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Decay of the supercurrent in tunnel junctions

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The current state in a Josephson junction is metastable. Thermal and quantum fluctuations destroy such a state and make its lifetime finite. By lowering the temperature the role of quantum fluctuations increases. If there is a gap in the excitation spectrum, then for quantum tunneling high-frequency processes (pair decay) are essential. Quantum tunneling is connected with a change of a large amount of electron states.

I. INTRODUCTION

A current state of a Josephson junction corresponds to a minimum of the free energy $U(\varphi)$ as a function of the phase difference between the two superconductors. Such minima are divided by a potential barrier. The lifetime of such a state is finite. At not too low temperatures such a current state decays due to thermal fluctuations. In this case the lifetime is proportional to $\exp(\Delta U/T)$ [where ΔU is the height of the potential barrier $U(\varphi)$].[1,2] At low temperature the quantum tunneling through the barrier becomes important. For a sufficiently large capacitance of the junction the time of tunneling through the barrier is large. In this case it is possible to use the adiabatic approximation for the potential $U(\varphi)$ and the tunneling probability can be found by usual quantum-mechanical formulàs.[3] In several experiments[4,5] a junction with a small capacitance was used. Quantum-mechanical tunneling in such a case is defined not only by one coordinate (phase difference φ), but by a large number of electron states. In this case both real and virtual processes are important.

In this work it is found that, after averaging over the electron states, the effective potential is a retarded potential. This potential is found and the problem of quantum-mechanical tunneling through such a retarded potential is solved. If the junction is shunted by a normal resistance or the superconductors have a large concentration of paramagnetic impurities then electron excited states with small energies are important and they lead to friction in classically allowed regions. The problem of quantum-mechanical tunneling in a system with friction was solved phenomenologically by Caldeira and Leggett.[6] Microscopic extension of the effective action was made in a work by Ambegaokar *et al.*,[7] but in this work the statistical sum or a transitional amplitude was averaged over the electron states. The probability is not equal to the modulus square of such averaged amplitude. In this paper we have found the transition probability averaged over the electron states.

For superconductors with large concentrations of paramagnetic impurities or in the case in which the junction is shunted by a normal resistor, the result obtained here coincides with the phenomenolgical result of Ref. 6 when a current is near its critical value.

For superconductors with a gap in the excitation spectrum, the resistance for a quasiparticle current at low temperature is exponentially large;[8] therefore, the nonadiabatic tunnel processes in this case are connected to the virtual decay of pairs. The reader is also referred to the previous results obtained by Widom *et al.*[9]

II. AVERAGE OF THE TRANSITION PROBABILITY ON THE ELECTRON STATES

A Josephson junction can be described by the Hamiltonian

$$\hat{H}=\hat{H}_L+\hat{H}_R+\hat{H}_T+\hat{Q}^2/2C \ , \tag{1}$$

where $\hat{H}_L,\hat{H}_R$ are the Hamiltonians of the left and right superconductors, C is the junction capacitance

$$\hat{H}_L=\int d^3r\, \psi^\dagger_{L\sigma}(r)\left[-\frac{1}{2m}\frac{\partial^2}{\partial r^2}-\mu\right]\psi_{L\sigma}(r) - \frac{g_L}{2}\int d^3r\, \psi^\dagger_{L\sigma}(r)\psi^\dagger_{L\sigma}(r)\psi_{L-\sigma}(r)\psi_{L-\sigma}(r) \ , \tag{2}$$

$\psi_{L\sigma}(r)$ is the annihilation operator for electrons with spin σ,

$$\hat{H}_T=\int d^3r_L d^3r_R[\hat{T}(r_L,r_R)\psi^\dagger_{L\sigma}(r_L)\psi_{R\sigma}(r_R)+\text{c.c.}] \ ,$$
$$\hat{Q}=\hat{Q}_L-\hat{Q}_R, \quad \hat{Q}_L=e\int d^3r\, \psi^\dagger_{L\sigma}(r)\psi_{L\sigma}(r) \ , \tag{3}$$

C is the capacitance of the tunnel junction. If at a time $-T$ the system is in a state i then the probability to find the system in a state f at $t=0$ is equal to

$$W_i^f = |\langle \psi^f | S | \psi^i \rangle|^2 ,$$
$$S = \exp\left[-i\int_{-T}^{0} \hat{H}(t)dt\right] . \tag{4}$$

