Static and dynamic magnetic response of spin-glass models with short-range interactions

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The static and dynamic magnetic response of the two-dimensional Edwards-Anderson model with a nearest-neighbor Gaussian exchange distribution is investigated by Monte Carlo simulation. A plateau in the equilibrium (slowly field cooled) susceptibility M_{eq}/H is found, which diverges for small fields with a power law $M_{\mathrm{eq}}/H \propto H^{1/\Delta}$, $\Delta=3.5\pm0.5$. The boundary of the plateau $H_c^{\mathrm{eq}}(T)$ tends to zero temperature as $H_c^{\mathrm{eq}}(T)\propto T^{\Delta}$, consistent with a scaling description appropriate for a static phase transition at $T=0$, and the associate scaling function is estimated. Surprisingly the data also are consistent with a scaling representation with nonzero freezing temperatures T_f, in striking similarity to experimental data, but $T_f=0$ is shown to be the correct choice. The zero-field-cooled susceptibility starts to differ from M_{eq}/H below a certain critical field $H_c(t)$, t being the time scale over which the field is applied. In the H-T plane $H_c(t)$ extrapolates to a time-dependent freezing temperature $T_f(t)$, and closely resembles the de Almeida—Thouless line. A tentative interpretation of these findings is attempted by combining scaling considerations with the reorientation of correlated clusters which have the size of the correlation length describing Edwards-Anderson order. The reduction of the free-energy barriers due to the magnetic field is treated in analogy with the interface free-energy reduction of ferromagnets in random fields.

I. INTRODUCTION

The nature of the freezing of magnetic moments in spin-glasses still receives considerable attention.[1] In particular, the question whether the spin-glass transition is a dynamic nonequilibrium process or a true phase transition with an infinite correlation length is controversial. There is a growing number of experiments which are interpreted as a clear evidence for a static phase transition. These include (i) a plateau in the field-cooled susceptibility as a function of temperature,[2-4] (ii) the singular phase diagram in the field-temperature plane,[3,5-9] (iii) scaling laws of the nonlinear field-cooled susceptibility,[10-12] and (iv) a breakdown of the Vogel-Fulcher law at the freezing temperature T_f.[12]

The most successful model of spin-glasses has been introduced by Edwards and Anderson (EA).[13] In fact, its *infinite-range* version has a static phase transition,[14] and many details of experiments (i)—(iv) have been predicted by this mean-field theory,[15,16] at least qualitatively. On the other hand, the EA model with *short-range* interactions in two dimensions has no static phase transition.[17-19] Nevertheless, this model also reproduces many experimental findings remarkably well.[20-23] Thus it appears to be necessary to investigate static and dynamic properties in much more detail to distinguish between nonequilibrium and static behavior.

In this paper, we continue the investigations of the two-dimensional EA model.[24] Thus the Hamiltonian is given by

$$\mathscr{H}=-\sum_{\langle ij\rangle} J_{ij}S_iS_j - H\sum_i S_i \ , \tag{1}$$

where $S_i=\pm1$ are Ising spins on a square lattice, J_{ij} are nearest-neighbor couplings distributed randomly by the Gaussian

$$P(J)\propto \exp-\left[\frac{J}{2(\Delta J)^2}\right] , \tag{2}$$

and H is an external applied field. For the dynamics we use usual Monte Carlo procedures of a 60×60 lattice with periodic boundary conditions (for details, see Refs. 21, 22, and 25). In Sec. II we present results of numerical simulations. Section II A contains static properties of the nonlinear susceptibility such as the plateau and scaling fits. In Sec. II B dynamic properties in the field-temperature plane are discussed. In Sec. II C we discuss the energy of metastable states. In Sec. III we present some tentative phenomenological relations, demonstrating how a correlation length which diverges at zero temperature is related to the nonlinear susceptibility and the characteristic relaxation time. In Sec. IV we summarize our findings and present our conclusions.

II. MONTE CARLO SIMULATIONS

A. Static magnetization

We now discuss the properties of the field-cooled magnetization. In thermal equilibrium the susceptibility is just a simple Curie law[26] (note that there is no phase transition in our model; thus the EA parameter $q=[\langle S_i\rangle]_{\mathrm{av}}$ is always zero). Thus one has

$$\lim_{H\to0}\frac{M_{\mathrm{eq}}(T,H)}{H}=\frac{1}{T} , \tag{3}$$

where M_{eq} is the equilibrium magnetization and T is the

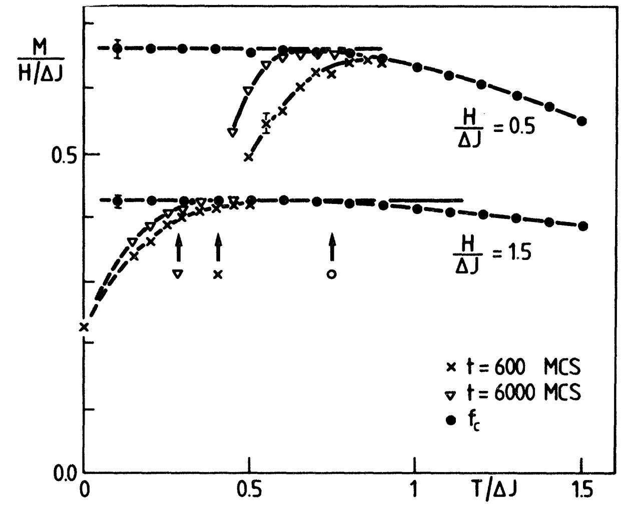

FIG. 1. Susceptibility M/H plotted vs temperature for two values of the field H. The solid circles show the field-cooled magnetization. The other symbols show the magnetization obtained from zero-field cooling and then applying the field H for a given time as indicated. Arrows identify critical fields as discussed in the text and shown in Figs. 2 and 3.

temperature ($k_B=1$). The field-cooled magnetization $M_{\rm FC}(T,H)$ is believed to be equal to $M_{\rm eq}$. But, as shown in Fig. 1, $M_{\rm FC}$ deviates strongly from the Curie law, in particular, one observes a plateau for low temperatures similar to experimental findings.[2-4]

There may be two reasons for this behavior: (i) One has $M_{\rm FC}=M_{\rm eq}$, but the field H is too large to see the limiting behavior of Eq. (3), or (ii) the system freezes into a nonergodic state and one has $M_{\rm FC}<M_{\rm eq}$. It turns out that both reasons are true.

