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Periodic flux dependence of the resistive transition in two-dimensional superconducting arrays

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New experimental data are presented which show a periodic variation of the resistance of two-dimensional arrays of superconductor–normal-metal-superconductor Josephson weak links with the magnetic flux per cell in units of the flux quantum, including a secondary minimum at the half-quantum points. Also presented is a simple model which accounts for the existence, shape, and magnitude of this periodic variation in terms of vortex core energies.

Voss and Webb[1] have reported a periodic variation of the resistance of large arrays of Josephson junctions with normal magnetic field, with resistance minima for integral numbers of flux quanta per unit cell of the array. Subsequently, Webb, Voss, Grinstein, and Horn[2] have observed secondary minima at the half-quantum field values, an observation which we have confirmed, as reported below. Monte Carlo simulations by Teitel and Jayaprakash[3] of phase transitions in frustrated two-dimensional X-Y models for a few flux values are consistent with these observations, but do not provide a general result. In this Communication, we present our measurements of the effect of normal magnetic fields on the resistance of Pb-Cu-Pb proximity-effect junction arrays. We show that the resistance modulation can be attributed to a periodic, field-dependent T_c, and we present a simple analytic model which accounts for the distinctive form of the periodic resistance variation, as well as its numerical magnitude.

The measurements reported here were made on samples similar to those discussed in a previous publication.[4] A typical sample, formed of 2000-Å-thick $PbBi_{0.05}$ squares on a 1500-Å-thick Cu film, is shown in Fig. 1(a). Samples were measured in a vacuum can, with electronic temperature regulation stable to 1 mK. Residual magnetic fields were reduced below 5 mG by means of two layers of mumetal; residual normal magnetic fields down to 1 mG could be detected and compensated for, with use of the sample itself as a magnetometer.

Upon cooling the samples below the island transition temperature T_{cs} ($\sim$7.3 K), the resistance dropped gradually as described previously.[4] As the transition to zero resistance at T_c was approached, oscillations in sample voltage versus magnetic field for fixed measuring current were observed in a narrow temperature range above T_c. Data for one sample are displayed in Fig. 2, showing sharp minima at integer flux values and secondary dips at the half integers in the traces with best signal-to-noise ratio. As shown in Fig. 3, $\Delta R(T)$, defined as the peak-to-peak amplitude of the periodic resistance modulation near zero applied field, closely follows the shape of dR/dT over most[5] of the resistive transition. Such a correspondence follows quite simply if we assume that T_c is a periodic function of applied magnetic field, as in the analysis[6,7] of the Little-Parks experiment on quantization in superconducting cylinders. Then, the constant of proportionality between ΔR and dR/dT is the shift in T_c due to the magnetic field; for the sample of Fig. 3, this shift is 0.075 K. Our model calculation (below) provides a simple phenomenological interpretation of this periodic modulation of T_c.

Our basic approach in modeling these effects is to assume that T_c scales with a "doubly renormalized" *mean* coupling energy $\bar{E}_J$, which includes a flux-dependent renormalization due to the phase deviations imposed by the field, in addition to the usual renormalization by random thermal fluctuations of the phases which takes E_J^0 into E_J. This reduction of the problem to consideration of an equilibrium quantity rather than a kinetic one obviously offers great simplification in analysis.

In the resistive state, macroscopic screening currents should be negligible, so that the flux per unit cell has a uniform value $f\Phi_0$ for all cells of the array. Here $f=Ba^2/\Phi_0$ is the flux per cell in units of the flux quantum, also termed

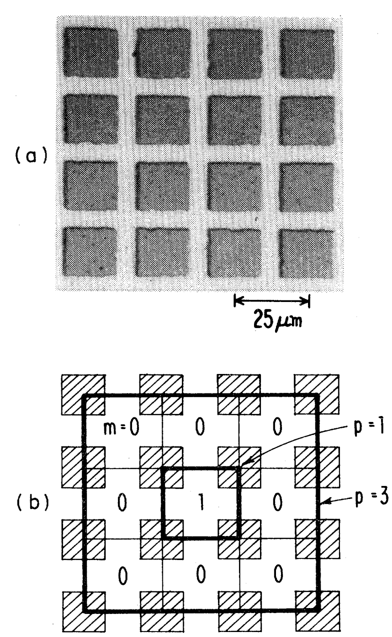

FIG. 1. (a) Micrograph of segment of typical 1000×1000 array, showing lead squares on copper film. (b) Schematic diagram, showing contours used in obtaining Eq. (2).