We shall use the Habbard-Stratonowich procedure to eliminate the term ψ^4 in the Hamiltonian. In analogy with the work[7] for the statistical sum, we obtain, for S' matrix,

$$S = \int \mathscr{D}^2\Delta_L \mathscr{D}^2\Delta_R \mathscr{D} V \hat{T} \exp\left[-i\int_{-T}^{0} \hat{H}_{\text{eff}}(t)dt\right] , \tag{5}$$

where $\Delta_{L,R}$ are complex functions,

$$\hat{H}_{\text{eff}} = \hat{H}_T - \frac{C}{2}V^2(t) + \hat{Q}V(t) + (\hat{H}_L)_{\text{eff}} + (\hat{H}_R)_{\text{eff}} ,$$

$$(\hat{H}_L)_{\text{eff}} = \int d^3r\, \psi^\dagger_{L\sigma}\left[-\frac{1}{2m}\frac{\partial^2}{\partial r^2} - \mu\right]\psi_{L\sigma}$$
$$+ \int d^3r [\Delta_L^*(r,t)\psi_{L\downarrow}(r)\psi_{L\uparrow}(r) + \text{c.c.}]$$
$$+ \frac{1}{g_L}\int d^3r\, |\Delta_L(r,t)|^2 .$$

We are interested in the probability w_i^f which is averaged over the initial and summed over the final electron states. In the zero approximation on barrier transparency, the functional integral over modulus $|\Delta|$ can be obtained by WKB method. In this case the modulus $|\Delta|$ is replaced by its equilibrium value which is independent of time and coordinates. In the same approximation

$$eV(t) = \frac{\partial\varphi(t)}{\partial t} , \tag{6}$$

where φ is the phase difference between the two superconductors. As a result, we have a functional integral over the phase $\varphi(t)$, which, in the interaction representation, has the form

$$W_i^f = \int \mathscr{D}\varphi(t) W[\varphi] \exp\left[i\int_{\circlearrowleft} dt \frac{C}{2e^2}\left[\frac{\partial\varphi}{\partial t}\right]^2\right] ,$$
$$W[\varphi] = \left\langle\left\langle \hat{T}_c \exp\left[-i\int_{\circlearrowleft} \hat{H}(t)dt\right]\right\rangle\right\rangle . \tag{7}$$

Here the double angular brackets indicate the averaging over the states of the Hamiltonian $(H_L)_{\text{eff}} + (H_R)_{\text{eff}}$. The integral $\int_{\circlearrowleft}$ stands for integration over the Keldish contour, that is, going from ($-T$ to 0) and back from (0 to $-T$). $\hat{T}_c$ is the ordering operator on this contour.[10] In Refs. 11 and 12 the transition probability was written as a function integral. In the problem of the Josephson junction in an external electrical field,[8,13] the phase $\varphi(t)$ on the two sides of the Keldish contour coincide. By taking into account the small difference of these two values in the classically permitted regions it is possible to obtain the Langevin equation together with the correlation function of random forces.[12] A completely different picture arises for the motion in the classical forbidden regions. Let us call the phase value on the upper contour $\varphi_1(t)$ and on the lower contour $\varphi_2(t)$. For small barrier transparency the value $W[\varphi]$ is given by

$$W[\varphi] = \exp\left[-\frac{1}{2}\left\langle\left\langle \hat{T}_c \int_{\circlearrowleft}\int_{\circlearrowleft} dt\, dt_1 \hat{H}_t(t)\hat{H}_t(t_1)\right\rangle\right\rangle\right] . \tag{8}$$

The averaging in formula (8) leads to the appearance of the Green functions of the left and right superconductors for zero barrier transparency. Therefore, the potential $eV(t) = \partial\varphi/\partial t$ leads just to a trivial phase factor. Separating this phase factor we obtain

$$\ln(W[\varphi]) = \int_{\circlearrowleft}\int_{\circlearrowleft}\sum_{\nu,\mu} |T_{\mu\nu}|^2 \{ \cos[\varphi(t)-\varphi(t_1)] \langle \hat{T}_c a_\nu(t) a_\nu^\dagger(t_1)\rangle \langle \hat{T}_c a_\mu(t_1) a_\mu^\dagger(t)\rangle$$
$$- \cos[\varphi(t)+\varphi(t_1)] \langle T_c a_\nu(t) a_\nu(t_1)\rangle \langle T_c a_\mu(t_1) a_\mu(t)\rangle \} dt\, dt_1 , \tag{9}$$

where the index ν,μ labels states in left and right superconductors, respectively. The average in formula (9) is made without the potential $V(t)$ using real values of the order parameters $\Delta_{L,R}$.