In Fig. 1, $M_{\rm FC}(T,H,\lambda)$ is shown for cooling from $T_0=1.5\Delta J$ to T linearly with time t at a rate of $\lambda=|dT/dt|=6.25\times10^{-5}\Delta J/\text{MCS}$ (MCS denotes Monte Carlo steps per spin) in an external field H. We have obtained the same results when our cooling rate was 4 times slower. This indicates that one has $M_{\rm FC}=M_{\rm eq}$ for $H>0.05\Delta J$ and $\lambda\leq6.25\times10^{-5}\Delta J/\text{MCS}$. However, for larger cooling rates we observe nonequilibrium values of $M_{\rm FC}$. For instance, for $H=0.05\Delta J$ and $\lambda=2.5\times10^{-4}\Delta J/\text{MCS}$ we have still obtained a plateau but with a smaller magnetization. Figure 2 shows the plateau values $M_{\rm FC}(T\to0,H,\lambda)/H$ as a function of field in a double-logarithmic plot. For fast cooling these values seem to saturate when $H\to0$. However, if λ is small enough, i.e., $M_{\rm FC}=M_{\rm eq}$, we observe a power-law divergence

$$M_{\rm eq}(T=0,H)/H\propto H^{-1/\Delta}\,, \tag{4}$$

with $1/\Delta=0.28\pm0.05$. Of course, Eq. (4) is consistent with the Curie law, Eq. (3).

It is interesting, that for fast cooling as well, one sees a plateau; we have no explanation for this fact, but for-

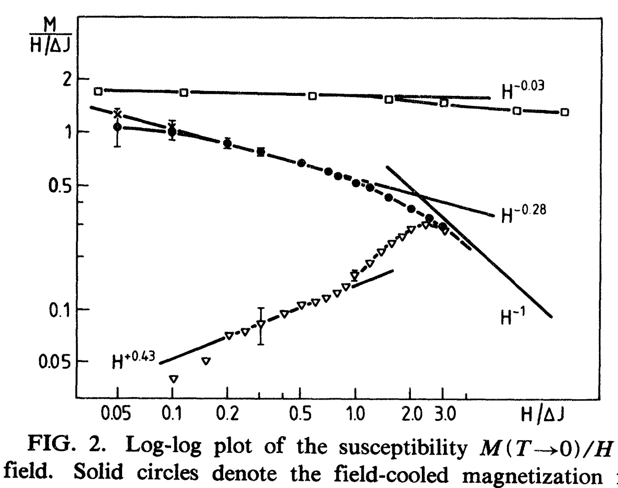

FIG. 2. Log-log plot of the susceptibility $M(T\to0)/H$ vs field. Solid circles denote the field-cooled magnetization for cooling rate $\lambda=|dT/dt|=2.5\times10^{-4}\Delta J/\text{MCS}$; crosses show it for $\lambda=6.25$ and $1.5\times10^{-5}\Delta J/\text{MCS}$. Open squares show experimental data of Monod and Bouchiat[3] for Ag-Mn(10 at. %) on arbitrary scales. Triangles show the zero-field-cooled magnetization at $T=0$.

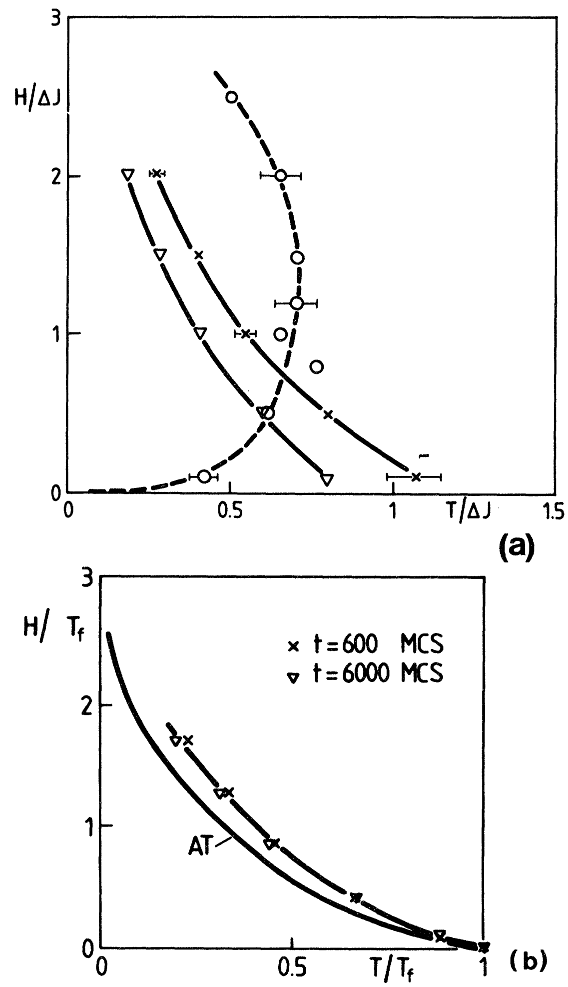

FIG. 3. (a) Static critical field $H_c^{\rm eq}(T)$, open circles, and dynamic critical fields $H_c(t)$ for $t=600$ MCS (crosses) and $t=6000$ (triangles) plotted vs temperature. (b) Normalized dynamical critical fields where $T_f(t)$ is obtained from extrapolating $H_c(t)$ to $H=0$. Our results are compared with the AT phase boundary of the mean-field theory.[15]

tunately this helps us to determine $M_{\rm eq}(T=0,H)$. As soon as the system is on the equilibrium plateau, the magnetization does not change. Thus we obtain $M_{\rm eq}(T\to 0,H)$, although for very low temperatures the system is not in equilibrium.[27]

From Fig. 1 we have defined a critical field $H_c^{\rm eq}(T)$ where the plateau begins. These fields are shown in Fig. 3(a) by open circles. From Eq. (3) one obtains $H_c^{\rm eq}(T)\to 0$ for $T\to 0$; in fact, from a scaling relation discussed in Sec. III, we expect

$$H_c^{\rm eq}(T)\propto T^{\Delta}\,. \tag{5}$$

We want to stress that in the absence of a phase transition, the plateau value $M_{\rm eq}/H$ has to diverge as $H\to 0$ and related to it $H_c^{\rm eq}(T)$ has to go to zero for $T\to 0$. In fact, recent experiments[3,4] are consistent with Eqs. (4) and (5). Figure 2 shows data from Monod and Bouchiat[3] which give $1/\Delta=0.03$. Barbara and Malozemoff[4] report $1/\Delta=0.013$ from fitting their data by Eq. (5). Both values of $1/\Delta$ are 1 order of magnitude smaller than our (two-dimensional) result. But note that (i) our simulations give much smaller $1/\Delta$ values if the cooling rate is too large, and (ii) the experimental data are also consistent with a nonzero static transition temperature T_f.[4] Thus clearly more detailed experiments on this question are necessary.

