

## Effect of uniaxial stress on the saturation of intervalence-band absorption in *p*-type Ge

R. B. James

*Solid State Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830*

D. L. Smith

*Honeywell Systems and Research Center, Minneapolis, Minnesota 55440*

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A calculation is presented of the effect of a compressive uniaxial stress on the saturation of the intervalence-band absorption in *p*-type Ge for light having a wavelength of 10.6  $\mu\text{m}$ . The intensity dependence of the absorption coefficient can be approximated by the expression for an inhomogeneously broadened two-level model where the value for the saturation intensity depends on the direction of applied stress, magnitude of the stress, and polarization of the light. Values of the saturation intensity are reported for compressive stress along the [001] direction and light polarized along the [001] and [100] directions, and for compressive stress along the [111] direction and light polarized along the [111] and [0 $\bar{1}$ 1] directions. For the directions of stress and light polarization investigated, we found that the saturation intensity is significantly smaller when the direction of polarization of the light is parallel to the stress direction. For light polarization perpendicular to the direction of applied stress, there is no significant change in the saturation intensity for compressive uniaxial stresses below about  $7 \times 10^9$  dyn  $\text{cm}^{-2}$ .

### I. INTRODUCTION

There is considerable interest in saturable absorbers due to their practical use in laser systems. Some of the applications of saturable absorbers to pulsed  $\text{CO}_2$  lasers include their use for the generation of passively mode-locked pulses of subnanosecond duration, and their use in the design of high-power  $\text{CO}_2$ -laser systems to provide adequate interstage isolation of the oscillator and amplifier. The attempts have been to find a material that is integrable into a beam transport system and has a broadband low-intensity absorption coefficient which saturates at high intensities. Several *p*-type semiconductors should exhibit these properties for light with wavelengths over the  $\text{CO}_2$  laser spectrum. Most of the experimental studies on the absorption saturation in *p*-type semiconductors have used samples of *p*-type Ge.<sup>1-6</sup> In a recent paper,<sup>7</sup> a theory was presented to calculate the intensity dependence of the absorption coefficient in several *p*-type semiconductors for light with a wavelength in the (9–11)- $\mu\text{m}$  region, which corresponds to the  $\text{CO}_2$  laser spectrum. The dominant absorption mechanism in this spectral region is intervalence-band transitions, where a free hole in the heavy-hole band absorbs a photon and makes a direct transition to the light-hole band. At sufficiently high light intensities, the hole population in the resonant region of the heavy-hole band becomes depleted, which leads to a decrease in the absorption coefficient with increasing intensity. The intensity dependence of the absorption coefficient was found to be closely approximated by the expression for an inhomogeneously broadened two-level model,

$$\alpha(I, \omega) = \frac{\alpha_0(\omega)}{[1 + I/I_s(\omega)]^{1/2}}. \quad (1)$$

Here  $\alpha_0(\omega)$  is the small-signal absorption coefficient for a photon energy  $\hbar\omega$ ,  $I$  is the light intensity, and  $I_s$  is the sa-

uration intensity. Values of the saturation intensity  $I_s$  were reported for several of the group-IV and III-V compound semiconductors.<sup>8,9</sup> The values of  $I_s$  for the different materials were found to vary over about 2 orders of magnitude. This variation in the saturation intensities for the different semiconductors is largely due to three factors: (1) the hole-phonon scattering rates, (2) the valence-band structures, and (3) the momentum matrix elements.

The presence of a uniaxial stress in a cubic semiconductor such as Ge reduces the degree of symmetry and gives rise to significant changes in the band structure (and momentum matrix elements), which leads to changes in the linear and nonlinear absorption associated with the intervalence-band transitions. In this paper we present calculations of the dependence of the saturation intensity for *p*-type Ge on a compressive uniaxial stress in order to determine how the saturation behavior can be tuned by applying a stress. The directions of the applied stress and polarization of the light are found to be important in determining the saturation characteristics. Explicit results are reported for a compressive stress along the [001] direction and light polarized along the [001] and [100] directions and for a compressive stress along the [111] direction and light polarized along the [111] and [0 $\bar{1}$ 1] directions. All results presented in this paper are for light having a wavelength of 10.6  $\mu\text{m}$ . The paper is organized in the following way: In Sec. II we discuss the theoretical approach which is used, in Sec. III we present the results for *p*-type Ge subjected to compressive uniaxial stress, and in Sec. IV we summarize our conclusions.

### II. THEORETICAL APPROACH

In semiconductors with the diamond or zinc-blende crystal structure, there are six bands (three sets of twofold-degenerate bands) near the valence-band maximum. Four of the bands are degenerate at  $k=0$  and the other two de-

generate bands are split to lower energy by the spin-orbit interaction. Away for  $k=0$ , the four degenerate bands split into two twofold-degenerate pairs called the heavy- and light-hole bands. At CO<sub>2</sub>-laser frequencies, optical transitions between the heavy- and light-hole bands dominate the absorption in  $p$ -type Ge. The spin-orbit splitting at  $k=0$  is 295 meV, so one-photon transitions to the split-off hole band are not energetically allowed for CO<sub>2</sub>-laser light.