The Green functions in formula (9) depend only on the energy of ν,μ states and have a sharp maximum near the Fermi surface. The matrix element $|T_{\mu\nu}|^2$ averaged over these states can be expressed in terms of the junction resistance in the normal state. The sum over the indexes μ,ν in formula (9) leads to the Green function integrated with respect to the energy ξ.

We divide the integration contour in two parts. For all the quantities laying on the upper (lower) contour we shall write the subscript 1 (2). Then it follows

$$\ln(W[\varphi]) = \frac{\pi}{2R_N e^2}\int\int_{-T}^{0} dt\, dt_1 \sum_{i,K=1,2} (-1)^{i+K} \{ K_{ik}(t-t_1)\cos[\varphi_i(t)-\varphi_K(t_1)] - L_{ik}(t-t_1)\cos[\varphi_i(t)+\varphi_K(t_1)] \} . \tag{10}$$

In formula (10)

$$K_{ik}(t)=g_{ik}^{(L)}(t)g_{Ki}^{(R)}(-t)\ ,$$
$$L_{ik}(t)=F_{ik}^{(L)}(t)F_{Ki}^{(R)}(-t)\ , \quad (11)$$

where

$$g_{ik}=\frac{i}{\pi}\int d\xi\, G_{ik}\ ,$$
$$G_{11}(t,t_1)=-i\langle \hat{T}\psi(t)\psi^\dagger(t_1)\rangle\ ,$$
$$G_{12}(t,t_1)=i\langle \psi^\dagger(t_1)\psi(t)\rangle\ , \quad (12)$$
$$G_{21}(t,t_1)=-i\langle \psi(t)\psi^\dagger(t_1)\rangle\ ,$$
$$G_{22}(t,t_1)=-i\langle \hat{T}^{-1}\psi(t)\psi^\dagger(t_1)\rangle\ .$$

Similar formulas determine the Gor'kov Green function F. The Green functions g_{ik} are connected with the retarded g^R, advanced g^A, and Keldish g^K functions by

$$g_{11}=g=\tfrac{1}{2}[g^K+(g^R+g^A)]\ ,$$
$$g_{22}=\tilde{g}=\tfrac{1}{2}[g^K-(g^R+g^A)]\ ,$$
$$g_{12}=g^<=\tfrac{1}{2}[g^K-(g^R-g^A)]\ , \quad (13)$$
$$g_{21}=g^>=\tfrac{1}{2}[g^K+(g^R-g^A)]\ .$$

In thermal equilibrium the Keldish Green function g^K is

$$g^K(\epsilon)=\tanh(\epsilon/2T)[g^R(\epsilon)-g^A(\epsilon)]\ . \quad (14)$$

Usually the Josephson junction is inserted in an electrical circuit which fixes the total current through the junction. For instance, large inductance at the external electrical circuit can fix the current. In this case it is necessary to add the magnetic energy $\mathcal{L}\mathcal{I}^2/2$ to the Hamiltonian (1). The phase difference on the contact is connected with the full magnetic flux. As a result, in the effective Hamiltonian will appear an additional term $-\varphi\mathcal{I}/e$. Then the transition probability W_i^f is

$$W_i^f=\int \mathcal{D}\varphi_1\mathcal{D}\varphi_2\exp\left\{\ln(W[\varphi])-i\int_{-T}^{0}dt\sum_{K=1,2}(-1)^K\left[\frac{C}{2e^2}\left[\frac{\partial\varphi_K}{\partial t}\right]^2+\frac{\mathcal{I}}{e}\varphi_K\right]\right\}\ . \quad (15)$$

III. TRANSITION TO IMAGINARY TIME

For a current value $\mathcal{I}$ lower than the critical one, the Josephson junction can remain a long time in a metastable state. We shall suppose that in this time the system can reach thermal equilibrium. In the following we confine ourselves within the limit of very low temperatures: The initial state i is a metastable state with minimum energy $\epsilon_{\rm min}$. For the sake of simplicity we put this energy equal to zero.

