as Ihas and Pobell.

The linked $\times$'s are the data of BSSZ. As we have discussed before, they represent an extrapolation of the measured superfluid density to the vanishing temperature by using two functions which are equally good at describing the data, but are inconsistent with each other in the asymptotic region. The $+$'s are the data of Schubert and Zimmermann,[51] who used the same technique as BSSZ but in addition obtained a shift temperature from the measurement of the superfluid-mass current. These two measurements are in good agreement as to the vanishing of ρ_s, except for the two $+$'s which are linked by the horizontal line.

The ∇ are the measurements of Giordano,[22] which are obtained from the change in the rate of helium flow through a Nuclepore filter. These are the only data plotted at the measured average pore diameter rather than the nominal manufacturer's value.[52]

The solid line in Fig. 9, fitted to the specific-heat data, gives an exponent of 0.562 ± 0.014, where we quote the

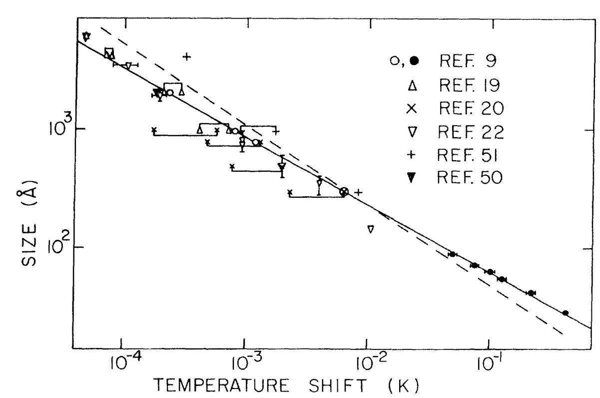

FIG. 9. Confinement size vs shift in transition temperature. Dashed line corresponds to scaling with the bulk correlation length. See text for meaning of linked symbols.

standard error. The dashed line is the expected scaling value of $\nu=0.675$. It is clear that if one ignores the specific-heat maximum of the films, most of the remaining data yield exponents which, although a bit low, are not completely inconsistent with scaling. Specifically, the individual data of Refs. 8, 19, 20, and 22 yield, respectively, 0.58 ± 0.05, 0.65 ± 0.02, 0.53 ± 0.08, and 0.65 ± 0.04. If one includes the data of the specific-heat maximum of the films, thereby gaining more leverage in drawing the straight line in Fig. 9, then the data definitely favor an exponent less than ν. This was also the conclusion of BSSZ in examining data for pore confinement only, but not limited to Nuclepore filters.

Overall, the evidence for bulk-correlation-length scaling, or lack of it on the basis of Fig. 9, is not very convincing. The use of a single feature of the thermodynamic response, the specific-heat maximum, or the vanishing of ρ_s can be very deceiving and is certainly subject to greater errors than the scaling of all the data as presented in Fig. 8. In the flow experiments, in particular, it is very difficult to establish whether the changes one sees are due to the variations in the superfluid density or the vanishing of the critical velocity.[51]

The temperatures at which the specific heat achieves a maximum or the superfluid density vanishes are but single points of a function describing all of the data. Except for historical reasons, they should carry no special weight in assessing finite-size scaling.

D. Analysis with other scaling functions

We have tried to scale the ρ_s data of helium confined in Nuclepore by using other functions which retain the scaling with the bulk correlation length. Specifically, if we write Eq. (1) in the equivalent form

$$\rho_s=\rho_{sb}\phi(\xi/d)\ , \tag{19}$$

we may expand ϕ for small ξ/d,

$$\phi=\phi_0+\phi_1\xi/d+\phi_2(\xi/d)^2+\Phi_3(\xi/d)^3+\cdots\ . \tag{20}$$

Since in the limit where ξ/d goes to zero $\rho_s=\rho_{sb}$, we must have $\phi_0=1$. Hence we have

$$(\rho_{sb}-\rho_s)/\rho_{sb}=-\phi_1\xi/d-\phi_2(\xi/d)^2-\phi_3(\xi/d)^3-\cdots\ . \tag{21}$$