The presence of a uniaxial stress splits the  $k=0$  degeneracy of the heavy- and light-hole bands and also modifies the valence-band structure away from  $k=0$ . As a result, the density of states for both the optical transitions and

phonon scattering transitions are changed by the application of uniaxial stress. In addition, the hole wave functions and thus the intervalence-band optical matrix elements are modified. These changes in the density of states and optical matrix elements lead to a stress-induced modification of both the low-intensity absorption coefficient and the saturation characteristics of  $p$ -type Ge.

The basic theoretical formalism used to describe uniaxially stressed material is the same as that for unstressed material (stress-modified wave functions and energy dispersion relations are input into the formalism). The absorption coefficient is then given by

$$\alpha(I, \omega) = \frac{4\pi^2}{(\epsilon_\infty)^{1/2} m^2 \omega c} N_h e^2 \sum_{\vec{k}} [f_h(\vec{k}) - f_l(\vec{k})] \sum_{\substack{b \text{ in } h, \\ c \text{ in } l}} |\vec{\eta} \cdot \vec{P}_{bc}(\vec{k})|^2 \frac{1/[\hbar\pi T_2(\vec{k})]}{[\Omega(\vec{k}) - \omega]^2 + [1/T_2(\vec{k})]^2}. \quad (2)$$

Here  $\hbar\omega$  is the photon energy,  $\epsilon_\infty$  is the high-frequency dielectric constant,  $N_h$  is the hole density,  $f_h(\vec{k})$  and  $f_l(\vec{k})$  are the intensity-dependent one-hole occupation probabilities for a state with wave vector  $\vec{k}$  in the heavy- and light-hole bands, respectively,  $\vec{\eta} \cdot \vec{P}_{bc}$  is the momentum matrix element between the bands  $b$  and  $c$  which is summed over the twofold-degenerate states in the heavy- and light-hole bands,  $\vec{\eta}$  is the polarization of the light,  $\hbar\Omega(\vec{k})$  is the energy difference between states with wave vector  $\vec{k}$  in the heavy- and light-hole bands, and  $T_2(\vec{k})$  is given by

$$\frac{2}{T_2(\vec{k})} = \sum_{c, \vec{k}'} (R_{h\vec{k} \rightarrow c\vec{k}'} + R_{l\vec{k} \rightarrow c\vec{k}'}), \quad (3)$$

where  $R_{a\vec{k} \rightarrow b\vec{k}'}$  is the rate at which a hole in band  $a$  with wave vector  $\vec{k}$  is scattered into a state with wave vector  $\vec{k}'$  in band  $b$ . The hole distribution functions  $f_h(\vec{k})$  and  $f_l(\vec{k})$  are calculated from the rate equations

$$\beta(\vec{k})[f_h(\vec{k}) - f_l(\vec{k})] = - \sum_{c, \vec{k}'} [R_{h\vec{k} \rightarrow c\vec{k}'} f_h(\vec{k}) - R_{c\vec{k}' \rightarrow h\vec{k}} f_c(\vec{k}')], \quad (4a)$$

$$\beta(\vec{k})[f_h(\vec{k}) - f_l(\vec{k})] = \sum_{c, \vec{k}'} [R_{l\vec{k} \rightarrow c\vec{k}'} f_l(\vec{k}) - R_{c\vec{k}' \rightarrow l\vec{k}} f_c(\vec{k}')], \quad (4b)$$

where

$$\beta(\vec{k}) = \frac{2\pi^2}{(\epsilon_\infty)^{1/2} m^2 \omega c} \frac{e^2 I}{\hbar\omega} \sum_{\substack{b \text{ in } h, \\ c \text{ in } l}} |\vec{\eta} \cdot \vec{P}_{bc}(\vec{k})|^2 \frac{1/[\hbar\pi T_2(\vec{k})]}{[\Omega(\vec{k}) - \omega]^2 + [1/T_2(\vec{k})]^2}. \quad (4c)$$

Stress enters the problem by changing the hole dispersion curves which modifies  $\Omega(\vec{k})$  [and hence the integration over wave vector in Eq. (2) and  $\beta(\vec{k})$  in Eq. (4c)] and the scattering rates in Eqs. (3), (4a), and (4b). The stress also changes the optical matrix elements which appear in Eq. (2) and in  $\beta(\vec{k})$ .

The one-hole energies in the undeformed crystal are calculated by second-order degenerate  $\vec{k} \cdot \vec{p}$  perturbation theory.<sup>10</sup> We apply the usual perturbation methods in the strained crystal, including first-order terms in the stress and second-order terms in  $k$  [there are no linear terms in  $k$  for  $E(\vec{k})$  near the band edge]. Taking account of the spin functions, the perturbation due to the uniaxial deformation can be represented by a  $6 \times 6$  matrix, which can be described by introduction of the deformation potentials  $a$ ,  $b$ , and  $d$ .<sup>11</sup> Numerically solving the secular equation gives the one-hole energies as a function of  $\vec{k}$  and the stress. In the analysis we include the effect of the split-off hole band in the manner of Ref. 12. The cyclotron resonance param-

eters used to describe the unstressed material are from Ref. 13 and the elastic compliance constants and deformation potentials used to describe the effect of stress are listed in Table I.