Equation (4) gave the singular behavior of $M_{\rm eq}$ at zero temperature. Now we consider $M_{\rm eq}$ at higher temperatures, $T>\Delta J$. Figure 4 shows the difference $\tilde{\Delta}(T,H)=1-TM_{\rm eq}/H$ of the equilibrium magnetization $M_{\rm eq}$ and the Curie law, Eq. (3), i.e., for $H\to 0$ one has $\tilde{\Delta}(T,H\to 0)\to 0$. In analogy to recent experiments,[10,11] we have tried to fit the data of Fig. 4 by scaling laws[28] which predict

$$\tilde{\Delta}(\epsilon,h)=\epsilon^{\beta}f\left[\frac{h}{\epsilon^{\gamma+\beta}}\right]=h^{1/\delta}g\left[\frac{h}{\epsilon^{\gamma+\beta}}\right], \tag{6}$$

where ϵ and h are independent scaling fields, β, γ, and δ are critical exponents, and $f(x)$ and $g(x)$ are universal scaling functions in complete analogy to standard critical phenomena. Following Omari *et al.*[11] we choose

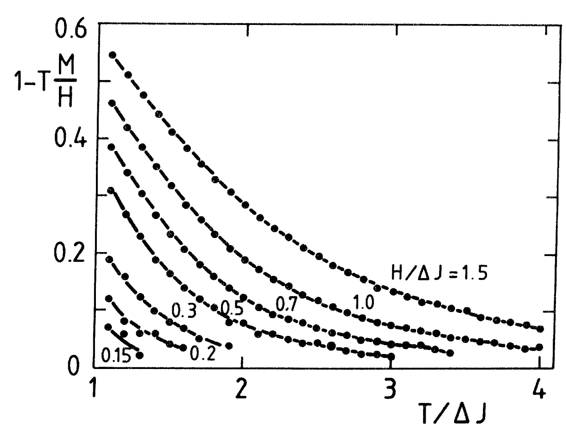

FIG. 4. Nonlinear susceptibility $\tilde{\Delta}(T,H)=1-TM_{\rm eq}/H$ vs temperature for different fields.

$$\epsilon=\frac{T-T_f}{T},\quad h=\left[\frac{H}{T}\right]^2. \tag{7}$$

Figure 5(a) shows the results for a scaling plot where $T_f=\Delta J$ was fixed, and γ and β have been determined by a least-squares fit. For comparison we have added the data of Ref. 11. Surprisingly, for our model, which has no static phase transition, this fit works rather well. But the fit is very insensitive to the values of T_f, γ, and β. This is shown in Fig. 6 where the least-squares routine stopped at a point $T_f=0.44\Delta J$, $\gamma=5.5$, and $\beta=0.4$. In fact, better fits are obtained for even lower T_f, higher γ, and unphysically negative β values. This proves that such scaling plots are no evidence for a phase transition.

If we expand $\tilde{\Delta}$ for small fields H, we obtain

$$\tilde{\Delta}(H,T)=\frac{a_3}{3}\left[\frac{H}{T}\right]^2-\frac{2a_5}{15}\left[\frac{H}{T}\right]^4+\cdots, \tag{8}$$

where $a_3(T)$ is related to the usual nonlinear susceptibility[22] $\chi_{\rm nl}(T)$ by $a_3=3T^3\chi_{\rm nl}$. $a_3(T)$ should diverge at a static phase transition temperature T_f if it exists. In fact, for our model it was already shown[22] that a_3 increases strongly with decreasing temperature. In the considered temperature range, $a_3(T)$ could equally well be fitted to power laws with either $T_f=0$ or $T_f=\Delta J$. Since the data of Ref. 22 were not obtained by slow field cooling, we repeat this analysis with the data of Fig. 4. Figure 7 shows $\tilde{\Delta}$ as a function of $(H/T)^2$; thus the slope at $H=0$ gives a_3. We see that $a_3(T)$ increases strongly with $T\to\Delta J$. Owing to large fluctuations at small fields, it is difficult to determine a_3, thus we have calculated a lower bound $a_3^{\rm eff}$ by

$$a_3^{\rm eff}\equiv 0.05T_0^2/H^2\,, \tag{9}$$

where T_0 is the temperature where $\tilde{\Delta}=0.05$ in Fig. 4. Figure 8 shows that in the temperature range shown, $a_3^{\rm eff}$ can be fitted by a power law

$$a_3^{\rm eff}\propto(T-T_f)^{-\gamma}\,, \tag{10}$$

where both $T_f=\Delta J,\gamma=1.95$ and $T_f=0,\gamma=1.7$ give good fits. However, for our model we know[17,18,22] that $T_f=0$ and $\gamma\simeq 4$.

Experimentally, a_3 and a_5 have recently been determined by Omari *et al.*[11] They fit $a_3(T)$ by Eq. (10) and obtain $\gamma=3.2\pm 0.1$, where T_f was taken as the temperature of the ac susceptibility cusp. They rule out the possibility of a power law, Eq. (10), with $T_f=0$. But as shown in Fig. 8, our data also seem to increase stronger than $T^{-\gamma}$ if we include the high-temperature expansion $a_3=1+12(\Delta J/T)^2+\cdots$ and the asymptotic low-temperature behavior $a_3=C_0/(T/\Delta J)^5$ with $C_0\approx 33$, as obtained from a scaling assumption discussed below. Thus again, we observe the same qualitative behavior as in experiments although quantitatively, due to the large γ value, the experimental a_3 data increase much stronger than in our model.