In the classically permitted regions, each state corresponds to a function $\varphi(t)$ which provides an extremum for the integral in the exponent of formula (15). If for the transition from i to f there is no extremal trajectory in real time, then the probability of such a transition is exponentially small and determined by the trajectory in the imaginary time. To find such a trajectory we shall continue the kernels K_{ik} and L_{ik} in the complex plane in time.

For temperature equal to zero the Green functions g and F have the following property: The function $g<(t)$ is an analytical function in the upper half-plane, while the function $g>(t)$ is analytical in the lower half-plane. The Green function $g(t)$ for $t>0$ coincides with the function $g>(t)$ and, for $t<0$, coincides with the function $g<(t)$. The Green function $\tilde{g}(t)$ for $t>0$ coincides with $g<(t)$ and for $t<0$ with $g>(t)$. Therefore, the Green functions $g(t),\tilde{g}(t)$ must be continued separately from the value $t>0$ and $t<0$. As a result of such continuation for the functions $g(\tau)$ and $\tilde{g}(\tau)$ on the imaginary axis we obtain

$$\begin{aligned} g(\tau)&=\tilde{g}(\tau)\\ &=-\mathrm{sgn}\tau\int_0^\infty\frac{d\epsilon}{2\pi}[g^R(\epsilon)-g^A(\epsilon)]\exp(-\epsilon\,|\tau|\,)\\ &=\begin{cases} g^<(\tau), & \tau>0\\ g^>(\tau), & \tau<0.\end{cases}\end{aligned} \quad (16)$$

The functions g and $\tilde{g}$ coincide with the Matsubara Green functions in the limiting case of zero temperature. Analogously for the Green function F on the imaginary axis we obtain

$$F(\tau)=\int_0^\infty\frac{d\epsilon}{2\pi}[F^R(\epsilon)-F^A(\epsilon)]\exp(-\epsilon\,|\tau|\,)\ . \quad (17)$$

After such continuation the first term in formula (10) can be written in the form

$$\begin{aligned} -\frac{\pi}{R_Ne^2}\Bigg[&\int\int_0^\infty d\tau\, d\tau_1 g_L(\tau-\tau_1)g_R(\tau-\tau_1)\sin^2\left[\frac{\varphi_1(\tau)-\varphi_i(\tau_1)}{2}\right]\\ &+\int\int_{-\infty}^0 d\tau\, d\tau_1\tilde{g}_L(\tau-\tau_1)\tilde{g}_R(\tau-\tau_1)\sin^2\left[\frac{\varphi_2(\tau)-\varphi_2(\tau_1)}{2}\right]\\ &-\int_0^\infty d\tau\int_{-\infty}^0 d\tau_1 g_L^<(\tau-\tau_1)g_R^>(\tau_1-\tau)\sin^2\left[\frac{\varphi_i(\tau)-\varphi_2(\tau_1)}{2}\right]\\ &-\int_{-\infty}^0 d\tau\int_0^\infty d\tau_1 g_L^>(\tau-\tau_1)g_R^<(\tau_1-\tau)\sin^2\left[\frac{\varphi_2(\tau)-\varphi_i(\tau_1)}{2}\right]\Bigg]\ .\end{aligned} \quad (18)$$

The two functions $\varphi_{1,2}$ can be replaced by one function $\varphi(\tau)$ coinciding with φ_1 in the half-space $(0,\infty)$ and with φ_2 in half-space $(-\infty,0)$. From formulas (10) and (16)–(18) for the probability W_i^f we obtain

$$W_i^f=\int \mathscr{D}\varphi(\tau)\exp\{-A[\varphi(\tau)]\}\ ,$$
$$A[\varphi(\tau)]=\int_{-\infty}^{\infty}d\tau\left[\frac{C}{2e^2}\left[\frac{\partial\varphi}{\partial\tau}\right]^2-\frac{\mathscr{I}}{e}\varphi(\tau)-\epsilon_0-\frac{\pi}{2R_Ne^2}\int_{-\infty}^{\infty}d\tau_1[F_L(\tau-\tau_1)F_R(\tau-\tau_1)\cos[\varphi(\tau)+\varphi(\tau_1)]\right.$$
$$\left.-2g_L(\tau-\tau_1)g_R(\tau-\tau_1)\sin^2\left[\frac{\varphi(\tau)-\varphi(\tau_1)}{2}\right]\right]\right]\ . \tag{19}$$