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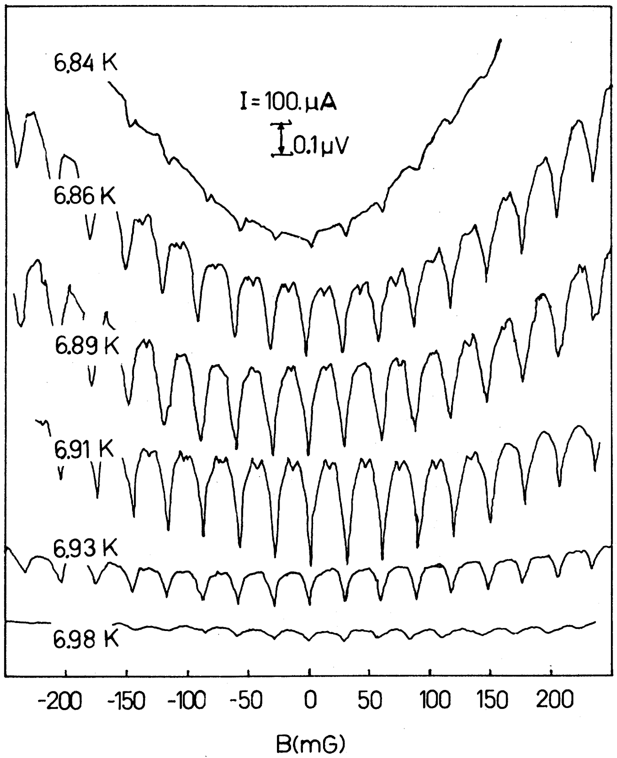

FIG. 2. Magnetic field dependence of resistance observed at various temperatures within the resistive transition of a two-dimensional array of superconductor–normal-metal–superconductor junctions.

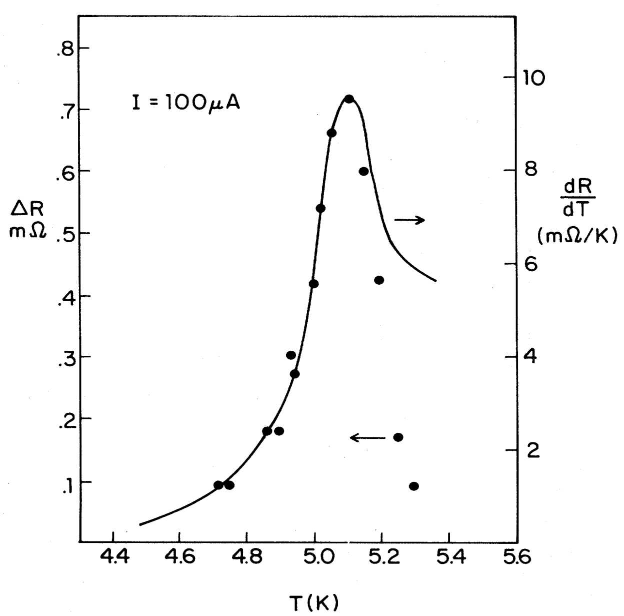

FIG. 3. Comparison of observed temperature dependences of the amplitude of the periodic resistance change ΔR and of dR/dT. (These data are from a different sample than those in Fig. 2.)

the lattice frustration. As in SQUID's (superconducting quantum interference devices),[8] the requirement that the phase of the superconducting wave function must vary through an integral multiple of 2π in going around a closed contour implies a constraint

$$\sum_{i=1}^{4} \theta_i = -2\pi f \ (\mathrm{mod}\, 2\pi) \equiv 2\pi(m-f) \ . \qquad (1)$$

Here the θ_i are the phase *differences* across the four Josephson links traversed in following around the perimeter of a unit cell, and the fluxoid quantum number m normally takes the integer value giving the lowest free energy. (Our analysis is given for a square lattice, in which each plaquette contains four Josephson junctions; similar behavior is found for a triangular lattice.)

