Effect of elastic anisotropy on the electromagnetic generation of ultrasound in potassium

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The free-electron theory of direct electromagnetic generation of ultrasound in metals is extended to include consideration of elastic anisotropy. It is shown that, for incidence along a direction for which transverse-acoustic waves exhibit anisotropy, the acoustic amplitude varies nonmonotonically with the intensity of an applied magnetic field B_0 independently of the scattering mechanism of the electrons at the surface of the metal. This is in contrast to the case of propagation along high-symmetry directions for which transverse-acoustic waves are degenerate where the nonmonotonic behavior of the acoustic amplitude with B_0 occurs only if the surface scattering is diffuse.

Recently Feyder, Kartheuser, Mohan, and Rodriguez[1] (in a paper hereafter referred to as I) presented a detailed analysis for the electromagnetic generation of ultrasound along the [100] direction of a cubic metal. The results of their study were compared with experimental work in potassium.[2,3] Potassium has an almost spherical Fermi surface and is therefore best suited for comparison with the free-electron theory of direct ultrasonic generation. Along the [100] direction of potassium the two linearly independent transverse-acoustic modes of vibration are degenerate, i.e., they travel with the same speed.

We will here modify the theory to include elastic anisotropy. In the [110] direction of potassium the two transverse-acoustic modes travel with different velocities—the fast shear mode polarized along the [001] direction (labeled as $\hat{1}$) has a speed $s_1 = 1.78\times10^5$ cm/sec while the slow shear mode polarized parallel to the $[1\bar{1}0]$ direction (labeled as $\hat{2}$) has a speed $s_2 = 6.56\times10^4$ cm/sec. We further assume that the polarization of the incident electromagnetic field is not in any preferred direction with respect to the crystallographic planes.

We consider a semi-infinite metal occupying the region $z \geq 0$. A static magnetic field B_0 is applied along the z direction (the [110] direction of the sample). A plane electromagnetic wave of angular frequency ω with the electric component along $\hat{x}$ and the magnetic component along $\hat{y}$ is incident normally on this surface from the region $z<0$ and propagates into the metal. We call θ the angle between $\hat{x}$ and $\hat{1}$.

We assume that all scattering mechanisms of the conduction electrons in the bulk of the metal can be described by a constant relaxation time τ. To account for the surface scattering of the electrons we follow the approach of Reuter and Sondheimer[4] and suppose that a fraction p of the electrons is reflected specularly from the surface while the remaining fraction $1-p$ is scattered diffusely.

The calculation of the electric field inside the metal was given in detail in I. The Fourier transform of the electric field inside the metal is obtained by solving numerically an integral equation; the results for both specular and diffuse scattering have been taken from I.

Consider now the positive ions in the metal. An acoustic wave traveling in the metal is described by displacements equal to the real part of

$$\vec{\xi}(\vec{r},t) = (\xi_1(z), \xi_2(z), 0)e^{-i\omega t} \ . \tag{1}$$

The equation of motion of the positive ions in the presence of external electric and magnetic fields as obtained in Ref. 5 is

$$M\frac{\partial^2\vec{\xi}}{\partial t^2} = \vec{\vec{G}}\cdot\frac{\partial^2\vec{\xi}}{\partial z^2} + \gamma e\left[\vec{E} + \frac{\dot{\vec{\xi}}\times\vec{B}_0}{c}\right] - \frac{\gamma m}{ne\tau}(\vec{j} + ne\dot{\vec{\xi}}) \ , \tag{2}$$

where n is the number of conduction electrons per unit volume having mass m and n/γ is the density of positive ions of mass M. $\vec{j}$ is the electron current density. $\vec{\vec{G}}$ is a tensor, diagonal when referred to the [001] and $[1\bar{1}0]$ axes of potassium, and is expressed as

$$\vec{\vec{G}} = \begin{bmatrix} Ms_1^2 & 0 \\ 0 & Ms_2^2 \end{bmatrix} \ . \tag{3}$$

The last term in Eq. (2) arising due to electron ion collisions is called the collision drag force.[5] The Lorentz force exerted by the rf magnetic field has been neglected.