By using the bulk scaling relation $\zeta=\nu$, this can be written as

$$\left[k-\frac{\rho_s}{\rho}t^{-\nu}\right]=-\phi_1'(dt^\nu)^{-1}-\phi_2'(dt^\nu)^{-2}-\phi_3'(dt^\nu)^{-3}-\cdots\ , \tag{22}$$

where we have used Eq. (2) and absorbed all constants in the ϕ's. This equation is the same as Eq. (11), which is based on a bulk plus surface contribution to ρ_s; if we have $\phi_1=0$, we accept bulk-surface scaling, Eq. (12), and simply truncate the expansion at ϕ_2. Fisher has pointed out[53] that the torsional oscillator data can be fitted with Eq. (22) if one sets $\phi_1=0$ and retains the next two terms. While this is true for these data, it is certainly not so for the data of BSSZ. In Fig. 10, we show a plot of the left-hand side of Eq. (21) vs $(dt^\nu)^{-1}$. The data should collapse on a universal curve when plotted this way. They do not.

VI. SUMMARY AND CONCLUSIONS

We have presented new data for the superfluid density of helium confined between sheets of Mylar at 4600 Å average separation. Unlike previous results, these data show the full range of behavior from bulk, to finite size, to 2D. In the latter regime, we have found good qualitative consistency with results on films of helium hundreds of angstroms thinner.

We have focused the data analysis on the finite-size region and have extended this analysis to data from several other experiments. To carry out this analysis, we have used a scaling function from the theory of Ginzburg, Pitaevskii, and Mamaladze as well as a scaling function which we suggest in analogy with the work of Josephson. The latter function may also be viewed as resulting from a series expansion of a general scaling function. In both of these approaches, one obtains a power law with a characteristic exponent which, according to finite-size scaling, is expected to be that of the bulk correlation length.

We find that the data do obey a power law, but the exponent is not that of the bulk correlation length. This is summarized in Table II. Furthermore, we find that the size scaling predicted by the Mamaladze theory is not correct, but that predicted in analogy with the work of Josephson agrees with the data.

We have compared our results with the earlier analysis of the specific heat of confined helium by Gasparini *et al.* in which disagreement with bulk-correlation-length scaling was also found. We suggest that the exponent which governs the scaling of the specific heat is related to that which governs the scaling of ρ_s. We find that this indeed is the case and obtain good numerical agreement among these data.

We have examined other data for the onset of superfluidity. Some of these data yield shift exponents which are consistent with bulk-correlation-length scaling. We find the overall picture, however, rather unconvincing.

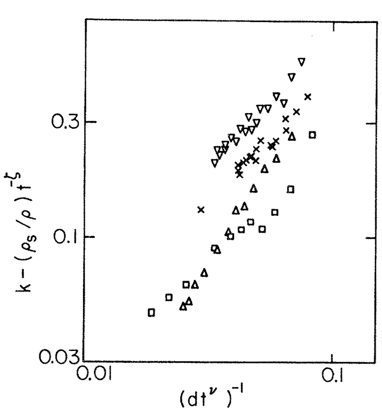

FIG. 10. Data of Ref. 20 plotted according to Eq. (21). Symbols have the same meaning as in Fig. 7.

More importantly, however, we emphasize that the onset temperature, or specific-heat maximum, is but a single point in the thermodynamic response of confined helium. They yield no more, or no less, information than any other point away from onset. We find the overall scaling of the data to give a much more compelling argument.

The conclusion which our analysis suggests is that the simplest picture of the bulk correlation length determining the behavior of confined helium is not correct. The presence of confining walls affect the behavior in a much more fundamental way than had been thought. In particular, the deviation from bulk behavior, to the extent that it is understood from a bulk-plus-surface contribution, suggests that there is a length associated with the surface which has a different, weaker, temperature dependence. This length could be a reflection of the modification induced by the walls on the order parameter. To the extent that this is true then, our results suggest that the failure of bulk-correlation-length scaling is associated with a failure to satisfy one of its assumptions, an "inert" confining boundary.

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