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Random-field effects on the critical behavior of an interacting Bose model

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The critical properties of a d-dimensional Bose model in the presence of a random quenched field are investigated by renormalization-group techniques via the replica trick. To first order in $\epsilon=6-d$ it is found that both at nonzero- and zero-temperature regimes the grand-canonical critical exponents are the same as those of the corresponding classical model so that thermal and quantum fluctuations appear to be irrelevant. Furthermore, at zero temperature, the scaling laws already established for the pure model are retrieved with the replacement of d with $d-4$, but the random critical exponents are not derivable from the pure ones with the dimensional shift $d \to d-4$.

I. INTRODUCTION

In recent years, after a wide variety of investigations on critical phenomena in pure classical[1] and quantum[2–14] systems, great interest has been shown for exploring the the influence of random impurities which always exist in any real system. In particular, several results have been obtained which display drastic effects of random quenched fields on phase transitions. Although it is not easy to imagine experimental realizations of such random fields, it is quite possible to have situations where they appear physically realizable.[15] For example, it has been shown that random fields can be experimentally generated by the application of a uniform field on an dilute uniaxially anisotropic antiferromagnet with quenched random exchange interactions[16] or as a result of the random bilinear coupling of two spin components near the transition from a uniaxially ordered phase to a mixed one.[17] In both cases, transitions may occur at very low temperatures where quantum effects may be relevant.

From the theoretical point of view, the amount of results is, at present, rich enough for classical models in the presence of random fields. Renormalization-group (RG) investigations[18] and exact predictions to all orders in perturbation theory[19] indicate an effective lowering of the space dimensionality due to the presence of the random field which becomes the dominant cause of disorder near the transition point instead of the thermal fluctuations.

In contrast, the problem of the effects of random quenched fields on critical behavior in quantum systems is not sufficiently explored. Lacour-Gayet and Toulouse[20] studied the ideal Bose gas at nonzero temperature in the presence of a random field as a special type of disorder (random sources and sinks of superfluid particles). They found that space dimensionalities of four and six play special roles and that the usual scaling laws are violated, as for classical systems.[18,19] More recently, Aharony *et al.*[21] have investigated the critical properties of a d-dimensional zero-temperature transverse Ising model in a longitudinal random field. For this model the critical exponents are the same as those of the zero-random-field case in $d-3$ dimensions and are identical to those of the classical transition which occurs at finite temperature. Thus, in the presence of a random field, the quantum fluctuations[3–14] seem irrelevant in the same way as the thermal fluctuations. A peculiar aspect of the approach of Ref. 21 is that it is crucially based on the equivalence of the d-dimensional pure system at zero temperature with the corresponding classical model in $d+1$ dimensions. Such a situation is known to be true only for quantum models for which a "dimensional crossover"[3,6,13] occurs in the zero-temperature limit. This is not the case for "bosonized systems,"[13] such as the interacting Bose gas,[7,13] the X-Y model in a transverse field,[5,14] and the spin-$\frac{1}{2}$ planar ferromagnetic model,[9] which show, at zero temperature, a Gaussian behavior for all dimensionalities, though a stable nontrivial fixed point exists for $d<2$. This unusual situation is not directly connected with a classical problem by a dimensional crossover, even if a dimensional shift $d \to d+2$ occurs in the zero-temperature limit. Rather, it may be explained in terms of a more complex crossover phenomenon $(d, n=2) \to (d+2, n=-2)$ also involving the symmetry of the system[7,13] (n is the number of order-parameter components). Therefore, it may be of some interest to investigate the effects of a random field in this anomalous situation. This is just the purpose of the present paper on the basis of a RG treatment, via the replica trick,[22] for a functional representation of an interacting Bose model in the presence of a short-range correlated random field. Of course, by means of an appropriate reinterpretation of the parameters involved,[13] all the results can also be extended to the above-mentioned spin models.[5,7–9]

The starting point lies on a second-quantized Hamiltonian and it is therefore microscopic in nature. This assures the correct inclusion of the quantum degrees of freedom so that the full effects of the quantum nature of the model can be accurately explored and directly compared with the thermal fluctuations and those induced by the random field.