The optical transition rate from band  $i$  to band  $j$  is proportional to the matrix element  $|\langle U_i | \vec{A} \cdot \vec{p} | U_j \rangle|^2$  for direct intervalence-band absorption, where  $\vec{A}$  is the vector potential of the light and  $\vec{p}$  is the momentum operator. The  $\vec{k} \cdot \vec{p}$  perturbation introduces a first-order correction to the wave function of parity opposite that of the band at  $k=0$ . In the deformed crystal the periodic part of the wave function correct to first order in the perturbation is given by

$$U_i = U_{0i} + \sum_j \frac{\hbar}{m} \frac{\langle U_{0j} | (\vec{k} \cdot \vec{p} + D) | U_{0i} \rangle}{E_i^0 - E_j^0} U_{0j}, \quad (5)$$

where  $D$  is the perturbation describing the uniaxial stress. The matrix element of  $\vec{A} \cdot \vec{p}$  between bands  $i$  and  $j$  is

TABLE I. Values for the nonzero elastic compliance constants in units of  $\text{cm}^2/\text{dyn}^a$  and the deformation potentials in units of  $\text{eV}^b$ . For cubic crystals,  $S_{12}=S_{13}=S_{23}$ .

$S_{11}$	$S_{12}$	$S_{44}$	$a$	$b$	$d$
$9.79 \times 10^{-13}$	$-2.68 \times 10^{-13}$	$1.47 \times 10^{-12}$	2.0	-2.2	-4.4

<sup>a</sup>H. B. Huntington, in *Solid State Physics*, edited by H. Ehrenreich, F. Seitz, and D. Turnbull (Academic, New York, 1958), Vol. 7, p. 213.

<sup>b</sup>J. D. Wiley, *Solid State Commun.* **8**, 1865 (1970).

$$\langle U_i | \vec{A} \cdot \vec{p} | U_j \rangle = \frac{\hbar}{m} \sum_j \frac{\langle U_{0i} | \vec{k} \cdot \vec{p} + D | U_{0j} \rangle \langle U_{0j} | \vec{A} \cdot \vec{p} | U_{0j} \rangle + \langle U_{0i} | \vec{A} \cdot \vec{p} | U_{0j} \rangle \langle U_{0j} | (\vec{k} \cdot \vec{p} + D) | U_{0j} \rangle}{E_i^0 - E_j^0} \quad (6)$$

The valence-band wave functions at  $k=0$  transform according to the representation  $\Gamma_{25}^+$ , and the momentum operator  $\vec{p}$  transforms according to  $\Gamma_{15}^-$ . Only those bands appearing in the decomposition of the direct product can have  $\vec{k} \cdot \vec{p}$  interactions with the valence band:

$$\Gamma_{25}^+ \times \Gamma_{15}^- = \Gamma_2^- + \Gamma_{15}^- + \Gamma_{12}^- + \Gamma_{25}^- \quad (7)$$

The stress components transform like  $\Gamma_{12}^+$  and  $\Gamma_{25}^+$ , and the bands which have  $D$  interactions with the valence band are given by

$$\Gamma_{25}^+ \times (\Gamma_{12}^+ + \Gamma_{25}^+) = \Gamma_{25}^+ + \Gamma_{15}^+ + \Gamma_1^+ + \Gamma_{12}^+ \quad (8)$$

We see that the bands which interact with the valence band at  $k=0$  through the stress perturbation have vanishing  $\vec{A} \cdot \vec{p}$  matrix elements, so that the products

$$\langle U_{0i} | D | U_{0j} \rangle \langle U_{0j} | \vec{A} \cdot \vec{p} | U_{0j} \rangle$$

are zero. Thus the  $\vec{A} \cdot \vec{p}$  matrix elements can be obtained by using the  $\vec{k} \cdot \vec{p}$  part of the Hamiltonian matrix (with  $\Delta=0$ ), and substituting  $k_l A_m + A_l k_m$  for  $k_l k_m$ .<sup>10</sup> The resulting matrix  $F$  must then be transformed to  $U^{-1}FU$  by the unitary matrix  $U$  that diagonalizes the Hamiltonian, which includes the uniaxial stress. Here the effect of stress enters in the determination of the transformation matrix  $U$ .

The hole-phonon scattering rates are calculated on the basis of the deformation-potential theory, where the deformation parameters are taken from the mobility fits of Brown and Bray.<sup>14</sup> In the calculation we assume that the matrix elements for hole-phonon scattering are independent of stress. The probability of scattering per unit time from a state  $\vec{k}$  to a state  $\vec{k}'$  also depends on the density of final states to which the hole can be scattered.<sup>15,16</sup> Thus the scattering rate for a hole with wave vector  $\vec{k}$  will depend on the magnitude and direction of the uniaxial stress due to the changes in the density of states. The density of

states in the heavy- and light-hole bands are found from the energy dispersion relations, and the scattering rates are calculated as a function of the hole energy and uniaxial stress.