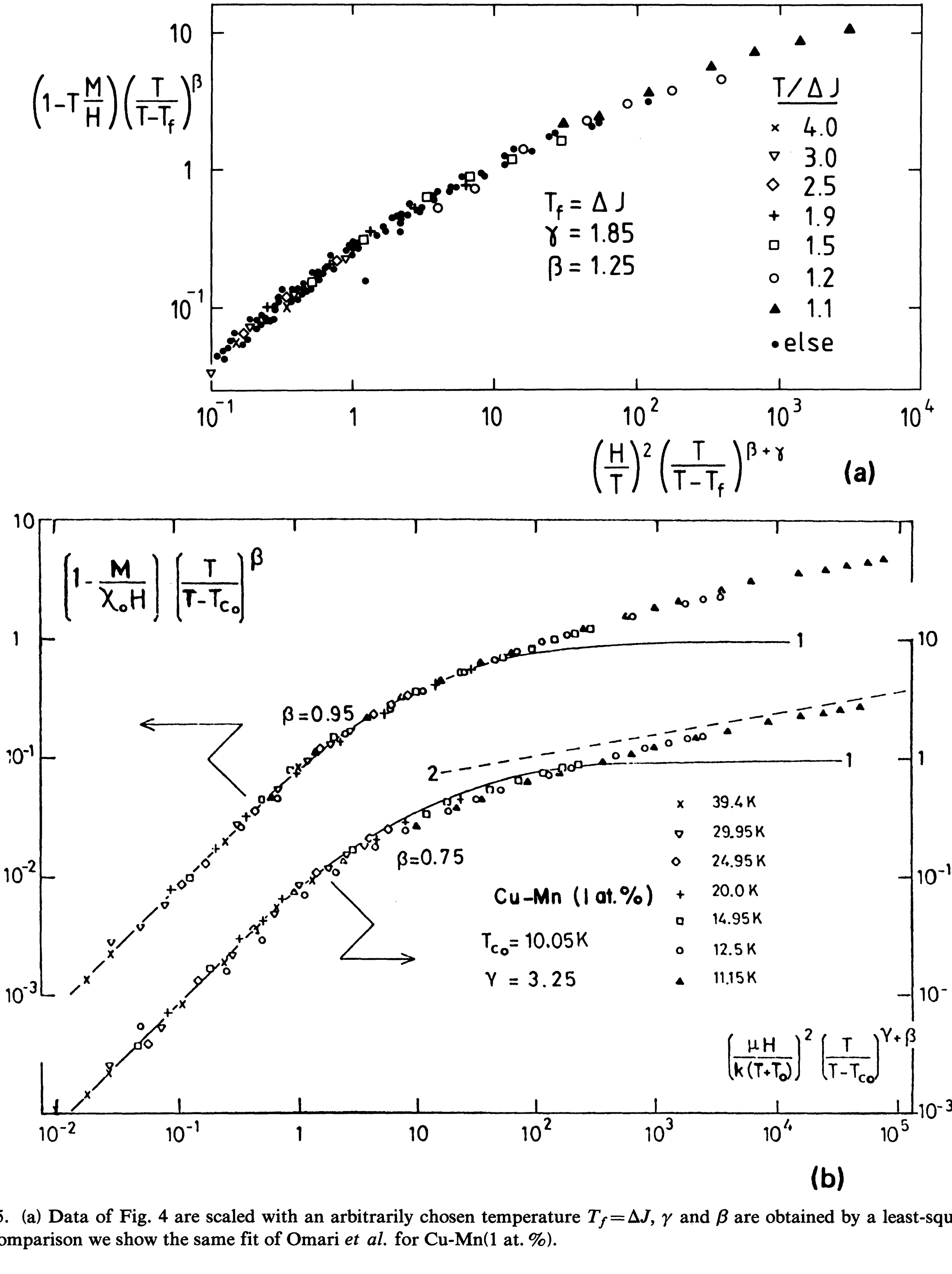

FIG. 5. (a) Data of Fig. 4 are scaled with an arbitrarily chosen temperature $T_f=\Delta J$, γ and β are obtained by a least-squares fit. (b) For comparison we show the same fit of Omari *et al.* for Cu-Mn(1 at. %).

B. Time-dependent magnetization

So far we have discussed the magnetization M_{eq} in thermal equilibrium. Now we will present results for the metastable, time-dependent magnetic response. Figure 1 shows M_{eq} obtained from slow cooling in an applied field H. In addition, we show the time-dependent magnetization $M(T,H,t)$ obtained from cooling in zero field to temperature T, applying a field H at time $t=0$, and recording M at time t. Since our model has no phase transition, we know that

$$\lim_{t\to\infty} M(T,H,t)=M_{\mathrm{eq}}(T,H) \ . \qquad (11)$$

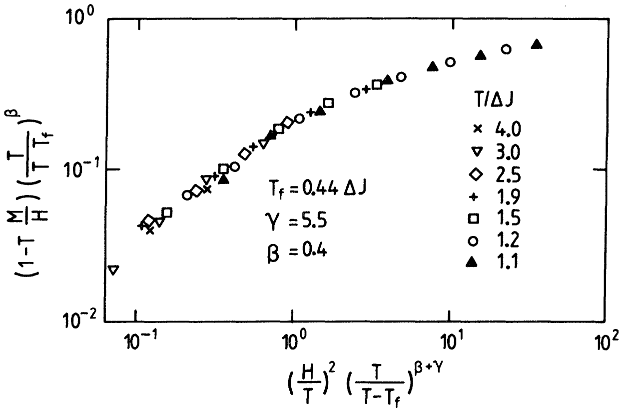

FIG. 6. Same as Fig. 5(a) but in addition T_f was obtained from the least-squares routine. Better fits are obtained for even smaller T_f, larger γ, and (unphysically) negative β values.

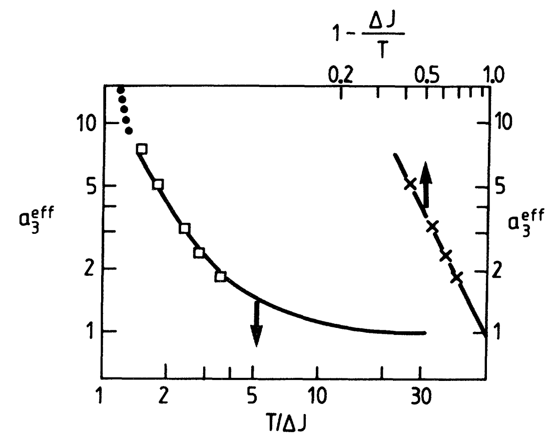

FIG. 8. Expansion coefficient a_3^{eff} determined from Fig. 4 with $\Delta(T,H)=0.05$ as a function of temperature. The solid line shows the first term of a high-temperature expansion of $a_3(T)$, and the dotted line shows the low-temperature behavior $a_3 \propto T^{2\Delta-2}$ expected from scaling laws.

However, at $T=0$ the slow relaxation is completely frozen out; the zero-field-cooled magnetization $M_{\text{ZFC}}(H) = M(0,H,t)$ is time independent and smaller than M_{eq}. Thus $M(T,H,t)$ has a rather singular behavior for $T \to 0$ and $t \to \infty$; in particular, one has

$$M_{\text{eq}}(0,H) = \lim_{T\to 0}\lim_{t\to\infty} M(T,H,t) \geq \lim_{t\to\infty}\lim_{T\to 0} M(T,H,t) = M_{\text{ZFC}}(H) \ . \tag{12}$$

Figure 9 shows $M_{\text{FC}}(0,H)$ [which for the fields and cooling rate shown is equal to $M_{\text{eq}}(0,H)$] and $M_{\text{ZFC}}(H)$ as a function of field. Only close to saturation $M \simeq 1$, that is for fields $H > 3\Delta J$ both magnetizations come together. For smaller fields one has a S-shaped $M_{\text{ZFC}}(H)$, while according to the result, Eq. (4), M_{FC} has an infinite slope at $H=0$. M_{ZFC} is also shown in Fig. 2 which indicates that $M_{\text{ZFC}} \propto H^{1+y}$, with $y \approx 0.43$ for intermediate fields where our data are most reliable. However, we do expect $M_{\text{ZFC}} \propto H$ for $H \to 0$ as will be discussed below. For the zero-field-cooled case, the susceptibility dM_{ZFC}/dH shown in Fig. 9 may be considered as a measure for the internal field distribution which has been calculated by Binder.[29] By increasing the field H to $H+\delta H$, only those spins are flipped which have an internal field H_i between $-(H+\delta H) \leq H_i < -H$. Of course this is strictly true only if the distribution of internal fields does not change when H is applied. This distribution is nonzero for $H \to 0$ and hence we also expect $dM_{\text{ZFC}}/dH \neq 0$ for $H=0$. Experimentally, a S-shaped curve of $M_{\text{ZFC}}(H)$ is typical for spin-glasses.[30] However, the behavior at $H=0$ has not yet been worked out in detail. In particular, it might be interesting to compare Ising with Heisenberg spin-glasses,[31] since both should change their magnetization in a different way (spin flip versus spin canting).

