Role of the de Haas—van Alphen effect in the quantum theory of high-frequency electromagnetic phenomena

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The role of the conductivity and the magnetization in electromagnetic wave propagation in a metal when an external magnetic field is present is reviewed. In the quantum case, the experimental results at different values of the frequency demonstrate that the approach of Lifshitz, Azbel', and Kaganov is not always valid. A theoretical analysis of the quantum absorption taking into account both the conductivity and the magnetization in the frame of the semiclassical theory is presented. The theory shows that the quantum oscillations at intermediate and high frequencies are dominated by the conductivity.

I. INTRODUCTION

When considering the problem of the propagation of an electromagnetic wave in a metal in the presence of an external magnetic field large enough to allow effects due to Landau-level quantization, one must take into account both the conductivity and the magnetization contribution. In a nonstationary field the Shubnikov—de Haas and the de Haas—van Alphen effects are *a priori* simultaneously present. Up until now[1] it has been usual to assume that the quantum oscillations of the surface impedance of a metal at very low and also at sufficiently high frequencies were due to the conductivity, while at intermediate frequencies they were assumed to be due to the magnetization.

The experimental results found by Vol'skii and Petrashov[2] at low frequencies, Zherebchevskii *et al.*[3] at intermediate frequencies, and Giura *et al.*[4] at microwave frequencies show that this approach is not always valid. Therefore the question of which effect dominates in the electromagnetic wave propagation must be theoretically reviewed.

In this paper we derive the conditions for the quantum absorption of the electromagnetic (EM) field when both the conductivity and the magnetization are present. We show that at intermediate and high frequencies the magnetization contribution, if it is present, reduces the quantum oscillation. In this case the Shubnikov—de Haas effect plays the main role as follows by the experimental results. To show this, we calculate the magnetization term at low but finite temperatures and introduce it together with the conductivity term previously found[5] in the Maxwell equation. Finally, we obtain an equation which allows us to display the conditions for the existence of the quantum EM wave propagation.

II. QUANTUM OSCILLATIONS IN AN EM FIELD AS A FUNCTION OF FREQUENCY

As is well known,[1] in the static case the quantum effects (related to the electrostatic and magnetostatic effects) which give rise to the Shubnikov—de Haas effect for the conductivity and the de Haas—Van Halphen effect for the magnetization, respectively, can be independently considered. In the problem of the propagation of the EM field in a metal in the presence of an external magnetic field $\vec{B}_0$, the microscopic field at a given point on the metal depends on the external magnetic field established by all the charges moving along orbits with radii of the order of the Larmor radius. The size of this radius is large compared with the distance between the charges, and therefore the field at a given point is an average field: One must consider the magnetic induction $\vec{B}$ and not the magnetic field $\vec{H}$. In an alternating EM field one measures the surface impedance Z of a metal. To find Z the Maxwell equations in which both the magnetization $\vec{M}$ and the current density $\vec{j}$ are present must be solved. The Maxwell equations, ignoring the displacement current as is usually permissible for good conductors, are

$$\mathrm{rot}\vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} ,$$

$$\mathrm{rot}\vec{H} = \mathrm{rot}(\vec{B} - 4\pi\vec{M}) = \frac{4\pi}{c}\vec{j} = \frac{4\pi}{c}\overleftrightarrow{\sigma}\cdot\vec{E} ,$$

where $\vec{j}$ is the conduction density current and $\overleftrightarrow{\sigma}$ the conductivity tensor. The Maxwell equations in this form are analogous to those revised in the literature with the difference that in our case the total current is divided in the "conductivity" part $\vec{j}$ and in the "diamagnetic" part $4\pi c\,\mathrm{rot}\vec{M}$.