Using the approximations of Ref. 5, Eq. (2) reduces to the following equations for the fast and slow mode:

$$\frac{\partial^2\xi_1}{\partial z^2} + \frac{\omega^2}{s_1^2}\xi_1 = -\frac{\gamma e}{Ms_1^2}\mathscr{E}_1 \ , \tag{4}$$

$$\frac{\partial^2\xi_2}{\partial z^2} + \frac{\omega^2}{s_2^2}\xi_2 = -\frac{\gamma e}{Ms_2^2}\mathscr{E}_2 \ , \tag{5}$$

where

$$\vec{\mathscr{E}} = \vec{E} - \vec{j}/\sigma_0 \ . \tag{6}$$

It is clear that ξ_1 and ξ_2, having different speeds, cannot be combined into circularly polarized components as was done in Ref. 5 for the completely isotropic case.

The components of $\vec{\mathcal{E}}$ in Eq. (6) are $\mathcal{E}_x$ and $\mathcal{E}_y$ which are obtained from the incident fields. $\mathcal{E}_1$ and $\mathcal{E}_2$ in Eqs. (4) and (5) can be expressed in terms of these components by a coordinate transformation. The solutions of Eqs. (4) and (5) for $z \rightarrow \infty$ are of the form

$$\xi_{1,2}(z) = \xi_{1,2}(\infty)\exp(i\omega z/s_{1,2}) \ , \tag{7}$$

where

$$\xi_1(\infty) = \xi_{1x}(\infty)\cos\theta - \xi_{1y}(\infty)\sin\theta \ , \tag{8}$$

$$\xi_2(\infty) = \xi_{2x}(\infty)\sin\theta + \xi_{2y}(\infty)\cos\theta \ . \tag{9}$$

Here $\xi_{1x}(\infty)$ and $\xi_{1y}(\infty)$ are solutions of Eq. (4) with $\mathcal{E}_1$ replaced by $\mathcal{E}_x$ and $\mathcal{E}_y$, respectively. $\xi_{2x}(\infty)$ and $\xi_{2y}(\infty)$ are similar sets of solutions obtained from Eq. (5). ξ_{1x}, ξ_{1y}, ξ_{2x}, and ξ_{2y} are calculated using the computational techniques discussed in I.

Figures 1–3 display $|\xi_1(\infty)|$, the acoustic amplitude for the fast mode as a function of magnetic field for both specular and diffuse scattering when the direction of incident polarization is parallel, perpendicular, and at an angle of $\pi/4$ to the fast shear axis. The values of the parameters used in the calculations are taken from Ref. 6; they are ω(angular frequency of the incident wave) $= 6.77\times 10^7$ sec^{-1}; l(mean free path of the electrons) $= 9.69\times 10^{-3}$ cm. The units used along the ordinates of the figures are reduced units in the sense described in I. The plots of $|\xi_2(\infty)|$, the acoustic amplitude for the slow shear mode as a function of magnetic

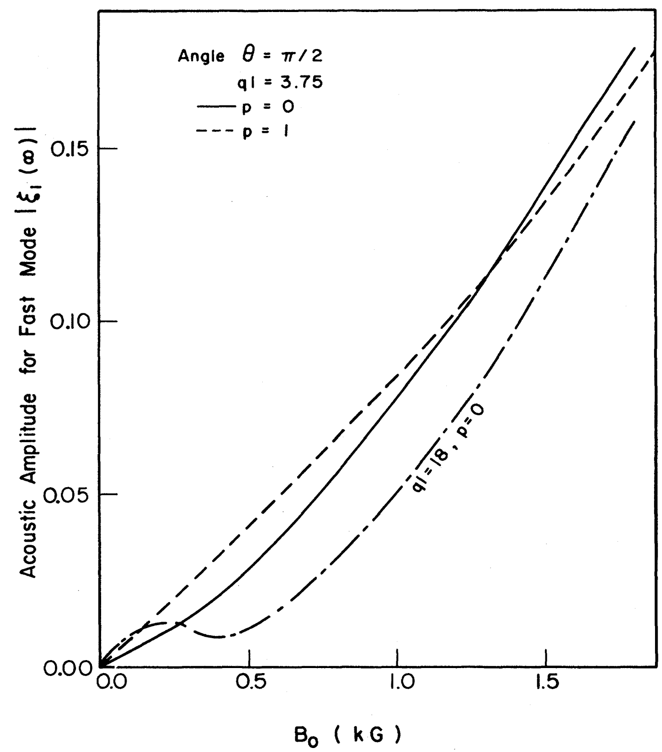

FIG. 2. Same as Fig. 1 for $\theta = \pi/2$.