At zero temperature the system is in a state with the fixed phase φ for a long time. The nonlocality of interaction in time is not essential in this case, and the energy of the initial state can be determined.

The energy ϵ_0 in formula (19) can be found from the condition that the energy in the initial state is zero. The exact exponential transition probability W_i^f can be found from the extremum of the functional $A[\varphi(\tau)]$,

$$W_i^f=\exp\{-A[\varphi_{\text{ext}}(\tau)]\}\ . \tag{20}$$

Formula (20) is valid, provided $A\gg 1$, i.e., if the current $\mathscr{I}$ is not too close to the critical current $\mathscr{I}_c$.

The function $\varphi_{\text{ext}}(\tau)$ can be found from the following equation:

$$\frac{\delta A[\varphi(\tau)]}{\delta\varphi(\tau)}=0\ . \tag{21}$$

For superconductors without paramagnetic impurities from Eqs. (16) and (17) we get

$$g(\tau)=-\frac{\text{sgn}\tau}{\pi}\int_{\Delta}^{\infty}\frac{d\epsilon\,\epsilon}{(\epsilon^2-\Delta^2)^{1/2}}\exp(-\epsilon|\tau|)$$
$$=-\frac{\Delta\,\text{sgn}\tau}{\pi}K_1(\Delta|\tau|)\ ,$$
$$F(\tau)=\frac{\Delta}{\pi}\int_{\Delta}^{\infty}\frac{d\epsilon}{(\epsilon^2-\Delta^2)^{1/2}}\exp(-\epsilon|\tau|)$$
$$=\frac{\Delta}{\pi}K_0(\Delta|\tau|)\ . \tag{22}$$

It follows from formula (22) that in the presence of a gap in the excitation spectrum, the Green functions $g(\tau)$ and $F(\tau)$ decrease exponentially at $\tau\Delta\gg 1$. As is shown below, in the vicinity of $\mathscr{I}_c$ this results in the capacity renormalization. At low temperatures the residual contact resistance is exponentially high in this case.

In some cases a Josephson junction is shunted by a normal resistance R_0. In this case the normal resistance R_N should be replaced by $\widetilde{R}_N$,

$$\widetilde{R}_N^{-1}=R_N^{-1}-R_0^{-1}\ , \tag{23}$$

which defines the critical-current value. Moreover in this case to the action A should be added the quantity

$$\frac{1}{\pi R_0e^2}\int\int_{-\infty}^{\infty}d\tau\,d\tau_1\frac{1}{(\tau-\tau_1)^2}\sin^2\left[\frac{\varphi(\tau)-\varphi(\tau_1)}{2}\right]\ . \tag{24}$$

Formula (24) has been obtained from the general expression (19), in which the Green function g is substituted by its value in a normal metal. We believe that in a shunt no gap exists in the excitation spectrum.

For currents near the critical one the phase changes in a small region near the value $\pi/4$. For sufficiently large capacitance the phase φ changes slowly in comparison with Δ^{-1}. Therefore, in a first adiabatic approximation it is possible to change $\varphi(\tau_1)$ in formula (19) by $\varphi(\tau)$. Then the action $A[\varphi]$ is equal,

$$A_0[\varphi]=\int_{-\infty}^{\infty}d\tau\left[\frac{C}{2e^2}\left[\frac{\partial\varphi}{\partial\tau}\right]^2-\frac{\mathscr{I}}{e}\varphi(\tau)-\frac{\mathscr{I}_c}{2e}\cos[2\varphi(\tau)]-\epsilon_0\right]\ . \tag{25}$$

