Response to "Comment on 'Specific sine-Gordon soliton dynamics in the presence of external driving forces' "

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Taking into account the statements raised by a recent controversy, we emphasize the specific features of the accelerated sine-Gordon soliton dynamics.

We note three basic features in the preceding Comment by Kaup: (i) The soliton [or, better said, the accelerated quasisoliton (AQS)] is not a point particle. (ii) There are different time scales in this AQS problem. (iii) Our definition of the AQS center differs from the definition commonly adopted in the literature, including Kaup's definition itself.

The first statement (i) has been extensively discussed in Ref. 1, where it was shown that the sine-Gordon problem in the presence of external driving forces [Eq. (1)] cannot, strictly speaking, lead to any kind of particlelike dynamics, since there appears in the Hamiltonian related to Eq. (1) a singularity. Interesting enough, with respect to Kaup's criticism of our performing only short-time scale calculations, this study was done assuming long-time AQS dynamics. It was also shown in this reference that the existence of two basic coupling regimes between the AQS wave and the external field R (the so-called coherent and incoherent interaction regime) strongly affects this singularity. Indeed, in the incoherent coupling regime, the Hamiltonian singularity appearing in the far wings of the AQS may be neglected from a physical point of view, while such is not the case in the coherent coupling regime (Ref. 2). Now remember that the particle concept is associated [in nonlinear field theory] with waves interacting in the incoherent coupling regime, and not in the coherent regime (in which the field concept dominates over the particle concept). Therefore, we believe that statement (ii) is strongly related to the existence of both these opposite coupling regimes, and we refer to Refs. 1 and 2 for a detailed discussion of (ii).

As for statement (iii), the calculation (7)–(9) is simply an extension to long-time scales of the perturbation treatment based on Rubinstein's eigenvalue problem (Ref. 3). However, the argument related to Eq. (11) is, as a matter of fact, independent of the time scale, as Kaup himself agrees in his interpretation of Eq. (17). It may be stated as follows for small-time scales (commonly assumed in standard eigenvalue perturbation problems such as Ref. 3): The position of the center of the AQS wave at time $t > t_0$ must be measured by the amplitude of the projection of the wave profile variation between t and t_0 onto the translation (Goldstone) eigenmode [Kaup's Eq. (8)]. This definition is not new (cf. Ref. 3) and has been discussed in detail in Ref. 2, where we pointed out that it is either equivalent to the definition of the center of mass of the system or strongly related to the canonical field momentum. An explicit calculation due to Scott and given in Appendix B of this reference shows the many physical difficulties occuring in any attempt to identify the AQS center with its center of mass. Actually, the distance between these points secularly increases. Hence we refer to Ref. 2 again for a detailed discussion of our definition of the AQS center taking into account comment (iii).

Finally concluding, we emphasize that we mostly agree with Kaup's comments 1–5. The discrepancy between our respective interpretations originates mostly from terminology. While Kaup's definition of the position of the AQS center seems a rather formal one, we preferred to focus on the pulse, which has a clear physical significance, inasmuch as it bears the greatest energy density of the AQS wave. We note that Kaup's interpretation of Eq. (18) is identical to ours, and this effect was actually the very purpose of our work concerning the specific sine-Gordon AQS dynamics.

Note added. We point out the recent publication of a paper by Yu S. Kivshar and M. Kosevich (Pis'ma Zh. Eksp. Teor. Fiz. 37, 542 (1983) [JETP Lett. 37, 648 (1983)]) which shows that the initial motion of the sine-Gordon soliton in the presence of so-called odd perturbations is non-Newtonian.

[1]J. J. P. Leon, G. Reinisch, and J. C. Fernandez, Phys. Rev. B 27, 5817 (1983).
[2]G. Reinisch and J. C. Fernandez, Phys. Rev. B 25, 7352 (1982).
[3]A. R. Bishop, J. A. Krumhansl, and S. E. Trullinger, Physica D 1, 1 (1980), and references therein.