The rest of the calculation proceeds as in Ref. 8. The absorption coefficient is calculated as a function of intensity and the numerical results are compared with Eq. (1). The numerical results satisfy Eq. (1) very closely. Values of the saturation intensity which are reported are determined by fitting Eq. (1) to the numerical results. The calculations are done for  $10.6\text{-}\mu\text{m}$  light and room-temperature conditions. Only hole-phonon scattering is included in the calculation of scattering rates. This procedure is appropriate for doping levels less than or equal to about  $3 \times 10^{15} \text{ cm}^{-3}$ .<sup>17</sup>

### III. RESULTS AND DISCUSSION

Although our numerical results for the intensity dependence of the absorption coefficient are quite close to the simple expression given by Eq. (1), they do not take exactly this functional form. Thus one cannot write an analytic expression for the saturation intensity that exactly describes our results. However, to qualitatively discuss the effect of uniaxial stress on the saturation intensity, it is highly desirable to have a reasonably simple expression for  $I_s$  which, although not exact, is capable of showing the important qualitative features of the stress dependence. Such an expression can be attained using the "first approximation for  $[f_h(\vec{k}) - f_l(\vec{k})]$ " discussed in Ref. 7. Comparing the result for the intensity dependence of the absorption coefficient in this approximation [see in particular Eq. (18) of Ref. 7] with Eq. (1), and taking the low-intensity limit so that the square root in Eq. (1) can be expanded [and the analogous square root in Eq. (18) of Ref. 7 can be expanded], one finds

$$\frac{1}{I_s} \approx \frac{\int_{\Omega(\vec{k})=\omega} dS / |\vec{\nabla}_{\vec{k}} \Omega(\vec{k})| \sum_{\substack{b \text{ in } h, \\ c \text{ in } l}} \{ |\vec{\eta} \cdot \vec{P}_{bc}(\vec{k})|^2 [f_h^e(\vec{k}) - f_l^e(\vec{k})] \} / l(\vec{k})}{\int_{\Omega(\vec{k})=\omega} dS / |\vec{\nabla}_{\vec{k}} \Omega(\vec{k})| \sum_{\substack{b \text{ in } h, \\ c \text{ in } l}} |\vec{\eta} \cdot \vec{P}_{bc}(\vec{k})|^2 [f_h^e(\vec{k}) - f_l^e(\vec{k})]}$$

where

$$I(\vec{k}) \equiv \frac{\hbar^2 c (\epsilon_\infty)^{1/2} m^2 \omega^2}{[T_h(\vec{k}) + T_l(\vec{k})] T_2(\vec{k}) 2\pi e^2 \sum_{\substack{b \text{ in } h, \\ c \text{ in } l}} |\vec{\eta} \cdot \vec{P}_{bc}(\vec{k})|^2} \quad (9b)$$

Here  $f_h^e(\vec{k})$  and  $f_l^e(\vec{k})$  are the equilibrium values of the distribution functions, the surface integral is over a surface of constant  $\Omega(\vec{k}) (= \omega)$  and

$$\frac{1}{T_h(\vec{k})} = \sum_{c, \vec{k}'} R_{h \vec{k} \rightarrow c \vec{k}'} \quad (10a)$$

and

$$\frac{1}{T_l(\vec{k})} = \sum_{c, \vec{k}'} R_{l \vec{k} \rightarrow c \vec{k}'} \quad (10b)$$

The factor  $I(\vec{k})$  corresponds to a contribution to the saturation intensity from states at wave vector  $\vec{k}$  and the contributions from the various states are weighted as described by Eq. (9a). Equation (9a) is written down for the purpose of qualitative discussion and was not actually used in the calculations. The calculations were performed as described in the preceding section.

The most important effect of stress on  $I_s$  is from the modification of the energy dispersion relations. This modification changes both the density of states which enter into the scattering times ( $T_h$ ,  $T_l$ , and  $T_2$ ) and the location of the resonant region [i.e., where the condition  $\Omega(\vec{k}) = \omega$  is satisfied]. The effect of stress on  $T_h$ , which describes scattering in the resonant region of the heavy-hole band, is particularly important.  $T_h$  is significantly greater than  $T_l$  and thus dominates in the expression ( $T_h + T_l$ ), and the stress-induced changes in  $T_h$  are significantly greater than the changes in  $T_2$ , which is dominated by scattering from the resonant region in the light-hole band to relatively high-energy states in the heavy-hole band. The optical matrix elements,  $|\vec{\eta} \cdot \vec{P}_{bc}(\vec{k})|^2$ , enter both in the expression for  $I(\vec{k})$  and in the weighting in Eq. (9a). Small optical matrix elements tend to increase  $I_s$ , but if the optical matrix element is small for a particular direction of  $\vec{k}$ , this direction is weakly weighted. Thus an important effect of the optical matrix elements is to weight the various directions in the integration over  $\vec{k}$  space. This weighting depends on the polarization and leads to a fairly strong polarization dependence of  $I_s$  under uniaxial stress.