Concerning the use of the replica trick, we must remember that, though widely used, it sometimes has problems of its own. Apart from these peculiar problems, which are the object of recent investigation,[23,24] many studies have proved that this trick constitutes a useful technique to obtain in a simple way correct physical results for random systems. Nevertheless, some analytic difficulties emerge in the ordered phase and some caution must be used,[24] but this is not the objective of the present work.

The paper is organized as follows. In Sec. II we introduce the model and, as result of the replica trick, we derive an "effective action," which is the basis of our RG approach. Furthermore, the general temperature-dependent RG equations are presented. In Sec. III they are discussed and an investigation of the random critical properties at nonzero- and zero-temperature regimes to first order in $\epsilon=6-d$ is made. Finally, in Sec. IV some conclusions are drawn.

II. THE MODEL AND THE RG EQUATIONS VIA THE REPLICA TRICK

Let us consider a d-dimensional, $(n/2)$-component Bose model[25] in a quenched random field $h(\vec{x})\equiv\{h^j(\vec{x}); j=1,\ldots,n/2\}$ described by the second-quantized grand-canonical Hamiltonian[20,26]

$$\hat{H}=\sum_{j=1}^{n/2}\int d^dx\,\Psi^{j\dagger}(\vec{x})\left[-\frac{\hbar^2}{2M}\nabla^2+r_0\right]\Psi^j(\vec{x})+\frac{u_0}{4}\sum_{i,j=1}^{n/2}\int d^dx\,\Psi^{i\dagger}(\vec{x})\Psi^{j\dagger}(\vec{x})\Psi^i(\vec{x})\Psi^j(\vec{x})$$

$$+\sum_{j=1}^{n/2}\int d^dx[h^j(\vec{x})\Psi^{j\dagger}+h^{j*}(\vec{x})\Psi^j(\vec{x})]\ . \tag{1}$$

Here $\Psi^{j\dagger}(\vec{x})$ and $\Psi^j(\vec{x})$ $(j=1,\ldots,n/2)$ are the usual Bose fields (operators for different j are assumed to be commutable), M denotes the mass of Bose particles, $\mu=-r_0$ is the chemical potential, and $u_0>0$. Concerning the random field $h(\vec{x})$, which described the disorder effects in the system,[20] we assume that $h^j(\vec{x})$ $(j=1,\ldots,n/2)$ or their Fourier components $h^j_{\vec{k}}=V^{-1/2}\int d^dx\,h^j(\vec{x})e^{-i\vec{k}\cdot\vec{x}}$ (V is the volume of the system) are Gaussian random variables with short-range spatial correlations:

$$[h^{j*}_{\vec{k}}]_{av}=[h^j_{\vec{k}}]_{av}=0,\quad [h^{i*}_{\vec{k}}h^j_{\vec{k}'}]_{av}=\delta_{ij}\delta_{\vec{k}\vec{k}'}h_0^2\ . \tag{2}$$

In (2) $\vec{k}$ denotes a wave vector for which a cutoff Λ is assumed,[2,7] and the bracket $[\ ]_{av}$ indicates an average over possible configurations of the random field.

For our purposes, it is convenient to introduce a functional representation of the model. For a given configuration of the random field, we can write for the grand-canonical partition function

$$\mathscr{Z}\{h\}=\mathrm{Tr}\,e^{-\hat{H}/T}=\int\mathscr{D}[\phi]e^{-\mathscr{H}\{\phi,h\}}\ , \tag{3}$$

where the "action" $\mathscr{H}\{\phi,h\}$ is defined by[12]

$$\mathscr{H}\{\phi,h\}=\int d^dx\int_0^{1/T}d\tau\,\vec{\phi}^*(\vec{x},\tau)\cdot\frac{\partial\vec{\phi}(\vec{x},\tau)}{\partial\tau}+\int_0^{1/T}d\tau\,H(\tau)\ . \tag{4}$$

In Eqs. (3) and (4) $\vec{\phi}(\vec{x},\tau)\equiv\{\phi^j(\vec{x},\tau); j=1,\ldots,n/2\}$ is a complex field such that $\vec{\phi}(x,0)=\vec{\phi}(x,1/T)$, T is the temperature, and $H(\tau)$ is the classical functional corresponding to $\hat{H}$, depending on τ through the fields ϕ,ϕ^*.