As shown in Fig. 1, for large times t, high temperatures T, and/or large applied fields H, the magnetization $M(T,H,t)$ will eventually be equal to the equilibrium one $M_{\text{eq}}(T,H)$. Thus we have defined dynamical critical fields $H_c(T,t)$ from the points where M_{eq} and $M(T,H,t)$ differ by 3%. The results are shown in Fig. 3(a). For each observation time t, we obtain a curve $H_c(T,t)$; for $t \to \infty$

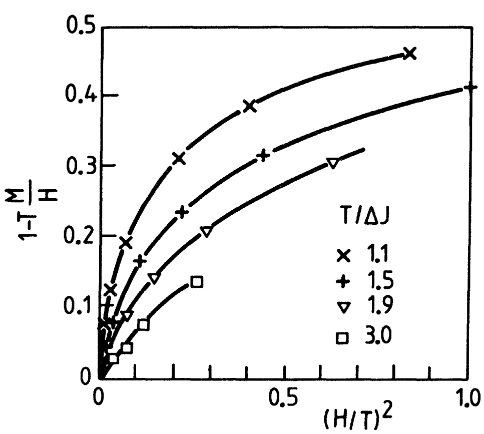

FIG. 7. Nonlinear susceptibility $\tilde{\Delta}(T,H) = 1 - TM_{\text{eq}}/H$ as a function of field H for different temperatures T. The slope at $H=0$ gives the expansion coefficient $a_3(T)$ of Eq. (8).

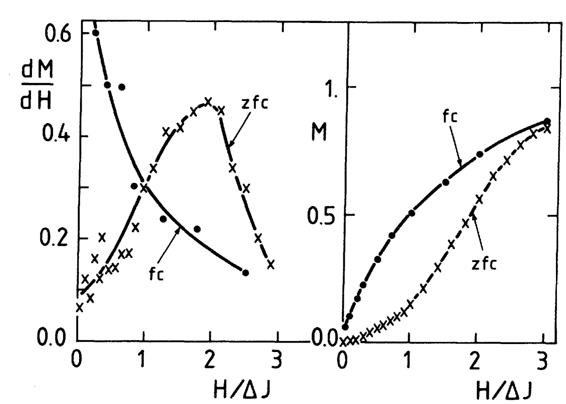

FIG. 9. Magnetization and its derivative at zero temperature for the field-cooled (circles) and zero-field-cooled (crosses) case. The same data are shown in Fig. 2.

these curves approach the origin $H=0, T=0$, as seen from Eq. (11).

In the infinite-range model, below the static transition temperature T_f, Eq. (11) does not hold since the system is truly nonergodic.[16,32] In fact, the difference $\tilde{\Delta}(\infty)$ with $\tilde{\Delta}(t)=M_{\text{eq}}-M(T,H,t)$ has been discussed as an order parameter for the spin-glass phase.[16] de Almeida and Thouless[15] (AT) have found the phase boundary $H_c(T)$ below which $\tilde{\Delta}(\infty)\neq 0$; close to the zero field T_f, one has

$$H_c(T)=A(1-T/T_f)^{\zeta}\,, \tag{13}$$

with $A=(\frac{4}{3})^{1.5}$ and $\zeta=\frac{3}{2}$. In our short-range model we have $\tilde{\Delta}(\infty)=0$. Nevertheless, we define $T_f(t)$ by extrapolating $H_c(T)$ in Fig. 3(a) to zero field, and then we scale H and T with $T_f(t)$. Figure 3(b) shows the result. Thus, surprisingly, the scaled dynamical critical fields behave similar to the AT result of the mean-field theory; in particular, we observe the singular behavior, Eq. (13), with $\zeta=\frac{3}{2}$, although we know that for very small fields due to the symmetry $H\to -H$ we must get the analytic behavior, Eq. (13), with $\zeta=\frac{1}{2}$.

Young[18] has recently obtained similar results, but in a different way. While we have defined $H_c(T,t)$ from the relaxation *into* the equilibrium state, Young considers the autocorrelation function *in* the equilibrium state and defines $H_c(T,t)$ as a line of constant relaxation time t. In both cases, one observes the AT behavior.

Our results of Figs. 3(a) and 3(b) are qualitatively similar to recent experiments.[3,5–9] In particular, $H_c(T,t)$ is time dependent and the scaled diagram follows the AT behavior. Only the amplitude A in Eq. (13), which in our model is close to the AT value, seems to depend strongly on the definition of $H_c(T,t)$ and the time scale t of the experiment. For short times, recent experiments[33] are consistent with a crossover from the AT behavior ($\zeta=\frac{3}{2}$) for large fields, $H>H_0$, to the analytic behavior ($\zeta=\frac{1}{2}$) for small fields, $H<H_0$. One observes $H_0\propto \ln t$.

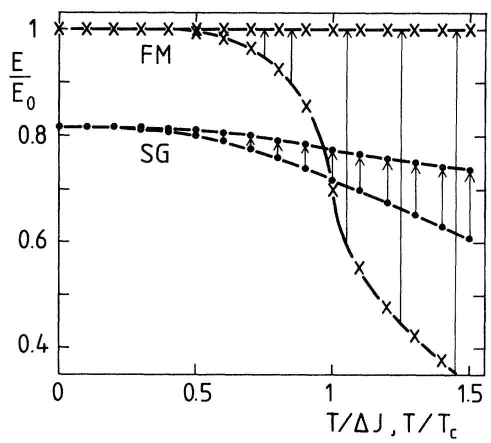

FIG. 10. Energy vs temperature for the ferromagnet and the spin-glass, respectively. $E_0=\sum_j |J_{ij}|$ would be the ground-state energy in the absence of frustration. The lower curves show the equilibrium energy at temperature T, while the upper curves denote the energy after quenching the system to $T=0$. T_c is the Onsager value of the critical temperature of the ferromagnet.

C. Metastable energy

We now compare the energy of the spin-glass with the one of the usual Ising ferromagnet on the same square lattice. In both cases the system was cooled slowly in zero field to a temperature T, then suddenly quenched to $T=0$. Figure 10 shows the energy scaled by $E_0=(1/N)\sum_{\langle ij\rangle}|J_{ij}|$ which should be the ground-state energy without frustration. Thus trivially one has $E(T=0)/E_0=1$ in the ferromagnet while in the spin-glass this ratio is reduced by 18%.[17] The ferromagnet—even above the critical temperature T_c—is always quenched to the ground state; we have never observed that domains are frozen in by our method of quenching to $T=0$. The spin-glass, however, behaves very differently. Even at low temperatures the quenched system was metastable and its energy increases with increasing temperature. Contrary to the ferromagnet one does not gain much energy by quenching the spin-glass to $T=0$. This demonstrates that the spin-glass changes its state continuously over a broad temperature range.[27] This may be either described by a cluster picture with smoothly increasing cluster size,[34] or by continuously bifurcating metastable valleys in a high-dimensional phase space.[35]

III. PHENOMENOLOGICAL RELATIONS

A. Scaling at $T=0$

In this section we want to make a few steps towards a phenomenological theory of spin-glasses. Firstly, we recall some facts which have emerged from Monte Carlo simulations and transfer-matrix calculation of the spin-glass model, Eqs. (1) and (2).