For a qualitative discussion, it is useful to have a simple form for the effect of stress on the heavy- and light-hole energy dispersion curves. A simple analytic form for these dispersion curves results if the split-off hole band is neglected and the  $4 \times 4$  Hamiltonian describing the heavy- and light-hole bands (and neglecting coupling to the split-off hole band) is diagonalized.<sup>11</sup> In this approximation

$$E_{h,l}(\vec{k}, \vec{\epsilon}) = Ak^2 + a \vec{l} \cdot \vec{\epsilon} \pm (E_{kk}^2 + E_{\epsilon\epsilon}^2 + E_{\epsilon k}^2)^{1/2}, \quad (11a)$$

where

$$E_{kk}^2(\vec{k}) = Bk^4 + C^2(k_x^2 k_y^2 + k_x^2 k_z^2 + k_y^2 k_z^2), \quad (11b)$$

$$E_{\epsilon\epsilon}^2(\vec{\epsilon}) = (1/2b^2)[(\epsilon_{xx} - \epsilon_{yy})^2 + \text{c.p.}] + d^2(\epsilon_{xy}^2 + \text{c.p.}), \quad (11c)$$

and

$$E_{\epsilon k}^2(\vec{k}, \vec{\epsilon}) = Bb [3(k_x^2 \epsilon_{xx} + \text{c.p.}) - k^2 \vec{l} \cdot \vec{\epsilon}] + 2Dd(k_x k_y \epsilon_{xy} + \text{c.p.}). \quad (11d)$$

Here  $A$ ,  $B$ ,  $C$ , and  $D$  are taken from cyclotron resonance experiments,<sup>13</sup>  $a$ ,  $b$ , and  $d$  are the deformation potentials,  $\vec{\epsilon}$  is the second-rank strain tensor which describes the stress magnitude and direction, and c.p. stands for cyclic permutation with respect to the indices  $x$ ,  $y$ , and  $z$ . The splitting of the heavy- and light-hole bands at  $k=0$  is contained in  $E_{\epsilon\epsilon}^2$ . The strain components are written in terms of the elastic compliance constants and stress components.<sup>18</sup> For the actual calculations, the dispersion curves were calculated from the  $6 \times 6$  Hamiltonian which includes coupling to the split-off hole band. Equation (11a) is written only for the purpose of discussion.

The term  $E_{\epsilon\epsilon}^2$  in Eq. (11) produces a splitting of the heavy- and light-hole bands at  $k=0$ . The splitting of the heavy- and light-hole bands at the zone center (including the split-off hole band) is shown in Fig. 1 for compressive stresses in the range of  $5.0 \times 10^8$  to  $1.8 \times 10^{10}$  dyn/cm<sup>2</sup>. The solid line is for a stress in the [001] direction, and the dashed line is for a stress in the [111] direction. At a wavelength of  $10.6 \mu\text{m}$ , the photon energy is 117 meV, thus at the highest stresses shown in Fig. 1, the zone-center splitting almost equals the photon energy. The stress at which this occurs is somewhat larger for the [111] direction than for the [001] direction.

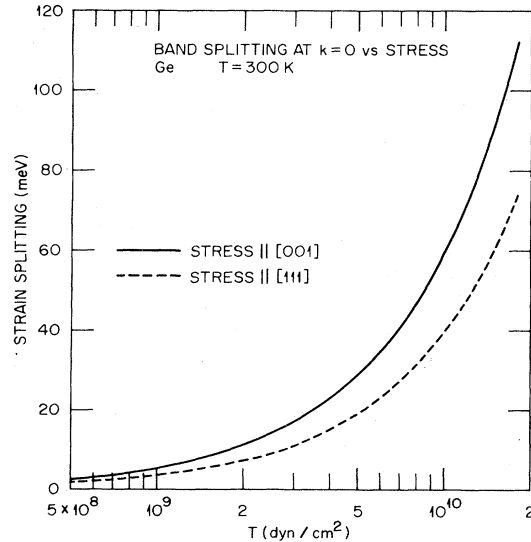


FIG. 1. Strain splitting of the heavy- and light-hole bands at  $k=0$  vs compressive stress for germanium at room temperature. The solid line shows the splitting in meV for stress in the [001] direction, and the dashed line shows the splitting for stress in the [111] direction.