From (1) it follows (in units for which $\hbar^2=2M=\Lambda=1$) that

$$\mathscr{H}\{\phi,h\}=\int d^dx\int_0^{1/T}d\tau\left[\,|\vec{\nabla}\vec{\phi}(\vec{x},\tau)|^2+r_0|\vec{\phi}(\vec{x},\tau)|^2+\vec{\phi}^*(\vec{x},\tau)\cdot\frac{\partial\vec{\phi}(\vec{x},\tau)}{\partial\tau}+\frac{u_0}{4}|\vec{\phi}(\vec{x},\tau)|^4\right.$$

$$\left.+[\vec{h}(\vec{x})\cdot\vec{\phi}^*(\vec{x},\tau)+\vec{h}^*(\vec{x})\cdot\vec{\phi}(\vec{x},\tau)]\right]\ . \tag{5}$$

Of course, since the disorder is quenched, the physical free energy of the system is defined by the average over the disorder[1]:

$$F=-T[\ln\mathscr{Z}\{h\}]_{av}\ . \tag{6}$$

By now using the replica trick,[22] for the averaged free energy we can write

$$F=-T\lim_{m\to 0}\frac{1}{m}\ln[\mathscr{Z}^m\{h\}]_{av}=-T\lim_{m\to 0}\frac{1}{m}\ln\mathscr{Z}_{eff}\ , \tag{7}$$

where

$$\mathscr{Z}_{\text{eff}}=\int \prod_{\alpha=1}^{m} \mathscr{D}[\phi_\alpha]e^{-\mathscr{H}_{\text{eff}}(\{\phi_\alpha\})} \tag{8}$$

and

$$e^{-\mathscr{H}_{\text{eff}}(\{\phi_\alpha\})}=\left[\prod_{\alpha=1}^{m} e^{-\mathscr{H}\{\phi_\alpha,h\}}\right]_{\text{av}} . \tag{9}$$

Thus the replica trick results in an effective action $\mathscr{H}_{\text{eff}}(\{\phi_\alpha\})$, which is a functional of m replications $\{\phi_\alpha;\ \alpha=1,2,\ldots,m\}$ of the original Bose field ϕ, connected with the physical properties of the random system by the appropriate limit (7).

Taking into account (2), one easily obtains

$$\mathscr{H}_{\text{eff}}(\{\phi_\alpha\})=\mathscr{H}^{(0)}_{\text{eff}}(\{\phi_\alpha\})+\mathscr{H}^{(I)}_{\text{eff}}(\{\phi_\alpha\}) , \tag{10}$$

where in the Fourier space

$$\mathscr{H}^{(0)}_{\text{eff}}(\{\phi_\alpha\})=\sum_{\alpha,\beta=1}^{m}\sum_{j=1}^{n/2}\sum_{\substack{q\\ 0<|\vec{k}|<1}} [(r_0+k^2-i\omega_l)\delta_{\alpha\beta}-(h_0^2/T)\delta\omega_{l,0}]\phi_\alpha^{j*}(q)\phi_\beta^{j}(q) , \tag{11}$$

$$\mathscr{H}^{(I)}_{\text{eff}}(\{\phi_\alpha\})=\frac{u_0T}{4V}\sum_{\alpha=1}^{m}\sum_{i,j=1}^{n/2}\sum_{\substack{\{q_\nu\}\\ 0<|\vec{k}_\nu|<1}} \phi_\alpha^{i*}(q_1)\phi_\alpha^{j*}(q_2)\phi_\alpha^{i}(q_3)\phi_\alpha^{j}(q_1+q_2-q_3) . \tag{12}$$

In (11) and (12) it is

$$\phi_\alpha^j(q)=\left[\frac{T}{V}\right]^{1/2}\int d^dx\int_0^{1/T}d\tau\,\phi_\alpha^j(\vec{x},\tau)e^{-i(\vec{k}\cdot\vec{x}-\omega_l\tau)} ,$$