(1) At zero magnetic field $H=0$, the EA correlation length defined by $[\langle S_iS_j\rangle^2]_{\text{av}}\sim\exp(-r/\xi_{\text{EA}})$ and the susceptibility $\chi_{\text{EA}}=(1/N)\sum_{ij}\langle S_iS_j\rangle^2$ vary with temperature as[17,18,22]

$$\xi_{\text{EA}}\propto(\Delta J/T)^2,\quad \chi_{\text{EA}}\propto\xi_{\text{EA}}^2\propto(\Delta J/T)^4,\quad \Delta J\gg T\,. \tag{14}$$

Of course, the results of Refs. 17, 18, and 22 do not unambiguously yield the exponents ν and γ describing the divergence of ξ_{EA} and χ_{EA}, where $\xi_{\text{EA}}\propto(\Delta J/T)^{\nu}$ and $\chi_{\text{EA}}\propto(\Delta J/T)^{\gamma}$ as $T\to 0$, since the estimates $\nu\approx 2$ and $\gamma\approx 4$ are derived from an extrapolation from the regime $\Delta J/T\approx 1$—where reliable numerical data are available—rather than from the asymptotic regime $\Delta J/T\gg 1$. In addition, one might expect that the correlation function might involve an exponent η as in standard critical phenomena, $[\langle S_iS_j\rangle^2]_{\text{av}}\propto r^{-(d-2+\eta)}$ for $r\ll\xi_{\text{EA}}$, which would in turn imply $\chi_{\text{EA}}\propto\xi_{\text{EA}}^{2-\eta}$ rather than Eq. (14). This latter case seems to be realized in the $\pm J$ model, where a power-law decay of the correlation function was suggested at $T=0$.[17]

(2) If a "ground state" (equal to a low-lying metastable state) $\{S_i^1\}$ of a spin-glass simulation in a system with N lattice sites is projected on any other ground sate $\{S_i^2\}$,[36] the typical projection $P=(1/N)\sum_i S_i^1S_i^2$ is of order $P\propto 1/\sqrt{N}$. By special preparation (heating to $T<\Delta J$), states with large projection (of order unity) onto each oth-

er can also be found: For any chosen value of P, one can find a ground state in the vicinity which differs at most by $\Delta P = \pm 1/\sqrt{N}$. This implies that the ordering described by ξ_{EA} is very degenerate.

(3) The "susceptibility" M/H ($T=0$) at zero temperature diverges for small fields according to Eq. (4).

(4) At zero magnetic field $H=0$, the relaxation time τ varies with temperature as[18]

$$\ln\tau \propto (\Delta J/T)^2 . \tag{15}$$

(5) For $H>0$ contours of constant relaxation time in the H-T plane can be interpreted as critical fields $H_c(T)$ described by Eq. (13), where $\zeta=\frac{3}{2}$ and T_f depends on τ according to Eq. (15) (Ref. 18 and 12),

$$H_c(T,\tau) \propto [1-T/T_f(\tau)]^{3/2} . \tag{16}$$

In the following, we try to correlate these facts with each other. However, we want to stress that our approach is very tentative; it is more meant to stimulate the reader to think in this direction than to present well-founded results.

Firstly, we discuss scaling at zero temperature. Note that according to Eq. (14) the static correlation length ξ_{EA} diverges at $T\to 0$. Thus in analogy to usual critical phenomena one might try to express any quantity A in terms of $\xi_{EA}^{x_A}$ with "critical" exponents x_A. Therefore, following Binder,[22] we assume a scaling of the static nonlinear response $\tilde{\Delta}(T,H)$ [see Eq. (6)],

$$\tilde{\Delta}(T,H) \equiv \left[\frac{T}{\Delta J}\right]^{\beta} \tilde{c}\left[\frac{H/\Delta J}{(T/\Delta J)^{\Delta}}\right] , \tag{17}$$

and require

$$\tilde{c}(x) = \begin{cases} c_0 x^2, & x \text{ small} \quad (18a) \\ 1-c_\infty x^{-1/\Delta}, & x \text{ large} . \quad (18b) \end{cases}$$

Equation (18a) reveals that $\tilde{c}(x)$ is analytic, and $\Delta(T,H)$ obeys the symmetry $H \leftrightarrow -H$. In order to see the plateau in $M(T)/H$, which diverges according to Eq. (4), we must

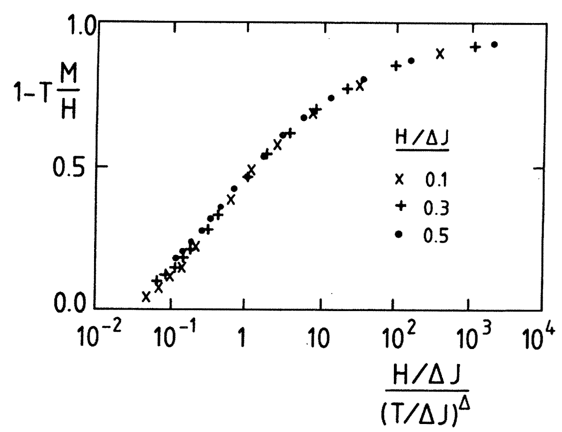

FIG. 11. Nonlinear susceptibility $\tilde{\Delta}(T,H)=1-TM_{eq}/H$ as a function of field H scaled by temperature T^{Δ} with $\Delta=3.5$. The data are from the range $0.1 \le H/\Delta J \le 0.5$ and $0.1 \le T/\Delta J \le 1$.

have $\beta=0$ and Eq. (18b) (Ref. 22) with $\Delta=3.5\pm 0.5$. In Fig. 11, we have plotted $\tilde{\Delta}(T,H)$ as a function of scaled fields and temperatures. In fact, for $H<0.5\Delta J$ and $T<\Delta J$ the scaling behavior, Eq. (17), is observed reasonably well over several orders of magnitude of the scaling variable x. Note that most of the data at high temperatures included in the scaling plots, Figs. 5 and 6, fall outside this true scaling regime, of course, and hence are not consistent with Eqs. (17) and (18).

A first consequence of scaling is a relation between the exponents appearing in Eqs. (4) and (14). Namely, expanding the magnetization of our symmetric spin-glass model for small fields H gives with Eq. (8),[37]

$$a_3(T) = 1 + 3 \sum_{j\,(\neq i)} [\langle S_i S_j\rangle^2]_{av} = 3\chi_{EA}(T, H=0) - 2 . \tag{19}$$

With Eq. (18a) we have $a_3(T) \propto T^{-2-2\Delta}$; thus from Eqs. (14) and (19) we obtain $\Delta=3$, which is consistent with the value quoted in Eq. (4) from the divergency of M/H at $T=0$. Another consequence of scaling is the behavior of χ_{EA} as a function of field. Namely, the scaling assumption gives for χ_{EA}

$$\chi_{EA}(T,H) = \left[\frac{T}{\Delta J}\right]^{2-2\Delta} \tilde{f}\left[\frac{H/\Delta J}{(T/\Delta J)^{\Delta}}\right] . \tag{20}$$

Changing the scaling variables we obtain

$$\chi_{EA}(T,H) = \left[\frac{H}{\Delta J}\right]^{(2/\Delta)-2} \tilde{g}\left[\frac{T/\Delta J}{(H/\Delta J)^{1/\Delta}}\right] , \tag{21}$$

where $\tilde{f}$ and $\tilde{g}$ are scaling functions. This gives, together with Eq. (14) and $\Delta=3$,

$$\xi_{EA}(T=0,H) \propto \left[\frac{H}{\Delta J}\right]^{-2/3} . \tag{22}$$

To test our assumption, Eq. (17), it would be very useful to check Eq. (22) numerically. This still has to be done. Another consequence of Eqs. (17) and (18) is that $a_5 \propto a_3^2$ as $T\to 0$. Experimental data of Omari *et al.*[11] seem to imply $a_5 \propto a_3^{2.25}$, while mean-field theory yields $a_5 \propto a_3^3$.