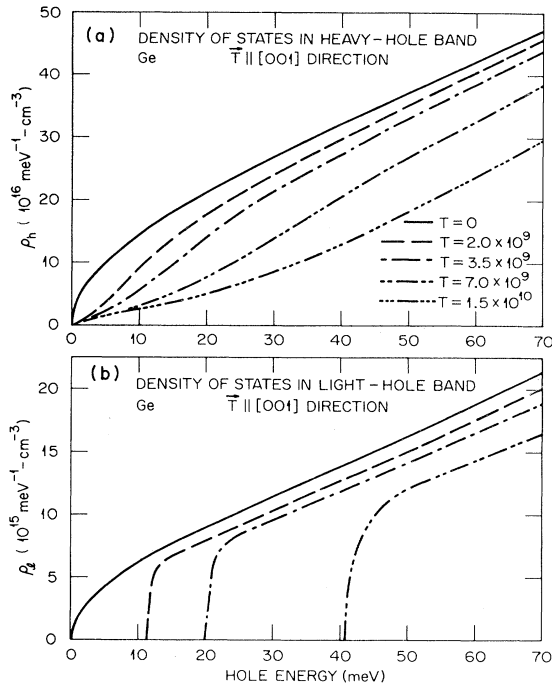


FIG. 2. Calculated values for the density of hole states vs compressive uniaxial stress for stress in the [001] direction. (a) shows the density of hole states for the heavy-hole band in units of  $10^{16} \text{ meV}^{-1} \text{ cm}^{-3}$ , and (b) shows the density of states for the light-hole band. The different curves are for the following values of the stress: —,  $T=0$ ; - - -,  $T=2.0 \times 10^9$ ; ---,  $T=3.5 \times 10^9$ ; — · —,  $T=7.0 \times 10^9$ ; and — · — · —,  $T=1.5 \times 10^{10} \text{ dyn cm}^{-2}$ .

The term  $E_{ek}^2$  in Eq. (11) depends on both the stress and the value of  $\vec{k}$ . It can be much smaller, comparable, or much larger than  $E_{ee}^2$  for states in the resonant region for  $10.6\text{-}\mu\text{m}$  light, depending on the particular direction in  $\vec{k}$  space and the applied stress. For example, for compressive uniaxial stress in the [001] direction (in the two-band approximation),  $E_{ek}^2$  is given by

$$E_{ek}^2 = Bb(\epsilon_{xx} - \epsilon_{zz})(k^2 - 3k_z^2). \quad (12)$$

The sign of  $E_{ek}^2$  depends on the particular direction in  $\vec{k}$  space through the  $(k^2 - 3k_z^2)$  term for  $T \parallel [001]$  direction. Thus it has a maximum value for  $k_z = 0$ , a minimum value for  $\vec{k} = |k| \hat{z}$ , and vanishes for  $k^2 = 3k_z^2$ . As a consequence, both  $E_{ee}^2$  and  $E_{ek}^2$  are important in determining the region in  $\vec{k}$  space at which the intervalence-band resonance occurs.

The positive term  $E_{ee}^2$  tends to increase the splitting between heavy- and light-hole bands and thus to push the resonance region to smaller values of  $|k|$ . The term  $E_{ek}^2$  can take on either sign; if it is positive, it also tends to push the resonance region toward the zone center, if it is negative, it tends to push the resonant region toward larger values of  $k$ .

The terms  $E_{ee}^2$  and  $E_{ek}^2$  affect the density of hole states in the heavy- and light-hole bands, which in turn affects the effective density of states in calculating the occupation

probabilities,<sup>19</sup> the joint density of states at the intervalence-band resonance, and the hole-phonon scattering rates. In Fig. 2 the calculated density of hole states (including the split-off hole band) is shown for several values of stress applied in the [001] direction. Figure 2(a) is the density of states in the heavy-hole band ( $\rho_h$ ) in units of  $10^{16} \text{ meV}^{-1} \text{ cm}^{-3}$ , and the bottom illustration shows the density of states in the light-hole band ( $\rho_l$ ) in units of  $10^{15} \text{ meV}^{-1} \text{ cm}^{-3}$ . The different curves in the figure are for zero stress and compressive stresses of  $2.0 \times 10^9$ ,  $3.5 \times 10^9$ ,  $7.0 \times 10^9$ , and  $1.5 \times 10^{10} \text{ dyn cm}^{-2}$ . The application of a compressive uniaxial stress causes significant deviation from the  $\epsilon^{1/2}$  dependence in  $\rho_h$  and  $\rho_l$  which follows from a parabolic band approximation. This deviation is greatest for hole energies ( $\epsilon$ ) comparable or less than the strain splitting. For small hole energies in the heavy-hole band,  $\rho_h(\epsilon, T)$  decreases by almost an order of magnitude at  $T = 1.5 \times 10^{10} \text{ dyn cm}^{-2}$  from the corresponding values of  $\rho_h$  in the unstressed material. For hole energies less than the strain splitting of the heavy- and light-hole bands, there are no allowed states in the light-hole band as noted in Fig. 2(b). For hole energies somewhat greater than the strain splitting, the density of states is approximately given by the unstressed value.

The scattering time  $T_h$  results primarily from holes in the resonant region of the heavy-hole band being scattered by phonons to another heavy-hole band state. Neglecting for the moment the fact that the resonant regions shift with stress (these shifts are actually quite important), the decrease in the heavy-hole density of states shown in Fig. 2 tends to lead to reduced scattering rates and hence a larger  $T_h$  and a smaller  $I_s$ . The hole-phonon scattering rates which dominate  $T_1$  and  $T_2$  are from scattering events where a hole in the resonant region of the light-hole band scatters to rather high-energy states in the heavy-hole band. The density of heavy-hole band states in this high-energy region is not much affected by stress; thus the scattering times  $T_1$  and  $T_2$  are not significantly changed except at the highest stress values. Finally, note that mov-