$q\equiv(\vec{k},\omega_l)$, and $\omega_l=2\pi lT$ ($l=0,\pm1,\pm2,\ldots$) are the Matsubara frequencies. As we see, the effective action introduces in the problem an additional quadratic term which describes the fictitious interaction between two replicas caused via the random field. This allows us to apply RG techniques quite parallel to the pure system,[7,10] taking $m\to0$ in the final results. The quantum RG procedure, where also the Matsubara frequencies are to be scaled as $\omega_l'=b^z\omega_l \Longrightarrow T'=b^zT$ (b is the spacial rescaling factor, z is the dynamical critical exponent), is readily applied to the "effective problem." Nevertheless, together with the "usual" diagrammatic techniques, the results

$$\langle\phi_\alpha^{i*}(q)\phi_\beta^{j}(q')\rangle_0^{(m)}=\delta_{ij}\delta_{qq'}\frac{1}{r_0+k^2-i\omega_l}\left[\delta_{\alpha\beta}+\frac{(h_0^2/T)\delta\omega_{l,0}}{r_0+k^2-i\omega_l-m(h_0^2/T)\delta\omega_l,0}\right] , \tag{13}$$

$$G(q)=[\langle\phi^{j*}(q)\phi^{j}(q)\rangle]_{\text{av}}-[\langle\phi^{j*}(q)\rangle\langle\phi^{j}(q)\rangle]_{\text{av}}=\lim_{m\to0}\frac{1}{m}\sum_{\alpha,\beta=1}^{m}\langle\phi_\alpha^{j*}(q)\phi_\beta^{j}(q)\rangle^{(m)} , \tag{14}$$

are to be taken into account in the smoothing process and for the appropriate choice of the rescaling factor ζ_b $(=b^{-(2-\eta)/2})$ for the replica fields $\{\phi_\alpha\}$, respectively. In (13) and (14) $\langle\ \rangle^{(m)}$ denotes an average with respect to the effective action (10) and the zero index in (13) indicates an averages with respect to $\mathscr{H}^{(0)}_{\text{eff}}(\{\phi_\alpha\})$.

We find that the temperature-dependent finite RG recursion relations in the limit $m\to0$ for the parameters r_0,h_0^2,u_0, suitable for a discussion of the random critical properties to first order in $\epsilon=6-d$ ($\eta=0$, $z=2$), are

$$\begin{aligned} r_0'&=b^2\left[r_0+\frac{n+2}{4}[u_0f_1(r_0,T;b)+u_0h_0^2f_2(r_0;b)]+O(u_0^2,u_0^2h_0^2,u_0^2h_0^4,\ldots)\right] ,\\ (h_0^2)'&=b^4[h_0^2+O(u_0^2h_0^6,\ldots)] ,\\ u_0'&=b^{[4-(d+z)]}\left[u_0-\frac{u_0^2}{4}[(n+6)f_3(r_0,T;b)+2f_4(r_0,T;b)]-\frac{u_0^2h_0^2}{2}(n+8)f_5(r_0;b)+O(u_0^2h_0^4,\ldots)\right] ,\\ T'&=b^zT ,\end{aligned} \tag{15}$$

where

$$f_1(r_0,T;b)=K_d\int_{b^{-1}}^{1}dp\frac{p^{d-1}}{e^{(r_0+p^2)/T}-1},\quad f_2(r_0;b)=K_d\int_{b^{-1}}^{1}dp\frac{p^{d-1}}{(r_0+p^2)^2},$$
$$f_3(r_0,T;b)=K_d\int_{b^{-1}}^{1}dp\frac{p^{d-1}/4T}{\sinh^2[(r_0+p^2)/2T]},\quad f_5(r_0;b)=K_d\int_{b^{-1}}^{1}dp\frac{p^{d-1}}{(r_0+p^2)^3},\tag{16}$$
$$f_4(r_0,T;b)=\frac{K_d}{2}\int_{b^{-1}}^{1}dp\frac{p^{d-1}}{r_0+p^2}\coth\left[\frac{r_0+p^2}{2T}\right],$$

and $K_d=2^{1-d}\pi^{-d/2}/\Gamma(d/2)$.

Equations (15) show that for $d>4-z$, u_0 is an irrelevant variable, i.e., it decays to zero for $b\to\infty$, whereas r_0 and h_0^2 are relevant. However, the recursion relations for these depend on products such as $u_0h_0^2$ and it is important to extract from (15) information about the behavior of the variable $w_0=u_0h_0^2$ under the RG process. Therefore, one must consider the recursion relation for this product, i.e.,

$$w_0'=b^{6-d}\left[w_0-\frac{u_0w_0}{4}[(n+6)f_3(r_0,T;b)+2f_4(r_0,T;b)]-\frac{w_0^2}{2}(n+8)f_5(r_0;b)+O(w_0^3,\ldots)\right],\tag{17}$$

together with the recursion relations for r_0, u_0, and T.