Lifshitz *et al.*,[1] in considering the quantum effects, take into account the relative contribution of both the conductivity and the magnetization in the propagation of an EM field. The magnetic term is proportional to the derivative of the magnetization with respect to the magnetic field and therefore involves a factor equal to the inverse of the period of quantum oscillations given by

$$\frac{2\pi}{\Delta(1/B_0)} = \frac{S_m}{e\hbar} = n + \tfrac{1}{2} ,$$

where S_m is the cross-sectional area of the constant-energy surface. In the case of not very high magnetic fields, the condition $n \gg 1$ is valid for the Landau quantum number n. At low frequency, $\omega\tau \ll 1$, one can use the instantaneous value of the alternating field in the static formulas, and therefore the de Haas—van Alphen term is more important. When the frequency increases, the contribution of the magnetization decreases. This is due, as Lifshitz *et al.*[1] assert, mainly to two reasons, the nonlocal effect at high frequencies and the fact that the magnetic moment does not instantaneously follow the variations of the alternating magnetic field. This fact gives rise to a factor $(\omega\tau)^{-1}$ in the expression of the magnetic moment. To continue, they assert that for a metallic sample of finite length, by increasing the frequency at the beginning, the Shubnikov—de Haas effect contributes to the quantum oscillations; at intermediate frequencies the de Haas—van Alphen effect is dominant and, finally, for high frequencies the Shubnikov—de Haas effect is again an important influence. The experimental results recently found in metals[2,3,4] do not always confirm these predictions. In the following section we will sketch the experimental situation.

III. EXPERIMENTAL RESULTS

Vol'skii and Pekashov[2] showed that at audio frequencies (approximately 160 Hz) oscillations in the EM absorption as a function of an external magnetic field exist. From the values of the periods they deduced that the oscillations have to be related to the cross-sectional areas of the tubes of the Fermi surface of aluminium in the third Brillouin zone. They concluded that the quantum oscillation observed in helicon propagation in a metal at low frequency is a manifestation of the de Haas—van Alphen effect. Recently, Zherebchevskii *et al.*[3] reported an experimental investigation of the surface impedance in a single-crystal sample of cadmium in the anomalous skin-effect regime. The external magnetic field is orthogonal to the sample surface and the EM frequency is in the MHz range. They showed that the experimental results are different for the antisymmetric and symmetric excitation method: For magnetic field $B_0 > 30$ kG, quantum oscillations are present with an amplitude 1 order of magnitude greater in the symmetric than in the antisymmetric case. They concluded that the experimental results demonstrate that the quantum oscillations are dominated by the conductivity. For higher frequencies (about 20 GHz) our measurements[4] have been interpreted as being due to the conductivity too.

To summarize, at low frequencies the magnetization gives the relevant contribution, and at intermediate and high frequencies the conductivity has the dominant role. The previous theory is in partial disagreement with these experimental results.

IV. CALCULATION OF $\vec{\mathrm{M}}$

In the nonstationary case, to obtain the magnetization $\vec{\mathrm{M}}$, we consider the expectation value m of the magnetic orbital momentum. The magnetization is the average value of m, and we find it by means of the distribution function obtained by the kinetic Boltzmann equation in semiclassical quantum conditions, as we have done for the calculation of the conductivity.[5] The orbital magnetic momentum is given by the derivative of the Hamiltonian with respect to the magnetic field. Neglecting the spin-orbit interaction in nonrelativistic conditions, and with the external magnetic field $\vec{\mathrm{B}}_0$ along the z axis, the expectation value of m in the energy representation is[6]

$$m = \frac{-e\Omega}{cm_c} r_1^2 = -\frac{2\epsilon_t}{cm_c\Omega'} = -\frac{2e}{cm_c}(n+\tfrac{1}{2})\hbar ,$$

with

$$r_1^2 = (p_x^2 + p_y^2)(m\Omega)^{-2} = 2\epsilon_t / m_c\Omega^2$$

the orbit radius, $\epsilon_t = (n+\frac{1}{2})\hbar\Omega$ the transverse energy, and $\Omega = eB_0/cm_c$ the cyclotron frequency.