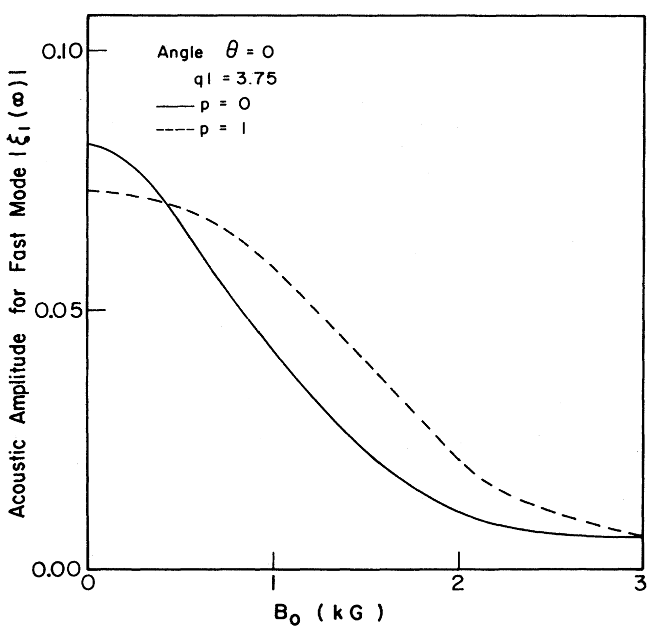

FIG. 1. Acoustic amplitude $|\xi_1(\infty)|$ for the fast shear mode as a function of B_0 for specular ($p=1$) and diffuse ($p=0$) scattering. Propagation is along the [110] direction of potassium. θ is the angle between the direction of the incident polarization and the fast shear axis. $ql = \omega l/s$, where s is the appropriate speed of the acoustic mode. The numerical values were obtained for potassium at a frequency of 10.77 MHz, assuming a mean free path $l = 9.69\times 10^{-3}$ cm and taking $\theta = 0$. The amplitude is expressed in the reduced units described in I.

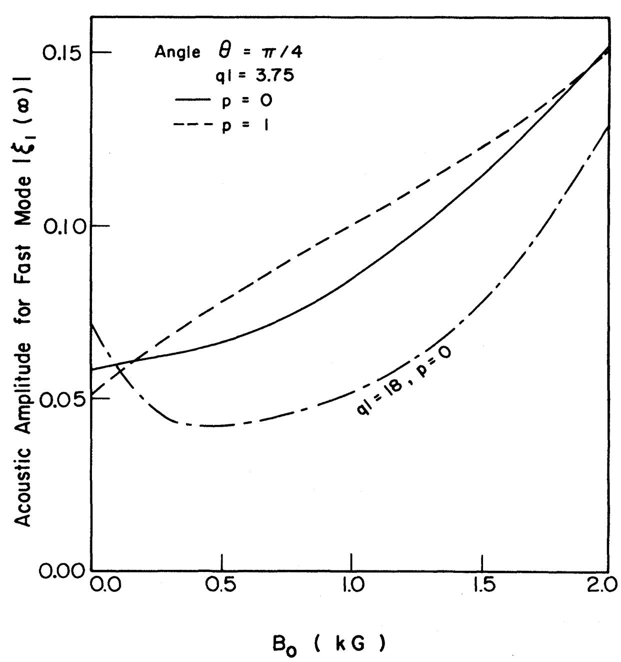

FIG. 3. Same as Fig. 1 for $\theta = \pi/4$.

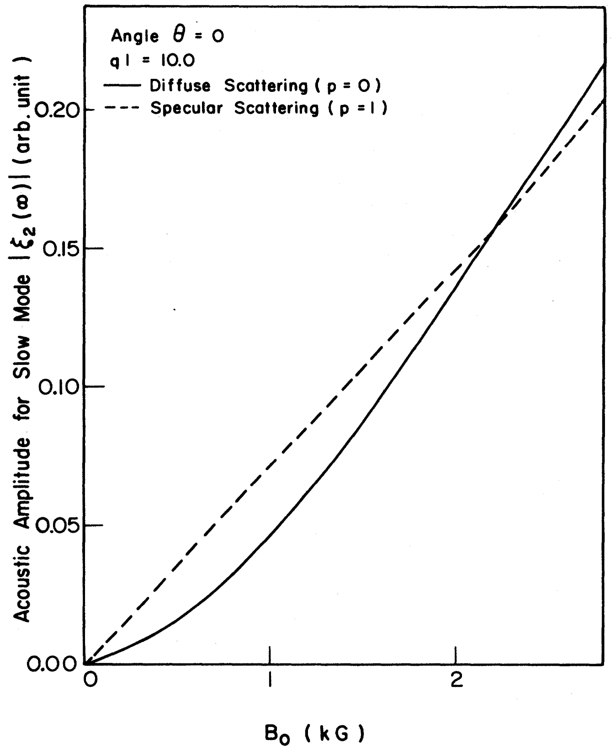

FIG. 4. Acoustic amplitude $|\xi_2(\infty)|$ for the slow shear mode as a function of B_0 for specular ($p=1$) and diffuse ($p=0$) scattering and for $\theta=0$. All other parameters are as in previous figures.