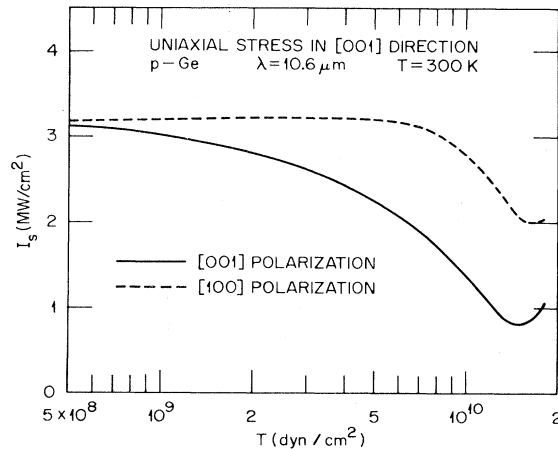


FIG. 3. Calculated values of the saturation intensity  $I_s$  in  $p$ -type Ge as a function of an applied uniaxial compressive stress for stress in the [001] direction at  $T = 300 \text{ K}$ . The solid (dashed) line is for  $10.6 \mu\text{m}$  radiation and light polarized along the [001] ([100]) direction.

ing the resonant region to larger  $k$  vectors (and higher hole energy states) leads to a larger density of final hole states and reduces  $T_h$  (and  $T_l$  and  $T_2$  to a lesser extent), whereas moving the resonant region to smaller  $k$  vectors (and lower hole energy states) increases  $T_h$ .

In Fig. 3 we present the calculated values for  $I_s$  as a function of uniaxial stress for stress in the [001] direction. The solid curve is for light polarized in the [001] direction, and the dashed curve is for light polarized in the [100] direction. All values shown are for room-temperature germanium and light having a wavelength of  $10.6 \mu\text{m}$ . In Fig. 4 the calculated values are presented for  $I_s$  for stress along the [111] direction and light polarized along the [111] and  $[0\bar{1}1]$  directions.

For both stress directions,  $I_s$  decreases with increasing stress for the polarization parallel to the direction of the stress. At the highest stress values, where the zone-center stress splitting becomes nearly equal to the photon energy,  $I_s$  begins to increase for [001] stress. A similar effect occurs for [111] stress at somewhat higher stress values than are shown in Fig. 4.

For the polarization orthogonal to the stress direction, the effect of stress on  $I_s$  is less than for the parallel case. For [001] stress,  $I_s$  decreases somewhat with increasing stress until the zone-center stress splitting becomes nearly equal to the photon energy and then it increases. For [111] stress,  $I_s$  first increases slightly and then decreases somewhat.

The polarization dependence of the stress effect on  $I_s$  results from the optical matrix elements which weight the various  $\vec{k}$  directions. Notice first that for zero stress the optical matrix element does not couple through the  $\Gamma_2^-$  intermediate state (the lowest-energy zone-center conduction-band state in Ge) for  $\vec{k}$  parallel to the polarization vector [see Eq. (6)]. Thus the optical matrix elements for  $\vec{k}$  parallel to the polarization vector are very small and the contribution to  $I_s$  from this direction in  $\vec{k}$  space is weakly weighted. On the other hand, for  $\vec{k}$  and  $\hat{\eta}$

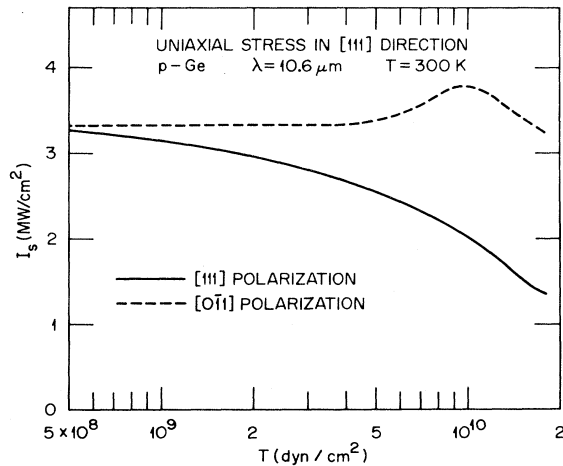


FIG. 4. Calculated values of  $I_s$  in  $p$ -type Ge as a function of uniaxial compressive stress for stress in the [111] direction. The solid (dashed) line is for light with a wavelength of  $10.6 \mu\text{m}$  and light polarized in the [111] ( $[0\bar{1}1]$ ) direction.

far from parallel, there is strong coupling of the optical matrix element through the  $\Gamma_2^-$  intermediate state and the optical matrix elements are large. Thus the contribution to  $I_s$  from these directions in  $\vec{k}$  space are strongly weighted.

For those directions in  $\vec{k}$  space nearly parallel to the stress direction,  $E_{ek}^2$  in Eq. (11) is negative and the resonant region tends to move to larger values of  $|k|$ . The increase in magnitude of the wave vector at the resonant region in these directions tends to increase the magnitude of the contribution to  $I_s$  from these states. For those directions in  $\vec{k}$  space nearly orthogonal to the stress direction,  $E_{ek}^2$  in Eq. (11) is positive and the resonant region moves in toward the zone center. The decrease in the magnitude of the wave vector at the resonant region in these directions tends to decrease the magnitude of the contribution to  $I_s$  from these states.