The magnetization M is given by

$$M = \int dp^3 mF = -\frac{2e}{cm_c}\int dp^3 (n+\tfrac{1}{2})\hbar(f_0+f) , \qquad (1)$$

where f_0 is the equilibrium distribution function and f is the first-order term of the development of F in the EM field amplitude. The expression found for f in Ref. 5 will be used, recalling that it is valid under the following conditions: A quadratic dispersion law model with high frequencies, $\omega\tau \gg 1$, low temperature, $\epsilon_F \gg k_B T$ (ϵ_F = Fermi energy), and high magnetic field intensities,

$$\hbar\Omega \gg \hbar\omega, k_B T .$$

In Eq. (1) we use the stationary value of the orbital magnetic moment jointly with the time-dependent distribution function f. In order to justify this assumption one must consider that the expectation value of the magnetic orbital moment in the energy representation refers to times of the order of the revolution time of electron orbits $2\pi/\Omega$, which in our case is much smaller than the relaxation time τ. In fact, up to microwave frequencies and for high magnetic field, we have $\Omega \gg \omega$; in addition, the Boltzmann-equation solution f is valid under the hypothesis $\omega\tau \gg 1$—collecting these conditions we have $\tau \gg 2\pi/\Omega$.

Let us calculate the equilibrium term M_0 in Eq. (1) to test the validity of the equation, because in the stationary state, one must obtain the known expression for the magnetization of the de Haas—van Alphen effect. With the use of the coordinates ϵ, p_z, φ, M_0 is given by

$$M_0 = -\frac{1}{(2\pi\hbar)^3}\frac{2e}{cm_c}\sum_{n=0}^{\infty}\int_{-\infty}^{\infty} dp_z \int_0^{2\pi} d\varphi\, \hbar^2\Omega(n+\tfrac{1}{2}) m_c f_0 , \qquad (2)$$

where the integral in $d\epsilon$, as is usually done in semiclassical quantization, is transformed according to

$$\int_0^{\infty} d\epsilon \cdots \hbar\Omega \sum_{n=0}^{\infty} \cdots .$$

In Eq. (2) the variable $\varphi = \Omega t$ is the angle that locates the point-P representative of the carrier state on the orbit (see

Fig. 2 of Ref. 5). Because the integrand does not depend on φ, Eq. (2) becomes

$$M_0=-\frac{2e\hbar^2\Omega}{(2\pi\hbar)^3}\frac{2\pi}{c}\sum_n\int dp_z\frac{n+\frac{1}{2}}{e^{(\epsilon_F-\epsilon)/k_BT}+1} . \tag{3}$$

The hypothesis $n \gg 1$ has to be assumed in order to compare the final result with that obtained by Lifshitz and Kosevich for the de Haas–van Alphen effect.[7] In the following we will use the usual development method with the Poisson's formula and consider only the magnetization part oscillating with the external magnetic field. Furthermore, we will suppose that the Landau quantum number n, considered to be a function of the energy ϵ and momentum p_z, has an extremum for which the following development is valid:

$$n(\epsilon,p_z)=n_m(\epsilon)+\frac{1}{2}\left[\frac{\partial^2 n}{\partial p_z^2}\right]_m(p_z-p_{zm})^2 . \tag{4}$$

Then, one obtains, for the oscillating part $\widetilde{M}_0$, the expression

$$\widetilde{M}_0=4\,\mathrm{Re}\left[\frac{2\pi e^2\hbar^2B_0}{c^2m_c(2\pi\hbar)^3}e^{\pm i\pi/4}n_m(\epsilon_F)\left[\frac{\partial n}{\partial\epsilon}\right]_{m,\epsilon_F}\left|\frac{\partial^2 n}{\partial p_z^2}\right|_{m,\epsilon_F}^{-1/2}\sum_{l=1}^{\infty}l^{-1/2}\int d\epsilon\frac{e^{i2\pi l n_m}}{e^{(\epsilon_F-\epsilon)/k_BT}+1}\right] , \tag{5}$$

where Re is the real part. The integration is carried out taking into account that only the exponential part is a rapidly varying function while the remaining factor in the integral can be treated as a constant. Let us use the rules of semiclassical quantization,