If we consider an array of N cells, there are $2N$ phase difference variables θ_i and N such constraints, leaving N (not $2N$) degrees of freedom for fluctuations. If $f=B=0$, all $\theta_i=0$ in the ground state. Taking account of thermal fluctuations, the mean coupling energy per link $-E_J^0\cos\theta_i$ is reduced to $-E_J=-E_J^0\langle\cos\delta\theta\rangle$. When $kT \ll E_J^0$, one can evaluate $\langle\cos\delta\theta\rangle$ by using the equipartition theorem in N quadratic degrees of freedom, reproducing the known[9] result $E_J \cong E_J^0 - kT/4$.

In the presence of a magnetic field, $f \neq 0$, and the θ_i are constrained to take values differing from zero. For any single cell, the lowest free-energy configuration (at least for $|f| < \frac{1}{2}$) is seen to be that in which (1) is satisfied by $\theta_i = 2\pi(m-f)/4$ for all i, so that the phase gradient is uniform around the ring. In an array, this prescription is not consistent with the constraints on neighboring cells (frustration), and more general solutions must be found to minimize the free energy. However, an important general observation can be made *without* explicit solutions: Given a configuration of θ_i satisfying (1) for a given f, exactly the same θ_i (and hence free energies) obtain if f changes by an integer and all m values change by the same integer. Hence, apart from finite junction size effects which make E_J field dependent and cause the quadratic background in Fig. 2, the free energy of the system will be strictly periodic in f. Since energies do not depend on the *sign* of phase differences, the free energy must also be even in f. Accordingly, the entire periodic dependence can be found from the variation from $f=0$ to $\frac{1}{2}$, and we restrict our attention to that range.

If the constraint (1) is summed over all cells in an array, all θ_i on internal links cancel since each appears twice, with opposite sign. Thus, to avoid having a macroscopic circulating screening current, it must be that $\overline{m}=f$, where $\overline{m}$ is averaged over all cells. Since we expect minimum energy when $|f-m|$ is as small as possible (to minimize phase differences), this implies that for $0<f<\frac{1}{2}$, a fraction f of the cells have $m=1$ and a fraction $(1-f)$ have $m=0$.

When $f \ll 1$, each $m=1$ cell is surrounded by many $m=0$ cells, and we can treat the array as composed of blocks containing $1/f$ cells with an $m=1$ cell in the center. For simplicity, consider nested $p\times p$ square contours centered on the $m=1$ cell, of area p^2, where $p=1,3,5,\ldots$ as illustrated in Fig. 1(b). The phase sum $\sum\theta_i$ around such a contour is $2\pi(1-fp^2)$, as is seen by summing (1) for all cells within the contour, or $\theta_p=(2\pi/4p)(1-fp^2)$ per link, assuming all θ_i are the same in the pth contour. Thus, the total energy increase relative to the field-free ground state summed over the links in these contours is

$$\Delta E = E_J \sum_{p=1,3,5}^{p_{\max}} 4p(1-\cos\theta_p)$$

$$= 8E_J \sum_p p\sin^2[\pi(1-fp^2)/4p] \ . \qquad (2)$$

Note that $p_{\max} \cong f^{-1/2}$, in order that $\overline{m} = f$; accordingly, the terms in the sum corresponding to the outer contours in the block where p approaches $p_{\max}$ are small because $(1-fp^2) \rightarrow 0$, as well as because of the p in the denominator when the sine is expanded. Because of this property, our results are rather insensitive to how the blocks of cells fit together, so that square, round, hexagonal, or irregularly shaped blocks would make little difference. [The same consideration explains why the difference in free energy of square and triangular flux arrays in type-II superconductors is so very small ($\sim$2%).] Thus we expect our method to be reasonably accurate despite its crudeness. This is confirmed by the fact that the vortex energy between contours at p_1 and p_2 given by (2) in the field-free case ($f=0$) approaches $(\pi^2/4)E_J\ln(p_2/p_1)$ for $p_1,p_2 >> 1$, which differs from the exact result[10] for this case, $\pi E_J \ln(p_2/p_1)$, only by a factor of $\pi/4$.