It is now convenient to operate with the differential version of the previous relations. It is immediate to see that the T-dependent RG differential equations for the renormalized variables $r(l)$, $w(l)=u(l)h^2(l)$, $u(l)$, and $T(l)$, appropriate for the random problem to first order in $\epsilon=6-d$, are

$$\frac{dr}{dl}=2r+K_d\frac{n+2}{4}[uA(r,T)+wB(r)],$$
$$\frac{dw}{dl}=\epsilon w-\frac{K_d}{4}\{uw[(n+6)C(r,T)+2D(r,T)]+2(n+8)w^2E(r)\},\tag{18}$$
$$\frac{du}{dl}=(-4+\epsilon)u-\frac{K_d}{4}\{u^2[(n+6)C(r,T)+2D(r,T)]+2(n+8)uwE(r)\},$$
$$\frac{dT}{dl}=2T,$$

where l is the logarithm of the RG length-rescaling factor and

$$A(r,T)=\{\exp[(1+r)/T]-1\}^{-1},$$
$$B(r)=(1+r)^{-2},$$
$$C(r,T)=\frac{1}{4T}\sinh^{-2}[(1+r)/2T],\tag{19}$$
$$E(r)=(1+r)^{-3},$$
$$D(r,T)=\frac{1}{2}\frac{1}{1+r}\coth[(1+r)/2T].$$

Of course, Eqs. (18) are to be solved with the initial conditions $r(0)=r_0$, $w(0)=w_0=u_0h_0^2$, and $u(0)=u_0$.

III. CRITICAL PROPERTIES

Here we are interested in investigating Eqs. (18) in the two temperature regimes, $T\neq0$ and $T=0$, and in describing the critical properties in terms of the chemical potential, which is the natural variable in our microscopic grand-canonical formulation.

For $T\neq0$, the fixed point $T^*=\infty$ of the RG equation for the temperature is just approached for $l\to\infty$ because repeated iteration of the RG transformation will lead to

$$T(l)=Te^{2l}\to T^*=\infty \text{ as } l\to\infty .$$

Thus, for (19) with $l\gg1$, Eqs. (18) reduce to

$$\frac{dr}{dl}=2r+K_d\frac{n+2}{4}\left[\frac{v}{1+r}+\frac{w}{(1+r)^2}\right],$$
$$\frac{dw}{dl}=\epsilon w-K_d\frac{n+8}{4}\left[\frac{vw}{(1+r)^2}+2\frac{w^2}{(1+r)^3}\right],\tag{20}$$
$$\frac{dv}{dl}=(-2+\epsilon)v-K_d\frac{n+8}{4}\left[\frac{v^2}{(1+r)^2}+2\frac{vw}{(1+r)^3}\right],$$

where we have introduced the appropriate variable $v=uT$ with $w=v\lambda=uh^2$ $(\lambda=h^2/T)$ and $du/dl=(-2v+dv/dl)/T$. As we see, Eqs. (20) are just the RG equations already obtained for the corresponding classical random problem[18,21,27] without the use of the replica trick. Thus we can say that near the critical dimensionality $d_c=6$, the $T\neq0$ critical behavior of the quantum model under study, in terms of variable $r_0=-\mu^7$, is the same as that for the corresponding classical random n-vector model. This behavior can be analyzed as usual by linearizing Eqs. (20) near the fixed points $(r^*=w^*=v^*=0)$ and

$$\left[r^*=-\frac{(n+2)\epsilon}{4(n+8)},\ w^*=\frac{2\epsilon}{K_6(n+8)},\ v^*=0\right]$$

with $\epsilon=6-d$. In particular, one finds that it is Gaussian for $d>6$ and not Gaussian for $d<6$. However, since the classical RG analysis for Eqs. (20) is well known, here we do not enter into details. Nevertheless, due to the different meaning of the parameter r_0 which enters the grand-canonical Hamiltonian, we wish to point out explicitly that in our case the chemical potential assumes the same role of the temperature in the classical model. Therefore, its critical value for $T\neq 0$ assumes relevance. It can be obtained by the criticality condition $t_r^{(c)}(0)=0$, where