$$n=\frac{c}{2\pi\hbar eB_0}S(\epsilon,p_z),\quad n_m=\frac{c}{2\pi\hbar eB_0}S_m(\epsilon),\quad \left[\frac{\partial^2 n}{\partial p_z^2}\right]_{m,\epsilon_F}=\frac{c}{2\pi\hbar eB_0}\left[\frac{\partial^2 S}{\partial p_z^2}\right]_{m,\epsilon_F},\quad \frac{\partial n}{\partial\epsilon}=\frac{cm_c}{\hbar eB_0} ,$$

where m_c is the cyclotron mass given by $(\partial S/\partial\epsilon)/2\pi$ and $S(\epsilon,p_z)$ is the cross-sectional area of the constant-energy surface $\epsilon(\vec{p})=\epsilon$ crossed with a plane orthogonal to $\vec{B}_0$ for a given value of p_z. With simple calculations Eq. (5) becomes

$$\widetilde{M}_0=-\frac{1}{(2\pi\hbar)^3}\left[\frac{e\hbar}{c}\right]^{3/2}\frac{B_0^{1/2}}{\sqrt{2\pi}}\sum_{l=1}^{\infty}\frac{S_m}{l^{3/2}m_c}\psi(l\lambda)\left|\frac{\partial^2 S}{\partial p_z^2}\right|_{m,\epsilon_F}^{-1/2}\sin\left[\frac{cS_m}{e\hbar B_0}l\pm\frac{\pi}{4}\right] , \tag{6}$$

with $\psi(l\lambda)=l\lambda/\sinh(l\lambda)$ and $\lambda=2\pi^2k_BTcm_c/eB_0\hbar$. Equation (6) is exactly the same expression reported in the literature for the de Haas–van Alphen effect.[1,7]

Let us now calculate the time-dependent part of M as follows:

$$M=-\frac{2e}{cm_c}\int dp^3(n+\tfrac{1}{2})\hbar f . \tag{7}$$

Integrals in the variable φ have to be calculated with the use of the Boltzmann-equation solution.[5,8] We assume that the axis z is directed along $\vec{B}_0$, that the axis x is parallel to the sample surface, and that the wave vector $\vec{k}$ is in the plane (y,z), and we then write

$$\int_0^{2\pi}d\varphi\int_{-\infty}^{\varphi}d\varphi'\exp(-\beta\varphi+iB\cos\varphi-iA\sin\varphi)[E_xv_x(\varphi')+E_yv_y(\varphi')+E_zv_z(\varphi')]\exp(\beta\varphi'+iA\sin\varphi'-iB\cos\varphi') , \tag{8}$$

where $\gamma_z=\alpha_zk_z/m_0$, $\beta=[\nu-i(\omega-\gamma_zp_z)]/\Omega$, and ω is the EM field frequency depending on the time as $\exp(i\omega t)$. A and B are functions of the wave vector $\vec{k}$, the energy ϵ, the magnetic field B_0, and the angle θ that the field direction makes with the normal to the sample surface. The expressions of A and B, in the case of a quadratic dispersion law, are given in the Appendix together with $v_x(\varphi)$, $v_y(\varphi)$, and $v_z(\varphi)$. In the following we shall use the exponential developments for the functions $\sin\varphi$ and $\cos\varphi$:

$$\exp(iD\sin\varphi)=\sum_{s=-\infty}^{\infty}J_s(D)\exp(is\varphi),\quad \exp(iD\cos\varphi)=\sum_{s=-\infty}^{\infty}i^sJ_s(D)\exp(is\varphi) . \tag{9}$$

One finds that the integral (8) contains terms, according to the EM field polarization, of the type

$$\begin{aligned}I_1&=\sum_{s,s',s'',s'''=-\infty}^{\infty}(-1)^{s'+s''}(i)^{s'+s'''}J_s(A)J_{s'}(B)J_{s''}(A)J_{s'''}(B)\frac{\pi\gamma\pm i\pi}{\gamma^2+1} ,\\ I_2&=\sum_{s,s',s'',s'''=-\infty}^{\infty}(-1)^{s'+s''}(i)^{s'+s'''}J_s(A)J_{s'}(B)J_{s''}(A)J_{s'''}(B)\frac{\pm\pi\gamma-\pi}{\gamma^2+1} ,\end{aligned} \tag{10}$$