We now observe that, although the long-range logarithmic energy effects are crucial for the ideal Kosterlitz-Thouless transition in zero field, for the case at hand the dominant energetic effect comes from the core plaquette because the range of the logarithm is cut off by the $(1-fp^2)^2$ factor in (2) to a $p_{\max} \cong 1/f^{1/2}$. Numerical examination of (2) shows that, even for f as small as 0.01, the core term contributes $\frac{2}{3}$ of the total energy shift, for $f=0.05$ the core contributes over 85%, while for $f \geqslant 0.1$ the core contribution is essentially 100%. Accordingly, apart from a small underestimate of the steepness of the initial rise within a few percent of the integer f values, the core contribution alone should give a good account of the experimentally observable effect of a magnetic field. Retaining only the core ($p=1$) term in (2), and normalizing to the total binding energy per vortex core, namely, $(2/f)E_J$, we obtain the average fractional reduction of the coupling energy by magnetic renormalization,

$$\overline{\Delta E}/E = 4f\sin^2\pi(1-f)/4 \ . \qquad (3)$$

The function (3) is not monotonic in the range $0 < f < \frac{1}{2}$, but rises linearly at first, reaches a maximum of 0.334 at $f=0.354$, and then drops to 0.293 at $f=\frac{1}{2}$. When reflected about $f=0$, and extended periodically, one obtains a form (see Fig. 4) which strikingly resembles the experimental data on $\Delta R(B)$ obtained in our laboratory (Fig. 2), and also by Webb *et al.*[2] on an array of tunnel junctions. In particular, (3) predicts a sharp dip by $\sim$12% to a secondary minimum at the half-integer values of f. Physically, this general shape results from the competition between a linear factor in f, proportional to the *number* of cores produced, and a factor falling roughly as $(1-f)^2$, which reflects the *reduction* of the energy shift for *each* core as more of the fluxoid quantum is taken up by flux, leaving less for phase difference across the junctions.

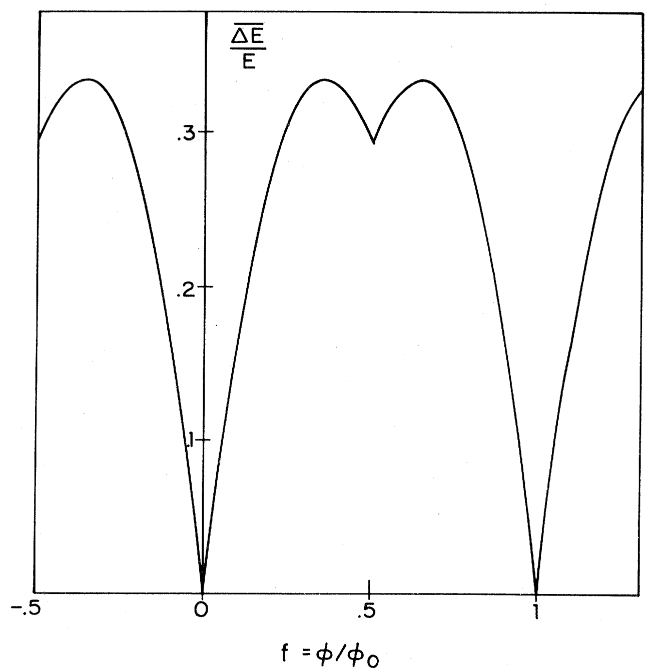

FIG. 4. Theoretical periodic dependence on magnetic flux per unit cell of fractional reduction in average coupling energy, as given by Eq. (3).

Although (3) considers only core energies, it is readily seen to be *exact* for the special case of $f=\frac{1}{2}$, where a simple "checkerboard" superlattice structure forms the ground state, and all links are core links for one, and only one, cell. As shown by Teitel and Jayaprakash,[3] more complex superlattice structures can be found to optimize the ground-state energies at other rational values of f. However, we argue that these higher superlattice effects only involve adjusting the small noncore energies, and hence that the resulting structure should be smaller than that given by (3). This conclusion is consistent with the results of the Teitel and Jayaprakash report of unpublished work, and with the predictions of Simonin, Wiecko, and Lopez,[11] for arrays of wire loops, both of which give results similar to our prediction (Fig. 4) apart from the addition of minor structure. Hence we expect that more complex superlattice effects will be much harder to observe.