For polarization in the stress direction, the directions of  $\vec{k}$  nearly parallel to the stress direction (that is, those states whose contribution to  $I_s$  increases with stress) are weakly weighted by the small optical matrix elements, whereas the directions of  $\vec{k}$  nearly orthogonal to the stress (those states whose contribution to  $I_s$  decreases with stress) is strongly weighted by large optical matrix elements. Thus for polarization parallel to the stress direction,  $I_s$  decreases with stress until the  $k=0$  stress splitting becomes nearly equal to the photon energy. At this point the resonant region in most directions is much closer to the zone center and the optical matrix elements begin to get very small (they vanish at the zone center), which pushes  $I_s$  up rapidly.

For polarization orthogonal to the stress direction, the directions of  $\vec{k}$  nearly parallel to the stress direction (and thus nearly orthogonal to the polarization) are strongly weighted by the large optical matrix elements. Some of

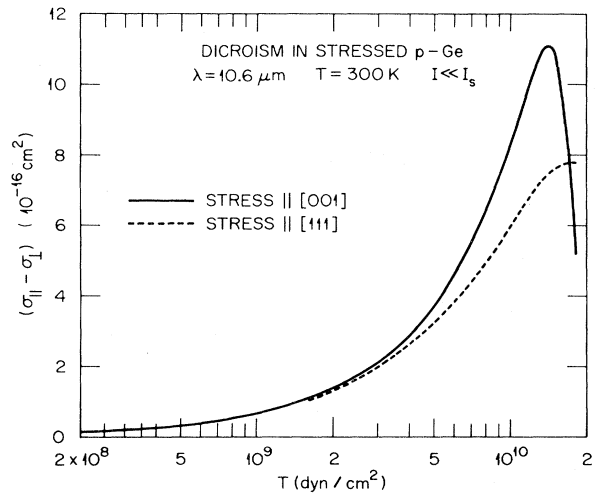


FIG. 5. Calculated dichroism of  $p$ -type Ge at 300 K as a function of compressive uniaxial stress for light with a wavelength of  $10.6 \mu\text{m}$  and an intensity much less than the saturation intensity. The solid line shows  $(\sigma_{\parallel} - \sigma_{\perp})$  for stress in the [001] direction and the dashed line shows the results for stress in the [111] direction.

the directions in  $\vec{k}$  space which are nearly orthogonal to the stress (those nearly parallel to the polarization) are weakly weighted by small optical matrix elements, and other  $\vec{k}$  directions which are nearly orthogonal to the stress (those which are also nearly orthogonal to the polarization) are strongly weighted by large optical matrix elements. As a result, there is significant cancellation among the contributions from the various  $\vec{k}$  directions to the stress-induced changes in  $I_s$ . Thus the effect of stress on  $I_s$  is less for polarization orthogonal to stress than for polarization parallel to stress. For [001] stress, the cancellation works out in such a way that  $I_s$  decreases slightly with increasing stress until the zone-center stress splitting nearly equals the photon energy. At this point  $I_s$  begins to increase because of the very small optical matrix elements for states near the zone center. For [111] stress, the cancellation works out in such a way that  $I_s$  first increases slightly and then decreases somewhat with increasing stress.

As discussed above, the effect of the compressive stress reduces the cubic symmetry, and a measurable low-intensity dichroism is predicted.<sup>16,20</sup> For the various combinations of stress and light polarizations examined, we find that when the small-signal absorption increases with increasing stress, the saturation intensity decreases. This should be expected since an increase in the excitation rates, which leads to a smaller  $I_s$ , also leads to a larger low-intensity absorption cross section ( $\sigma$ ). Explicit values for the calculated dichroism are shown in Fig. 5 as a function of the stress. Here  $\sigma_{\parallel}$  is the small-signal cross section for light polarized parallel to the stress axis and  $\sigma_{\perp}$  is the small-signal cross section for light polarized perpendicular to the uniaxial stress direction. The dichroism is measured in units of  $10^{-16}$  cm<sup>2</sup>. The solid (dashed) curve shows the dichroism for compressive uniaxial stress in the [001] ([111]) direction. For stress parallel to the [001] direction, both  $\sigma_{\parallel}$  and  $\sigma_{\perp}$  decrease significantly at stresses greater than about  $1.5 \times 10^{10}$  dyn cm<sup>-2</sup>. This drop is what

causes the saturation intensity to begin to increase at the corresponding values of stress (see Fig. 4).

#### IV. CONCLUSION

We have calculated the linear and nonlinear absorption coefficient in *p*-type Ge at 10.6  $\mu$ m as a function of uniaxial stress. Explicit values for the saturation intensity are presented as a function of applied compressive stress for room temperature conditions. Changes in the valence-band structure, momentum matrix elements, and hole-phonon scattering rates are included in the analysis. We find that the absorption saturation behavior depends on the direction of the light polarization, the direction of the stress, and the magnitude of the stress. For most combinations of stress and directions of light polarization, the values of  $I_s$  remain approximately unchanged or tend to decrease with increasing compressive stress. For the cases where the small-signal absorption increases with increasing stress, the saturation intensity monotonically decreases with stress, and for the cases where the small-signal absorption decreases with increasing stress, the saturation intensity is approximately unchanged except at the highest stresses considered.

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