Scaling behavior in amorphous and disordered metals

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We show that the low-temperature resistivity of amorphous and disordered metals is described by a square-root variation with temperature, as suggested by various scaling theories of the metal-insulator transition. The coefficient Δ (the correlation gap) has the identical dependence on resistivity, $\Delta \sim \rho^{-2}$, reported for materials with resistivity up to three orders of magnitude greater.

The temperature dependence of the resistivity of amorphous metals has been the subject of considerable research and some controversy. At room temperature, the temperature derivative α correlates roughly with the resistivity (the Mooij correlation[1]), being negative above 150 $\mu\Omega$ cm and positive below, an effect usually explained by the diffraction model based on the Faber-Ziman liquid metal theory.[2] At low temperatures the derivative is negative in the very great majority of amorphous metals, irrespective of its room-temperature value, a behavior so widespread as to suggest a universal origin. The variation in the low-temperature region has been crudely characterized as logarithmic, which has led a number of authors to invoke the Kondo effect[3] as the cause. However, the general insensitivity of α to applied magnetic field has caused others (including the present authors) to prefer an explanation of structural origin, most notably scattering from two-level tunneling states.[4] Neither of the models has found universal acceptance, nor, on close examination, are they seen to provide an exact quantitative description of the data. At the present there is, in fact, no model which embraces all aspects of the low-temperature resistivity.

In the present Rapid Communication we present new data, and a reanalysis of old, which show that a far better description of ρ is a $\sqrt{T}$ dependence, a form which is predicted by several recent theories for high-resistivity conductors.[5]

Some recent experiments on conductivity and tunneling in amorphous NbSi (Ref. 6), GeAu (Ref. 7), crystalline SiP (Ref. 8), and granular Al (Ref. 9) have indicated that, as the metal-insulator transition is approached the conductivity passes smoothly to zero. Recent scaling theories of the metal-insulator transition,[10,11] which take account both of localization[12] and electron-electron interactions,[5] give just such a result. On the insulating side of the transition a correlation gap Δ opens in the one-electron density of states so that the conductivity vanishes exponentially with temperature. On the conducting side the correlation gap is anticipated by a square-root anomaly in the density of states at $T=0$:

$$N(E)=N_0(E)(1+\sqrt{E}/\Delta) \quad , \tag{1}$$

which is smoothed out at finite temperature leading to a conductivity:

$$\sigma(T)=\sigma(0)(1+\sqrt{T}/\Delta) \quad . \tag{2}$$

The experiments cited above (Refs. 6–9), in materials covering approximately the resistivity range 1 to 10^{-3} Ω cm, are in good agreement with these results and yield, moreover, a correlation gap which varied as ρ^{-2} indicating strong rather than weak coupling.[9]

It is therefore of interest to examine the extent to which the main body of amorphous metals, whose resistivities are generally an order of magnitude lower than the materials cited above, fit into the classification. That there exists low-temperature-resistivity anomalies has been well documented[4,13] particularly for metal-metalloid systems. Indeed Rapp, Baghat, and Gudmundsson[13] point out a $\sqrt{T}$ dependence for ρ for a number of such alloys, but the presence of magnetic elements in their materials introduces some uncertainty into the generality of this result. On the other hand, many nonmagnetic amorphous metals turn out to be superconducting which introduces the possibility of the influence of superconducting fluctuations.

To provide unequivocal evidence for the low-temperature behavior we have made and measured several Y-Al metallic glasses which, down to 60 mK, show neither superconductivity nor magnetic effects. The alloys were prepared by melt spinning under inert gas atmosphere (details will be given elsewhere[14]) and have the additional advantage of very high resistivities (Table I)—among the highest for amorphous metals. The temperature derivative of the resistivity is negative at all temperatures; below 10 K it varies as $\sqrt{T}$ down to our lowest temperature in the absence of the applied magnetic field. These data are illustrated in Fig. 1 along with data for several other representative alloy systems. Several points should be underlined:

(i) The alloys cover almost an order of magnitude in resistivity.

(ii) They include all types of amorphous alloys, including those with positive and negative α at room temperature. Indeed from Fig. 1 it is impossible to predict the high-temperature behavior.

(iii) The crystalline alloy *Pd*Cr, which shows a nonmagnetic resistance minimum,[15] falls naturally into this classification.

TABLE I. Analysis of the temperature dependence of the resistivity according to Eq. (2) for representative alloys.

Sample	Reference	ρ ($\mu\Omega$ cm)	$T^{1/2}$ slope ($K^{-1/2}$)	Δ (eV)
$Y_{77.5}Al_{22.5}$	14	275	12.4×10^{-4}	56
$Y_{67}Al_{33}$	14	259	11.2	68
$Fe_{40}Ni_{40}(P,B)_{20}$	18	211	12.7	54
$Cu_{60}Zr_{40}$	15	195	8.2	128
$Fe_{29}Ni_{49}(Si,P,B)_{22}$	14	173	10	186
$Fe_{75}(P,B,Al)_{25}$	13	162	6.8	186
$Fe_{40}Ni_{40}(P,B)_{20}$	13,21	186	7.8	142
$Fe_{80}B_{20}$	21	125	5.8	256
$Fe_{79}Cr_1B_{20}$	16	122	7.22	165
$Co_{75}P_{25}$	19	100	7.0	176
$Ni_{75}P_{25}$	4	105	3.9	567
$Pd_{82}Cr_{18}$ [a]	22	64	3.1	958
$Mg_{80.4}Cu_{19.6}$ [b]	20	57	2.5	1380

[a]Crystalline alloy. [b]Estimate only.