$$t_r = r - r^* + \mathscr{A}(w-w^*) + \mathscr{B}v \tag{21}$$

with

$$\mathscr{A}=\frac{K_6}{8}(n+2)\left[1+\frac{n-4}{n+8}\frac{\epsilon}{2}\right],$$

$$\mathscr{B}=\frac{K_6}{16}(n+2)\left[1+\frac{n-4}{n+8}\frac{\epsilon}{4}\right],$$

is the only relevant scaling field for $\epsilon=6-d>0$ in the linear analysis of Eqs. (20) near the random fixed point. We find to first order in ϵ

$$\mu_c(T)=\frac{K_6}{16}(n+2)\left[u_0T+2u_0h_0^2 + \frac{n-4}{4(n+8)}(u_0T+4u_0h_0^2)\epsilon\right] \tag{22}$$

and we can write $t_r(0)=r_0-r_{0_c}(T)=\mu_c(T)-\mu$. Thus the approach to the critical point in terms of the chemical potential is realized for $\mu\to\mu_c(T)$ and all the thermodynamic quantities will be expressed in terms of $\mu_c(T)-\mu$.[28] Finally, we observe that, since

$$\Delta\omega=2\pi T(l)=2\pi e^{zl}T\to \text{ as } l\to\infty ,$$

only the Matsubara frequency $\omega_0=0$ survives near the criticality and the original Bose problem reduces exactly to a classical problem. Therefore, the previous statement made for $T\neq 0$ remains true to arbitrary order in ϵ (Ref. 19) and, by approaching the criticality in terms of μ, the critical exponents and the scaling laws can be obtained from those of the pure case[7] (e.g., $2-\alpha=d\nu$) with the dimensional shift $d\to d-2$.

Let us consider now the $T=0$ regime where we might have quantum effects. In this limit Eqs. (18) reduce to

$$\frac{dr}{dl}=2r+K_d\frac{n+2}{4}\frac{w}{(1+r)^2},$$

$$\frac{dw}{dl}=\epsilon w-\frac{K_d}{4}\left[\frac{uw}{1+r}+2(n+8)\frac{w^2}{(1+r)^3}\right], \tag{23}$$

$$\frac{du}{dl}=(-4+\epsilon)u-\frac{K_d}{4}\left[\frac{u^2}{1+r}+2(n+8)\frac{uw}{(1+r)^3}\right].$$

As we see, some differences with respect to Eqs. (20) appear. In particular, it is evident that the presence of a random field introduces the order-parameter dimensionality in the $T=0$ Bose problem. It is known in fact that, in the zero random field, any dependence on n is absent in the quantum regime to arbitrary order in the perturbation series.[10] However, the peculiarity of the $T=0$ random problem is the coefficient $-4+\epsilon$ in the third RG equation in contrast with $-2+\epsilon$ in the corresponding classical equation. This is a manifestation of the dimensional shift $d\to d+2$ which occurs in the pure case when $T\to 0$.

By inspection of Eqs. (23), we find that the random fixed point exists:

$$r^*=-\frac{n+2}{4(n+8)}\epsilon,\quad w^*=\frac{2}{K_6(n+8)}\epsilon,\quad u^*=0, \tag{24}$$

which is the same as that at $T\neq 0$, except for the change of $u^*=0$ in $v^*=0$. The eigenvalues of the linearized RG equations about the Gaussian fixed point and the random fixed point and the corresponding linear scaling fields are, respectively,

$$\lambda_r^{(G)}=2,\quad \lambda_w^{(G)}=\epsilon,\quad \lambda_u^{(G)}=-4+\epsilon,$$
$$\lambda_r=2-\frac{n+2}{n+8}\epsilon,\quad \lambda_w=-\epsilon,\quad \lambda_u=-4, \tag{25}$$

$$t_r^{(G)}=r+a_Gw,\quad t_w^{(G)}=w,\quad t_u^{(G)}=u,$$
$$t_r=(r-r^*)+a(w-w^*)+bu, \tag{26}$$
$$t_w=(w-w^*)-cu,\quad t_u=u,$$

where

$$a_G=\frac{K_6}{8}(n+2)(1+\epsilon/2),\quad a\equiv\mathscr{A},$$
$$b=\frac{K_6}{48}\frac{n+2}{n+8}\frac{\epsilon}{2},\quad c=\frac{\epsilon}{8(n+8)}. \tag{27}$$