with $\gamma=\beta+i(s+s')$. In the case $\omega\tau\gg 1$ we calculate the terms $\pi\gamma\pm i\pi/(\gamma^2+1)$ and $\pm\pi\gamma-\pi/(\gamma^2+1)$ by means of the identity $\nu=1/\tau$,

$$\lim_{\nu\to 0}\frac{1}{x-i\nu}=P(1/x)+i\pi\delta(x)\ .$$

For the oscillating part, because of the $\delta(x)$ term, we obtain, from Eq. (10),

$$\tilde{I}_1=2\pi^2\Omega\delta(\omega-\gamma_z p_z)J_0J_1\frac{A}{(A^2+B^2)^{1/2}}\ ,\quad \tilde{I}_2=2\pi^2\Omega\delta(\omega-\gamma_z p_z)J_0J_1\frac{B}{(A^2+B^2)^{1/2}}\ . \tag{11}$$

After some straightforward calculations the quantum part amplitude is

$$\begin{aligned}\tilde{M}&=-\frac{2e^2\hbar^2\Omega}{c(2\pi\hbar)^3}\sum_{n=0}^{\infty}\int dp_z(n+\tfrac{1}{2})\frac{df_0}{d\epsilon}2\pi^2\delta(\omega-\gamma_z p_z)\left[\frac{\alpha_z p_z}{m_0}E_z+\frac{k^2(n+\frac{1}{2})\hbar}{2\Omega}\left[g(\theta)\Phi(\theta,\vec{\mathrm{E}})-\frac{\alpha_z p_z}{m_0}E_z\right]\right]\\&=\frac{2e^2\hbar^2\Omega}{ck_BT(2\pi\hbar)^3}\frac{2\pi^2}{\gamma_z}\sum_{n,i}(n+\tfrac{1}{2})\cosh^{-2}\left[\frac{\epsilon_F-(n+\frac{1}{2})\hbar\Omega-(1/2m_z)(\omega/\gamma_z)_i^2}{k_BT}\right]\\&\quad\times\left\{\left[\frac{\omega}{k_z}\right]_i E_z+\frac{k^2(n+\frac{1}{2})\hbar}{2\Omega}\left[g(\theta)\Phi(\theta,\vec{\mathrm{E}})-E_z\left[\frac{\omega}{k_z}\right]_i\right]\right\}.\end{aligned} \tag{12}$$

The index i runs over the possible values of ω/k, i.e., it depends on the EM dispersion law $\omega(k)$. $\Phi(\theta,\vec{\mathrm{E}})$ is a function of θ and the EM field amplitude $g(\theta)$ depends on θ and the effective masses; both functions are given in the Appendix. From Eq. (12) one can see that the quantum part of the time-dependent magnetization is of the first order in the wave field, because $\Phi(\theta,\vec{\mathrm{E}})$ is a linear function of $\vec{\mathrm{E}}$.

V. SOLUTION OF THE MAXWELL EQUATIONS AND CALCULATION OF THE ABSORPTION

Let us designate $\vec{\mathrm{B}}$ the time-dependent magnetic field, $\vec{\mathrm{B}}_0$ the static magnetic field, and $\vec{\mathrm{M}}(\vec{\mathrm{B}})$ the time-dependent magnetization. Since $B \ll B_0$ the expansion[3]

$$\begin{aligned}\vec{\mathrm{M}}(\vec{\mathrm{B}})&=\vec{\mathrm{M}}(\vec{\mathrm{B}}_0+\vec{\mathrm{B}})-\vec{\mathrm{M}}(\vec{\mathrm{B}}_0)\cong\left[\frac{\partial\vec{\mathrm{M}}}{\partial\vec{\mathrm{B}}}\right]_0\vec{\mathrm{B}}\\&=\sin^2\theta\left[\frac{\partial\vec{\mathrm{M}}}{\partial\vec{\mathrm{B}}}\right]_0\vec{\mathrm{B}}\end{aligned} \tag{13}$$

is valid. The Maxwell equations are

$$\begin{aligned}&\mathrm{rot}\left\{\vec{\mathrm{B}}\left[1-4\pi\sin^2\theta\left[\frac{\partial\vec{\mathrm{M}}}{\partial\vec{\mathrm{B}}}\right]_0\right]\right\}=\frac{4\pi}{c}\vec{\mathrm{j}}=\frac{4\pi}{c}\vec{\sigma}\cdot\vec{\mathrm{E}}\ ,\\&\mathrm{rot}\vec{\mathrm{E}}=-\frac{1}{c}\frac{\partial\vec{\mathrm{B}}}{\partial t}\ .\end{aligned} \tag{14}$$