Turning to the magnitude of the effect, we noted above that the measured amplitude of the periodic ΔR of one sample implied a periodic $\Delta T_c = 0.075$ K at $T_c \sim 4.2$ K. This amplitude ratio $\Delta T_c/T_c = 0.018$ is much less than our calculated $\overline{\Delta E}/E$, but that is expected because the exponential variation of the coupling energy with T in our proximity effect bridges reduces $\Delta T_c/T_c$ relative to the equivalent $\overline{\Delta E}/E$ by a factor of $(1+d/2\xi_N)^{-1}$, where d is the length of normal metal separating superconducting islands, and ξ_N is its coherence length. [Use of this conversion factor is essentially equivalent to use of the dimensionless temperature $T' \equiv kT/E_J(T)$ introduced in Ref. 10 to take account of the strong temperature dependence of the coupling energy.] For the measured parameters of our sample, the predicted $\overline{\Delta E}/E$ of ~ 0.3 scales down to a predicted $\Delta T_c/T_c$ of ~ 0.024, close to the observed value of 0.018. This observed value is actually a lower limit, since ΔR was found to be substantially larger in the limited amount of data taken at lower measuring currents. Considering the simplified nature of the model and the limitations of the available data, we consider this degree of consistency very satisfactory.

After this paper was submitted, B. Pannetier, J. Chaussy, and R. Rammal reported [J. Phys. (Paris) Lett. (in press)] experimental and theoretical work on a honeycomb lattice of superconducting wires, showing a feature at $f=\frac{1}{3}$. In our model, this feature arises for the honeycomb lattice (but not in square or triangular lattices) because in it, for $f>\frac{1}{3}$, adjacent cells must be occupied. When our summation of individual core energies is augmented by inclusion of the resulting clusters, a marked feature is found at $f=\frac{1}{3}$. Details will be reported in a later publication.

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[1]R. F. Voss and R. A. Webb, Phys. Rev. B 25, 3446 (1982). [Related, but distinct, effects had been observed earlier in perforated films by A. T. Fiory, A. F. Hebard, and S. Somekh, Appl. Phys. Lett. 32, 73 (1978).]

[2]R. A. Webb, R. F. Voss, G. Grinstein, and P. M. Horn, Phys. Rev. Lett. 51, 690 (1983).

[3]S. Teitel and C. Jayaprakash, Phys. Rev. B 27, 598 (1983); also (unpublished).

[4]D. W. Abraham, C. J. Lobb, M. Tinkham, and T. M. Klapwijk, Phys. Rev. B 26, 5268 (1982).

[5]The fact that $\Delta R(T)$ no longer follows dR/dT at the high-temperature end has a simple explanation. Just below T_{cs} the reduction in resistance is caused by a temperature-dependent spreading of the proximity effect superconductivity into the Cu surrounding the Pb islands, but without phase locking between islands. This effect is not sensitive to magnetic field induced phase shifts, so gives rise to no periodic resistance variation.

[6]W. A. Little and R. D. Parks, Phys. Rev. Lett. 9, 9 (1962).

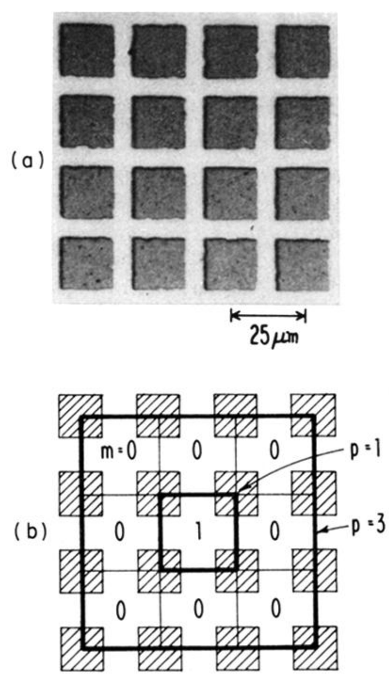
[7]M. Tinkham, Phys. Rev. 129, 2413 (1963).

[8]See, for example, M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1975), pp. 196–202.

[9]T. Ohta and D. Jasnow, Phys. Rev. B 20, 139 (1979).

[10]C. J. Lobb, D. W. Abraham, and M. Tinkham, Phys. Rev. B 27, 150 (1983).

[11]J. M. Simonin, C. Wiecko, and A. López, Phys. Rev. B 28, 2497 (1983).



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