From (25) it immediately follows that the random fixed point and the Gaussian fixed point are stable for $\epsilon>0$ and $\epsilon<0$, respectively. Furthermore, the random critical surface for $d<6$ is determined by setting $t_r^{(c)}(0)=0$, and the $T=0$ critical chemical potential is given by

$$\mu_c=-r_{0c} =\frac{K_6}{8}(n+2)\left[u_0h_0^2 + \frac{1}{2(n+8)}\left[\frac{u_0}{6}+(n-4)u_0h_0^2\right]\epsilon\right]. \tag{28}$$

Of course, $t_r(0)=r_0-r_{0_c}=\mu_c-\mu$ measures the deviation from the criticality.

We are now, in a position to calculate the $T=0$ critical exponents. For example, for the correlation length $\xi\sim(\mu_c-\mu)^{-\nu}$, we find

$$\nu=1/\lambda_r=\tfrac{1}{2}[1+(n+2)/2(n+8)\epsilon]$$

for $d<6$, as for $T\neq 0$. It is also easy to verify that, as in the classical random problem,[18] the vanishing of u^*, which is a consequence of the irrelevance of u for $d>4-z$, is responsible for the failure of the $T=0$ scaling laws already found for the nonrandom Bose system.[7]

Let us consider the physical grand-canonical free-energy density at $T=0$ defined by

$$\mathscr{F}(r_0,w_0,u_0)=\lim_{\substack{T\to 0,\\ V\to\infty}}\left[\lim_{m\to 0}\frac{\mathscr{F}_{\text{eff}}}{m}\right], \tag{29}$$

where $\mathscr{F}_{\text{eff}}=-(T/V)\ln\mathscr{Z}_{\text{eff}}$. Then it is immediate to show that $\mathscr{F}$, under the RG transformation, scales as

$$\mathscr{F}(r_0,w_0,u_0)=e^{-(d+2)l}\mathscr{F}(r(l),w(l),u(l)) \ . \tag{30}$$

Thus, near the criticality and in terms of the linear scaling fields, the singular part of $\mathscr{F}$ scales as

$$\mathscr{F}_s(t_r^0,t_w^0,t_u^0)\approx e^{-(d+2)l}\mathscr{F}_s(t_r^0e^{\lambda_r l},t_w^0e^{\lambda_w l},t_u^0e^{\lambda_u l})$$
$$\approx(r_0-r_{0c})^{(d+2)\nu}f\left[\frac{t_w^0}{(r_0-r_{0c})^{\phi_w}},\frac{u_0}{(r_0-r_{0c})^{\phi_u}}\right] \tag{31}$$

due to the arbitrariness of the rescaling parameter $l\gg1$. In (31) $f(x,y)=\mathscr{F}_s(1,x,y)$, $t_j^0=t_j(0)$ $(j=r,w,u)$, and

$$\phi_w=\frac{\lambda_w}{\lambda_r}\simeq-\frac{\epsilon}{2},\quad \phi_u=\frac{\lambda_u}{\lambda_r}\simeq-2\left[1+\frac{n+2}{2(n+8)}\epsilon\right], \tag{32}$$

are the "crossover exponents" associated with the irrelevant fields t_w^0 and $t_u^0=u_0$. Observe now that if $f(x,y)$ were regular for $x\to0$, $y\to0$, for the critical exponent α defined by

$$\frac{\partial^2\mathscr{F}_s}{\partial r_0^2}\sim(r_0-r_{0c})^{-\alpha} \ ,$$

we should have the scaling law $2-\alpha=(d+2)\nu$ already obtained for the pure model at $T=0$.[7] To see that this does not occur in the random case, making use of "traditional diagrammatic" techniques, we can construct the cumulant perturbation series for $\mathscr{F}$:

$$\mathscr{F}=\mathscr{F}_0-\sum_{\nu=1}^{\infty}\frac{(-1)^\nu}{\nu!}\lim_{\substack{T\to0,\\ V\to\infty}}\frac{T}{V}\left[\lim_{m\to0}\frac{\langle(\mathscr{H}_{\text{eff}}^{(I)})^\nu\rangle_{0,c}^{(m)}}{m}\right] . \tag{33}$$