Equations (14), in the harmonic regime $\exp i(\vec{\mathrm{k}}\cdot\vec{\mathrm{r}}-\omega t)$, become[5,8,9]

$$\left\{k^2\delta_{\alpha\beta}\left[1-4\pi\sin^2\theta\left[\frac{\partial M}{\partial B}\right]_0\right]-\frac{4\pi i\omega}{c^2}\sigma_{\alpha\beta}\right\}E_\beta+2E'_\beta(0)=0\ . \tag{15}$$

Introducing the complex variable $\omega/kv_a=\alpha+i\beta$ with $v_a=(B_0^2/4\pi^2m_0\delta N)^{1/2}$, the Alfvèn velocity, an equation $F(\xi)=0$ (with $\xi^2=\alpha^2-\beta^2$) is obtained from Eq. (15), whose solution gives the allowed values for the wave vector $\vec{\mathrm{k}}$.

In order to obtain the function $F(\xi)$, we shall introduce into Eq. (15) the expressions for the conductivity tensor components $\sigma_{\alpha\beta}$ found in Ref. 5 and the expression of the magnetization we derived in this paper. The consistency conditions of Eqs. (15), in terms of the variables α and β are

$$\begin{aligned}&1-(\alpha^2-\beta^2)[1+c_2G_u^2(\alpha,\beta)]+c_1\beta G_u(\alpha,\beta)=0\ ,\\&2\beta[1+c_2G_u^2(\alpha,\beta)]+c_1G_u(\alpha,\beta)=0\ .\end{aligned} \tag{16}$$

The expressions c_1 and c_2 are given in the Appendix. The term $G_u(\alpha,\beta)$ is equal to the term defined by Eq. (7) of Ref. 10 divided by

$$I_3=1-4\pi\sin^2\theta\left[\frac{\partial M}{\partial B}\right]_0 . \tag{17}$$

To find the explicit expression of $G_u(\alpha,\beta)$ in Eq. (16), we have to insert into Eq. (17) the expression for the magnetization. As we have stressed at the end of the preceding section, the time-dependent term of M is of the first order in the EM field, therefore it is negligible in a first-order theory [see Eq. (15)]. For the stationary term of M, Eq. (6) cannot be used since it has been obtained in the limit $n \gg 1$, no more valid than when $\hbar\Omega \gg \hbar\omega, k_BT$. In this case transitions between Landau levels in the photon absorption are forbidden ($\Delta n=0$). The $\tilde{M}_0$ evaluation will be carried under the conditions $\hbar\Omega \gg \hbar\omega$ and very low, but finite, temperature, as follows:

$$\frac{\partial \widetilde{M}_0}{\partial B_0} = -\frac{2e\hbar^2}{c(2\pi\hbar)^3} 2\pi\Omega \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} dp_z (n+\tfrac{1}{2}) \frac{df_0}{d\epsilon}\frac{d\epsilon}{dB}$$

$$= -\frac{2e\hbar^2}{c(2\pi\hbar)^3}\frac{2\pi\hbar e\Omega}{cm_c k_B T} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} dp_z (n+\tfrac{1}{2})^2 \cosh^{-2}\left[\frac{\epsilon_F-(n+\tfrac{1}{2})\hbar\Omega-\dfrac{p_z^2}{2m_z}}{k_B T}\right] . \tag{18}$$