The evidence indicates that this behavior is universal and that it reflects the high resistivity rather than the amorphous state. This conclusion is reinforced upon further analysis of the data shown in Fig. 1 together with a reanalysis of other results available in the literature (Refs. 4 and 13–22). Table I summarizes our findings in terms of the correlation gap parameter Δ of Eq. (1). As shown in the $\log\Delta$-$\log\rho$ plot in Fig. 2 all these materials fall on the *same* $\Delta\sim\rho^{-2}$ line which represents the more sensitive granular Al and amorphous NbSi. The inverse square dependence of Δ on ρ leads[6] to a particularly simple law of corresponding states for the *conductivity*:

$$\sigma(T)-\sigma(0)\simeq 6\sqrt{T}\,(\Omega\,\text{cm})^{-1},$$

independently of the sample conductivity over more than four orders of magnitude. We believe that such scaling behavior provides clear evidence for interpreting the low-temperature resistivity anomaly as a percursor effect of the localization transition in a three-dimensional interacting electron system.

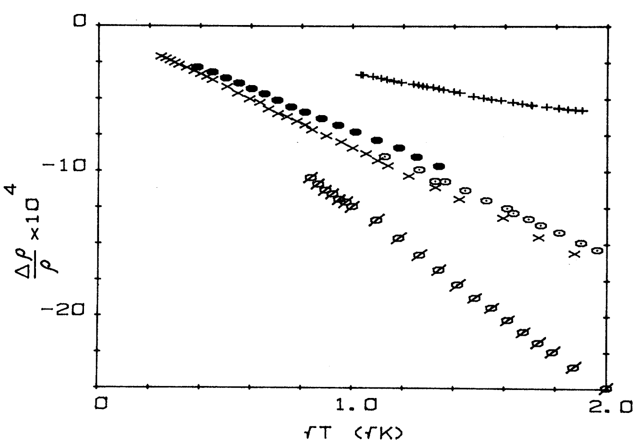

FIG. 1. Relative resistivity plotted as a function of $\sqrt{T}$: ϕ, $Y_{77.5}Al_{22.5}$; $\times$, $Fe_{40}Ni_{40}(P,B)_{20}$; $\odot$, $Cu_{60}Zr_{40}$; $\bullet$, $Fe_{79}Cr_1B_{20}$; $+$, $Pd_{82}Cr_{18}$.

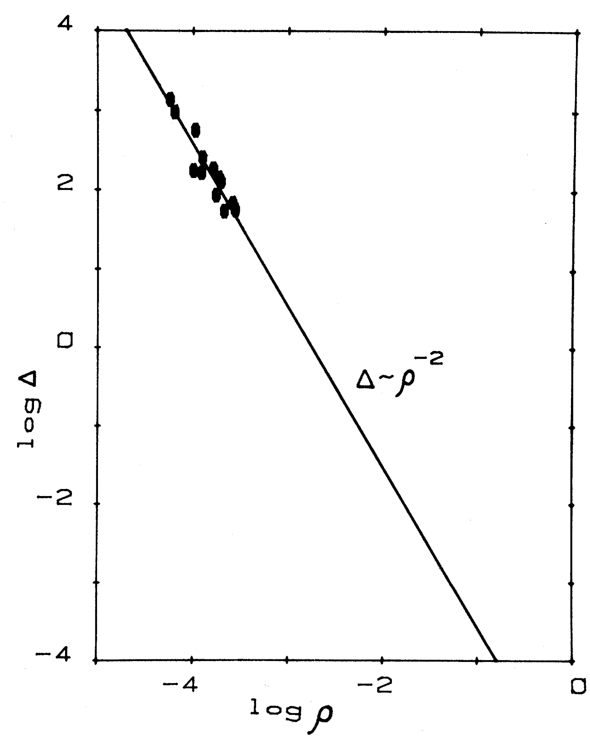

FIG. 2. Log-log plot of the correlation gap Δ (eV) vs the resistivity ρ (Ω cm). The solid line is taken directly from Fig. 3 of Ref. 6.

We conclude with several comments on the consequences of these results. The present state of the theory[11] indicates a scaling on the metallic side of the transition to the weak-coupling limit,[5] $\sigma(0) \sim \Delta^{1/3}$. The experimental evidence over a wide range of disordered materials calls for the introduction of additional mechanisms, such as dynamic screening,[11] to convert the exponent from 0.3 to 0.5. The scaling behavior also provides a basis to examine where dynamical interactions of magnetic or other origin assume comparable importance to the electron-electron term. Examples such as $Cu_{50-x}Gd_xZr_{50}$ $(1<x<4)$ (Ref. 15), $Fe_{80-x}Cr_xB_{20}$ $(20<x<30)$ (Ref. 16), and PdSi (Ref. 17) show significant magnetic contributions at low temperatures. At higher temperatures electron-phonon and other interactions can introduce different characteristics, such as the Mooij correlation.

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