Near the critical current the minimum of the functional (25) is obtained by the function $\varphi(\tau)$,

$$\varphi(\tau)-\varphi_i=\frac{3(1-x^2)^{1/2}}{2}\frac{1}{\cosh^2(\omega_0\tau/2)}\ , \tag{26}$$

and is equal to

$$A_0=\frac{C}{e^2}\int_{-\infty}^{\infty}d\tau\left[\frac{\partial\varphi}{\partial\tau}\right]^2$$
$$=\frac{6}{5}(1-x^2)^{5/4}\left[\frac{2C\mathscr{I}_c}{e^3}\right]^{1/2}\ ,$$

where

$$x=\mathscr{I}/\mathscr{I}_c,\quad \varphi_i=\tfrac{1}{2}\sin^{-1}x\ ,$$
$$\omega_0=(1-x^2)^{1/4}\left[\frac{2e\mathscr{I}_c}{C}\right]^{1/2}\ . \tag{27}$$

To first order on the adiabatic parameter the value of the functional $A[\varphi]$ can be found by inserting the zero solution (26) into (19)

$$A[\varphi]=A_0\left[1+\frac{45\zeta(3)}{2\pi^3R_0C\omega_0}+\frac{3\times 2^{1/2}\pi(\Delta_L\Delta_R)^2}{16R_NC(\Delta_L^2+\Delta_R^2)^{5/2}}\times F\left[\frac{5}{4},\frac{7}{4},2,\left[\frac{\Delta_L^2-\Delta_R^2}{\Delta_L^2+\Delta_R^2}\right]^2\right]\right]\ . \tag{28}$$

The second term in formula (28) is connected with low-frequency dissipation during the tunneling process. This term coincides with the value obtained in Ref. 6. If the shunt resistance is of the order of the normal resistance R_N and the adiabatic parameter Δ/ω_0 is much larger than unity, then this term gives the main correction. Shunt resistance can be larger than R_N. In this case high-frequency processes with frequency $\omega \sim 2\Delta$ are important. The last term in (28) is determined by such a process. High-frequency processes can give essential contribution to the transition probability, but their contribution to the viscosity in classically permitted regions is exponentially small. If the adiabatic parameter is not small, then the minimum of the action can be found from the solution of the integral equation (21).

In Ref. 14 the action was found by a variational method. The viscosity was taken into account only by the low-frequency contribution in the action [formula (24)]. Therefore, the result of Ref. 14 is valid only if the shunt resistor is small with respect to R_N.

In Ref. 5 shunting resistance was in some cases larger than R_N, and the capacitance was small. In this case the term with capacitance in Eq. (19) may be omitted and the high-frequency processes become essential. An order of magnitude of the action in this case is given by

$$A \sim (1-x^2)^{5/4}(\mathscr{I}_c/\Delta e) . \tag{29}$$

For $\mathscr{I}$ near the critical value $\mathscr{I}_c$ the integral equation (21) becomes a differential one, and for arbitrary value of the capacitance C, we have

$$A[\varphi]=\tfrac{6}{5}(1-x^2)^{5/4}\left[\frac{2\mathscr{I}_c C^*}{e^3}\right]^{1/2} ,$$
$$C^* = C + \frac{3\times 2^{1/2}\pi(\Delta_L\Delta_R)^2}{8R_N(\Delta_L^2+\Delta_R^2)^{5/2}} F\left[\frac{5}{4},\frac{7}{4},2,\left[\frac{\Delta_L^2-\Delta_R^2}{\Delta_L^2+\Delta_R^2}\right]^2\right] . \tag{30}$$

Thus we see that in a large temperature region, $T \sim T_c$, the quantum-tunneling probability is of the same order of probability to overcome the barrier with the help of thermal fluctuations. This corresponds to the experiment of Ref. 5. Exponentially small probability of tunneling in this case is connected with the fact that changes of the collective variable φ imply a change of a very large amount of quasiparticle states. Overlap of many such electron states is exponentially small.

By each phase slippage, the magnetic flux changes by one quantum. This means that in the electrical circuit an average voltage will appear,

$$\langle eV \rangle \sim \omega_0 \exp(-A) . \tag{31}$$

Formula (31) is only valid provided that after tunneling the system stops at the neighboring potential minimum, which is possible at a sufficiently small value of the shunt resistance only. At high voltages quantum and thermal fluctuations are less important. They determine only the width of radiation line.[15]

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