B. Relaxation time in zero field

We suppose that the relaxation time τ in Eq. (16) is due to a reorientation process of a cluster containing $(\xi_{EA})^d$ spins.[34] This reorientation involves a crossing of an energy barrier. We assume that this occurs by moving a wall through the cluster.[38] In a ferromagnet or in a Mattis spin-glass model,[39] a wall between the only two possible ground states would cost a free energy $\Delta F \propto \Delta J \xi_{EA}^{d-1}$. However, in our frustrated spin-glass model, Eqs. (1) and (2), one has to flip $\xi_{EA}^{d/2}$ spins only to go from one ground state to a typical other one.[40] This assumption we infer from the property (2) noted above, that is, at $T=0$, where $\xi_{EA}^d=N$, two ground states differ by overturning at least $\sqrt{N}$ spins. Thus a wall would cost a free energy of the order of

$$\Delta F \propto \Delta J (\xi_{EA}^{d-1})^{1/2} . \tag{23}$$

Assuming now that the relaxation time is given by an Arrhenius formula, we obtain for $d=2$ and with Eq. (14),

$$\ln\tau=\Delta F/T \propto \Delta J/T\xi_{\rm EA}^{1/2} \propto (\Delta J/T)^2 \,, \tag{24}$$

in agreement with the numerical findings, Eq. (16).

Our treatment thus uses the fact that a correlated cluster can exist in more and more local free-energy minima as the temperature is lowered and the size of this cluster increases, and hence the free-energy barriers between these minima increase with a smaller power of cluster size than in an Ising ferromagnet. This behavior may be considered as a remnant of what happens in the mean-field limit of spin-glasses below T_f, where also more and more "valleys" in phase space representing different ordered states become available as the temperature is lowered, with the free-energy barriers between them being infinite.

A more general "dynamic scaling" formulation is obtained by requiring that the typical free-energy barriers in the system scale as $\xi_{\rm EA}^{z-1/\nu}$, where z is a "dynamic exponent," which would yield

$$\ln\tau=\Delta F/T \propto (\Delta J/T)\xi_{\rm EA}^{z-1/\nu} \propto T^{-\nu z}$$

instead of Eq. (24) (where $z=1$). This result, in turn, implies, for the frequency dependence of the freezing temperature, setting $\tau \propto 1/\omega$, that $T_f^{-1}(\omega) \propto |\ln\omega|^{1/\nu z}$.

C. Dynamic properties in nonzero magnetic field

When a magnetic field is applied, the free-energy barrier which is due to the wall between two neighboring states in phase space is lowered by displacing the interface locally: *With respect to the spin-glass alignment, a uniform magnetic field acts as a random field would act on a ferromagnet.*[41,42]

We now estimate how much free energy can be won by displacing the wall a distance w in the presence of a field. In the absence of a field, there will already be fluctuations described by the probability law

$$P_T(w) \propto \exp(-\text{const}\, w^2/\xi_{\rm EA}) \,, \tag{25}$$

where we assume that the distribution of domain-wall widths w is a Gaussian, and that the typical width is proportional to the linear dimension $(\xi_{\rm EA})^{1/2}$ itself or smaller [if we allow for a temperature dependence of the constant in Eq. (25)]. Namely, as mentioned before, two neighboring valleys in phase space are assumed to differ by overturning a cluster of only $(\xi_{\rm EA}^d)^{1/2}$ spins. Hence, the energy barrier is associated with moving a wall of linear dimension $\xi_{\rm EA}^{1/2}$ through such a cluster. (All lengths are measured in units of the lattice spacing.)

In the absence of a field, the maximum of $P_T(w)$ occurs for $w=0$. In this case, the energy barrier is given by Eq. (23) [for $d=2$, $\Delta F^*|_{H=0}=\xi_{\rm EA}^{1/2}(H=0)\Delta J$]. In the presence of a field, the maximum occurs for finite w, and the free energy is reduced.

Since neighboring states taking a volume V differ by overturning $\sqrt{V}$ spins, the displacement of the interface a distance w over an area $\xi_{\rm EA}^{(d-1)/2}$ is affected by overturning $(\xi_{\rm EA}^{(d-1)/2}w)^{1/2}$ spins. The typical field energy then will be

$$\Delta U_H=-\text{const}' H\xi_{\rm EA}^{(d-1)/4}w^{1/2} \,, \tag{26}$$

and hence the probability distribution for w at nonzero H is

$$\begin{aligned} P_{T,H}(w) &\propto \exp\left[-\text{const}\frac{w^2}{\xi_{\rm EA}}\right]\exp\left[-\frac{\Delta U_H}{T}\right] \\ &=\exp\left[-\text{const}\frac{w^2}{\xi_{\rm EA}}+\text{const}'\frac{H}{T}(\xi_{\rm EA}^{(d-1)/2}w)^{1/2}\right] . \end{aligned} \tag{27}$$

The maximum occurs at

$$\begin{aligned} &\frac{d}{dw}\left[-\text{const}\frac{w^2}{\xi_{\rm EA}}+\text{const}'\frac{H}{T}(\xi_{\rm EA}^{(d-1)/4}w^{1/2})\right]=0 \,, \\ &\frac{w}{\xi_{\rm EA}} \propto \frac{H}{T}\xi_{\rm EA}^{(d-1)/4}w^{-1/2} \,, \end{aligned} \tag{28}$$

and hence

$$w^{3/2} \propto \frac{H}{T}\xi_{\rm EA}^{(d+3)/4}, \quad w^{1/2} \propto \left[\frac{H}{T}\right]^{1/3}\xi_{\rm EA}^{(d+3)/12} \,, \tag{29}$$

then we find, for the energy change ΔU_H with this maximum,

$$\Delta U_H \propto -H^{4/3}T^{-1/3}\xi_{\rm EA}^{d/3} \,. \tag{30}$$

Of course, Eq. (30) holds only for large enough fields such that w as given by Eq. (29) is much larger than a lattice spacing. In this regime, we find, with

$$\xi_{\rm EA}=T^{-\nu}\tilde{\xi}(H/\Delta J/(T/\Delta J)^{\Delta}) \propto H^{-\nu/\Delta} \,,$$

that

$$\Delta U_H \propto -T^{-1/3}H^{4/3}H^{-d\nu/3\Delta} \,. \tag{31}$$

If we assume that we can apply the usual hyperscaling relation[22] $(\beta=0,\eta=2-d)$ $d\nu=\gamma=2\Delta-2$, we obtain $2d\nu/\Delta=4-4/\Delta$, and hence