For instance, to first order we find

$$\langle\mathscr{H}_{\text{eff}}^{(I)}\rangle_0^{(m)}=m\frac{u_0}{16}(n+2)n\frac{V}{T}\times\left[\frac{1}{V}\sum_{\vec{k}}\frac{1}{e^{(r_0+K^2)/T}-1}+\frac{h_0^2}{V}\sum_{\vec{k}}\frac{1}{(r_0+k^2)(r_0+k^2-mh_0^2/T)}\right]^2 \tag{34}$$

and the corresponding contribution to $\mathscr{F}$ behaves as $u_0h_0^4=u_0^{-1}w_0^2$. An analogous calculation for higher terms in series shows that, for finite w_0 and infinitesimal u_0, the dominant contributions behave as u_0^{-1} as well. This indicates that, after iteration of the RG procedure, near the random fixed point where $r\simeq r^*\neq0$, $w\simeq w^*\neq0$, and $u\simeq u^*=0$, the singular part of $\mathscr{F}$, as a function of r,w,u, behaves as u^{-1} for $u\to0$. This allows us to conclude that in (31) $f(x,y)\simeq f(0,y)\sim y^{-1}$ for $x\to0$ and $y\to0$ and the revised scaling law follows:

$$2-\alpha=[(d+2)-4]\nu$$
$$=[(d-4)+2]\nu \ , \tag{35}$$

i.e., in the quantum pure relation d is replaced by $d+\lambda_u=d-4$. Similarly, one can show that the other $T=0$ scaling laws for the pure Bose model[7] are modified in an identical way.

The previous discussion indicates that, for a Bose system at $T=0$, the effects of the introduction of a random field are as follows:

(i) to change drastically the Gaussian behavior found for all dimensionalities in the pure case[7,10];

(ii) to shift the upper critical dimensionality from 2 to 6;

(iii) to convert d in the scaling laws of the nonrandom case to $d+\lambda_u=d-4$.

Of course, the knowledge of ν, η, and λ_u is sufficient to determine all the relevant random critical exponents, which result in the same way as those of the $T\neq0$ case.

IV. CONCLUSIONS

From our investigation based on a RG treatment via the replica trick, some conclusive remarks can be drawn. Firstly, at $T\neq0$, the random quantum model is equivalent to a classical n-vector model in a random quenched field and the critical exponents can be obtained from those for a pure system with the dimensional shift $d\to d-2$. Furthermore, in agreement with the results of Ref. 21 obtained for the transverse Ising model with a different approach, also for random Bose model at $T=0$, the critical exponents are the same as those of the transition occurring at finite temperature. Therefore the presence of a random field destroys the classical to quantum crossover for $T\to0$ which occurs, in contrast, when the field is absent.[12] Thus, in any case, the quantum fluctuations, in the same way as the thermal fluctuations, seem to be irrelevant relative to those caused by the random field.

However, we wish to point out a substantial difference existing between the transverse Ising model and the Bose model in the $T=0$ regime when a random field is present. Whereas in the first case the critical exponents are derivable from those of the pure model at $T=0$ with the dimensional shift $d\to d-3$, the $T=0$ critical exponents for a random Bose model cannot be obtained from the pure exponents (which are mean-field-like) with the dimensional shift $d\to d-4$. This is due to the anomalous behavior of the pure model at $T=0$, which is Gaussian also for $d<2$ [to arbitrary order in $\epsilon=2-d$ (Ref. 10)] though a stable non-Gaussian fixed point exists. This aspect of the problem is characteristic of all bosonized systems[7,13] where, when pure, no dimensional crossover occurs for $T\to0$ and corresponds to the peculiar anisotropy existing between the wave-vector components and the Matsubara frequencies in the quantum action. In this connection we speculate that, in contrast, for pure quantum systems with dimensional crossover $d\to d+z$ when $T\to0$,[3,6,13] the introduction of a random field at $T=0$ generates an "inverse dimensional crossover $d\to d-(2+z)$," that is, in the pure exponent d is replaced by $d-(2+z)$.

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