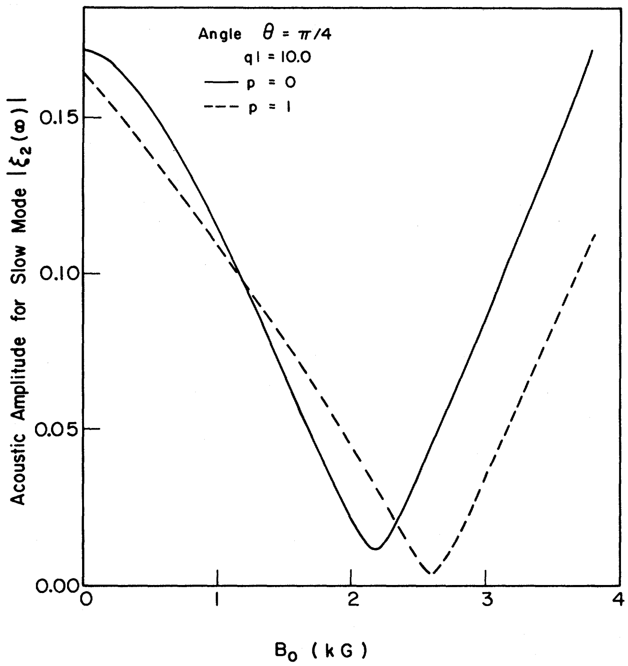

FIG. 6. Same as Fig. 4 for $\theta=\pi/4$.

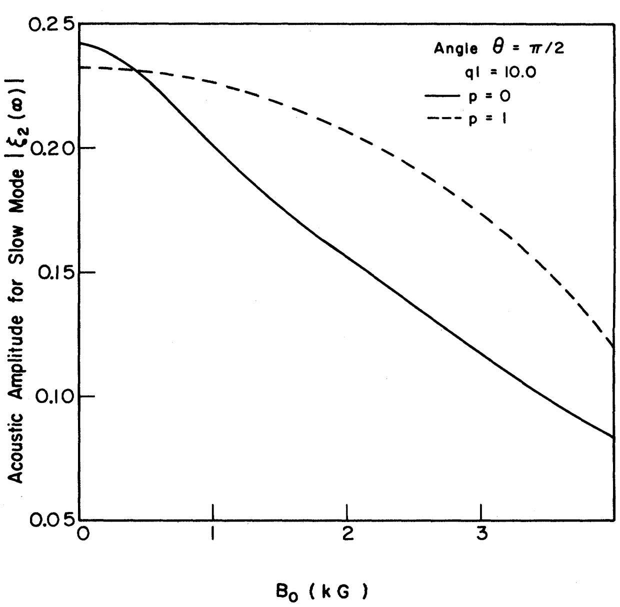

FIG. 5. Same as Fig. 4 for $\theta=\pi/2$.

field for both specular and diffuse scattering and for the three cases described above are shown in Figs. 4–6.

CONCLUSIONS

The acoustic amplitudes of the fast shear mode calculated on the basis of the free-electron model and plotted in Figs. 1 and 2 as a function of magnetic field are in reasonable agreement with the experimental results of Wallace, Gaerttner, and Maxfield.[3]

The curves of the acoustic amplitude for the slow shear mode for both specular and diffuse scattering shown in Fig. 6 show some of the essential features of the experimental results in Ref. 6. It is of particular interest to note in this case that the results obtained assuming specular scattering exhibit similar behavior as those obtained for diffuse scattering. This is in contrast to the case of elastic isotropy (the [100] direction of potassium) where the nonmonotonic nature of the acoustic amplitude as a function of the magnetic field could only be explained by diffuse scattering of the electrons from the surface. The reason for the behavior of the slow shear modes when $\theta \neq 0$ or $\pi/2$ is that the driving force $\mathscr{E}_2$ [see Eq. (5)] has a minimum as a function of B_0 because of the difference in phase of the incident electric field and the electric current density as functions of the applied magnetic field.

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