In the hypothesis $T\to 0$, as $(df_0/d\epsilon)=\delta(\epsilon_F-\epsilon)/k_B T$, one obtains the expression

$$\frac{\partial \widetilde{M}_0}{\partial B_0} = \frac{2e\hbar^2}{c(2\pi\hbar)^3}\frac{2\pi e\hbar\Omega}{cm_c k_B T} \sum_{n=0}^{n_F} (n+\tfrac{1}{2})^2 m_z \{2m_z[\epsilon_F-(n+\tfrac{1}{2})\hbar\Omega]\}^{-1/2} ,$$

which is divergent when a Landau level crosses the Fermi level. Replacing in Eq. (18) the variable p_z with ϵ by means of $\epsilon=(n+\frac{1}{2})\hbar\Omega+p_z^2/2m_z$, and by means of partial integrations, we have

$$\int_{-\infty}^{\infty} dp_z \cosh^{-2}\left[\epsilon_F-(n+\tfrac{1}{2})\hbar\Omega-\frac{p_z^2}{2m_z}\right] = -4\int_{(\epsilon_F-b_n)/k_B T}^{-\infty} [2m_z(\epsilon-b_n-k_B T)]^{1/2}\frac{\sinh z}{\cosh^3 z}dz , \tag{19}$$

where $b_n=(n+\frac{1}{2})\hbar\Omega$. As the main contribution to the preceding integral is due to the exponential part, we can substitute the integrand function with its value when the exponential is maximum. Equation (19) then becomes

$$\int_{-\infty}^{\infty} dp_z \cdots = -4\frac{\{2m_z[\epsilon_F-b_n-k_B T\,\mathrm{arctanh}(\frac{1}{3})^{1/2}]\}^{1/2}}{\cosh^2[(\epsilon_F-b_n)/k_B T]} ,$$

and

$$\frac{\partial \widetilde{M}_0}{\partial B_0} = -\frac{2e^3}{\pi^2}\frac{\sqrt{2m_z}}{c^3 m_c^2 k_B T}\sum_{n=0}^{\infty}\frac{(n+\frac{1}{2})^2[\epsilon_F-(n+\frac{1}{2})^2\hbar\Omega+0.51k_B T]^{1/2}}{\cosh^2[(\epsilon_F-(n+\frac{1}{2})\hbar\Omega)/k_B T]} . \tag{20}$$

From Eq. (19) one obtains, for $F(\xi)$,

$$F(\xi)=1-\xi^2[1+c_2G_u^2(\xi)]-\frac{c_1^2G_u^2(\xi)}{2[1+c_2G_u^2(\xi)]} , \tag{21}$$

where $G_u(\xi)$ is obtained by introducing into Eq. (15) $(\partial\widetilde{M}_0/\partial B_0)$, as given by Eq. (20), and the tensor coefficients of the conductivity $\sigma_{\alpha\beta}$, as given in Ref. 5. The expression for $G_u(\xi)$ is

$$G_u(\xi)=\frac{\sum_{n=0}^{\infty}(n+\frac{1}{2})^2\cosh^{-2}[a(x_n^2-\xi^2)]}{1+4\pi L\sin^2\theta\, c_3\sum_{n=0}^{\infty}[(n+\frac{1}{2})^2(x_n^2+x_0)^{1/2}/\cosh^2(ax_n^2)]} , \tag{22}$$

with

$$\epsilon_F-(n+\tfrac{1}{2})\hbar\Omega=\frac{v_a^2 m_0}{2\alpha_z\cos^2\theta}x_n^2 ,$$

$$x_0=\frac{0.51k_B T}{v_a^2 m_0}2\alpha_z\cos^2\theta ,$$

$$c_3=\frac{e^3(2m_z)^{1/2}}{2\pi^4c^2m_c^2k_B T}\frac{B_0^3}{\overline{\delta}N\,2\alpha_z\cos^2\theta} .$$

The numerical solution of equation $F(\xi)=0$ is obtained specifying the values of parameters (effective-mass coefficients, Fermi level, carrier number, etc.) for bismuth. The magnetization term is also multiplied by a numerical factor L which must be considered as a free parameter in the numerical fit.