$$\Delta U_H \propto -T^{-1/3}H^{2(1+\Delta^{-1})/3} \,. \tag{32}$$

Then a contour $\ln\tau(T,H)=\text{const}$ in this regime is given by

$$\Delta F^*|_{H=0}+\Delta U_H=\text{const} \,. \tag{33}$$

Since the extrapolation of this contour to $H=0$ defines $T_f(t)$, we obtain

$$H^{1/\zeta} \propto 1-T/T_f(t) \,, \tag{34}$$

with $1/\zeta=\frac{2}{3}(1+\Delta^{-1})$. Instead of the A-T behavior, $\zeta=\frac{3}{2}$, we rather obtain $\zeta\simeq1.48$ for the experimental value $\Delta\simeq75$ and $\zeta\simeq1.17$ for $\Delta\simeq3.5$ obtianed from our computer simulations, Eq. (4). In this context we note that it recently has been suggested[43] that the exponent ζ of the A-T line for finite-range spin-glasses is $\zeta=\frac{3}{2}$ for $d\geq8$ only, while for $6<d<8$, ζ changes with d and becomes $\zeta=1$ for $d=6$, while in this regime the exponents β and γ still have their mean-field value. Thus there is no reason to expect that ζ is still $\frac{3}{2}$ at the lower critical dimension. Similarly, if one defines a critical field from the susceptibility maximum[44] one finds that near T_f it varies according to Eq.

(34) with $\zeta=(\gamma+\beta)/2$, in the non-mean-field case. We emphasize that our treatment does not imply that the contours of constant relaxation time in the H-T plane can be scaled in a universal form by writing $H/T_f(t) \propto [1-T/T_f(t)]^{\zeta}$, since the proportionality constant in Eq. (34) is temperature dependent. In fact, Fig. 3(b) reveals systematic deviations between the data for $t=600$ and 6000 MCS rescaled in this way. A detailed analysis of dynamic scaling of the relaxation time $\tau(T,H)$ will be given in Ref. 45.

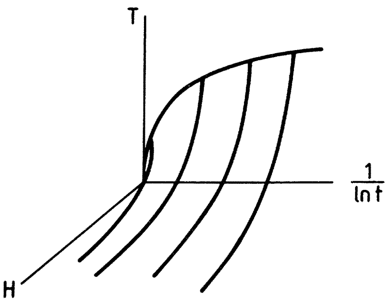

FIG. 12. Schematic view of the characteristic relaxation time t as a function of field and temperature.

IV. SUMMARY AND CONCLUSIONS

We have investigated the static and dynamic magnetic response of the two-dimensional EA model. Computer simulations showed a plateau in the equilibrium (slowly-field-cooled) susceptibility M_{eq}/H which diverges at zero temperature and small fields with the power law $M_{eq}/H \propto H^{1/\Delta}$, $\Delta=3.5\pm0.5$. Accordingly, a static "critical" field $H_c^{eq}(T)$, defined by the boundary of the plateau, decreases with temperature; we expect $H_c^{eq} \propto T^{\Delta}$. Although our model has no static phase transition with a nonzero critical temperature T_f, we find that $M_{eq}(T,H)/H$ can be fitted by scaling functions with a nonzero T_f and reasonable critical exponents γ and β in a broad temperature and field regime.

From the points where the difference in the static and dynamic magnetic response vanishes, we obtain time-dependent critical fields $H_c(T,t)$ and, by extrapolation to $H=0$, a time-dependent freezing temperature $T_f(t)$ with $T_f(\infty)=0$. Surprisingly, our results for the scaled non-equilibrium fields $H_c(T,t)/T_f(t)$ follow the singular behavior of the mean-field theory which has an equilibrium phase boundary $H_c(T)$ found by AT.

At zero temperature, the zero-field-cooled magnetization $M_{ZFC}(H)$ has a S-shaped behavior which is typical for spin-glasses. For $H>3\Delta J$, it approaches $M_{eq}(T=0, H)$ while for small fields we find $M_{ZFC}(H) \propto H^{2.4}$. If we quench our system from temperature $T=T_i$ to $T=0$, we find that the energy still varies with T_i. This shows that the spin-glass changes its state continuously down to zero temperature.

In the second part we try a few steps towards a phenomenological description of spin-glasses. Thus the behavior of the equilibrium susceptibility may be described in terms of scaling at zero temperature and zero field. This relates the divergence of the nonlinear susceptibility for $T\to0$ to the exponent Δ. Furthermore, this predicts that the static correlation length $\xi_{EA}(T,H)$ diverges as $\xi_{EA}(T=0) \propto H^{-\nu/\Delta} \approx H^{-2/3}$ for two dimensions.

A characteristic relaxation time is related to ξ_{EA}. By making use of the fact that the spin-glass ground state is highly degenerate, we find for the dynamic freezing temperature $T_f(t) \propto (\ln t)^{1/\nu z}$ with $\nu z \approx 2$. The dynamics in an applied field is discussed in context with the random-field problem. This approach yields that the relaxation time varies as $H^{z'}$ where z' is related to Δ.

Thus the picture we now have for the two-dimensional Ising spin-glass is illustrated in Fig. 12, which shows a characteristic relaxation time t as a function of field H and temperature T. For a given observation time t and sufficiently large fields, one observes a behavior similar to the A-T behavior $H_c \propto [T-T_f(t)]^{3/2}$. At $H=0$ the freezing temperature $T_f(t)$ depends on the observation time t as $T_f(t) \propto (\ln t)^{1/2}$, i.e., it is zero for $t\to\infty$, since the model has no static phase transition at a nonzero T_f. For nonzero H values the correlation length ξ_{EA} is finite [Eq. (22)], and by using Eq. (24) one obtains $T_f(H\neq0, t) \propto (\ln t)^{-1}$. Thus the surface of Fig. 12 is very singular at the point $T=0, H=0$ and $t=\infty$. At this point we expect scaling in terms of a (diverging) static correlation length ξ_{EA}.

Experimentally this surface defines a dynamic freezing transition of spin-glasses; e.g., compare with Fig. 3. The singular behavior at the $T=0$ and $t=\infty$ axis is reflected in Fig. 9; approaching this axis below and above the surface gives M_{ZFC} and $M_{FC}=M_{eq}$, respectively, which behave completely different as a function of field especially with respect to the singular behavior as $H\to0$.

As in our previous computer simulations the results of the present work are in remarkable qualitative agreement with experiments. So some of the experimental data can be fitted by power laws with $T_f=0$ and, reversely, some of our equilibrium results can be fitted with a nonzero T_f. This demonstrates that the recent interpretations of experimental data in terms of mean-field theory and critical phenomena have to be considered with greater care. Quantitatively, the corresponding exponents of a $T_f=0$ fit seem to differ by more than 1 order of magnitude between experiments and our two-dimensional results.

At present it is not clear whether this is due to (i) time effects in experiments, (ii) quantitative differences between the two- and three-dimensional models, or (iii) a true static phase transition in real spin-glasses. Clearly, more detailed experiments and computer simulations in three dimensions are necessary.

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