In Figs. 1 and 2 the behaviors of the function $F(\xi)$ as a function of ξ^2 are reported for different values of the magnetization parameter L. The angle θ is 88°. One can see that for $L=1$ (value appropriate for bismuth) the quantum waves are present. For Fig. 1 the external magnetic field is $B_0=5$ kG, and for Fig. 2, $B_0=3$ kG.

By increasing L, the number of solutions of the equation $F(\xi)=0$ reduces, and for $B_0=5$ kG, when $L>10$, there is no intersection of $F(\xi)$ with the axis ξ. In other words, the equation $F(\xi)=0$ is no more satisfied. From a physical point of view it means that by increasing the de Haas–van Alphen term, quantum propagation disappears and a sole solution exists for $\xi^2=1$, which gives the classi-

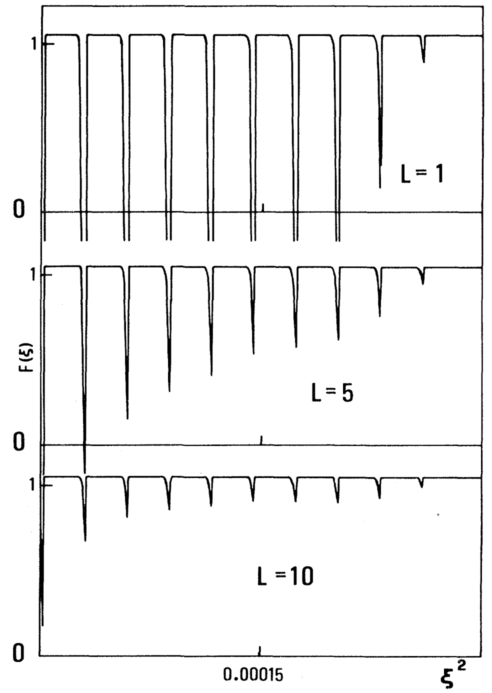

FIG. 1. Behavior of $F(\xi)$ as a function of ξ^2 for different values of the magnetization parameter L with $\theta=80°$ and $B_0=5$ kG.

cal Alfvèn waves with a propagation velocity v_a. When $B_0=3$ kG the equation $F(\xi)=0$ has no solution for $L>5$ (Fig. 2). Therefore the quantum oscillations are only of the Shubnikov–de Haas type, because the magnetization contribution, if it exists, reduces the quantum effect and does not substitute for the conductivity term. The role of magnetization with regard to the disappearance of quantum oscillations, increases under the same conditions for L for lower values of the external magnetic field.

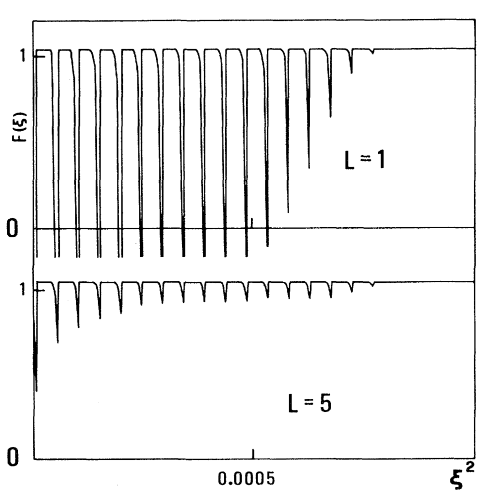

FIG. 2. Same as Fig. 1 with $B_0=3$ kG.

An experimental feature[4] peculiar to oscillations related to the quantum EM waves of high frequency is the disappearance of oscillations by increasing the external magnetic field B_0. As we mentioned, the magnetization term does not explain this effect because of the role played by the intensity of the magnetic field. The disappearance of oscillations for high values of the external magnetic field has been previously interpreted by considering only the conductivity contribution; the results of this paper confirm this hypothesis.

One can conclude that the present theory correctly explains the role played by the conductivity in the quantum EM oscillations as experimentally found by Zabrisky *et al.* and ourselves.[3,4] A direct contribution of the time-dependent magnetization might exist, as it follows from Eqs. (12) and (15), only to second order in the EM field.

APPENDIX

Assuming an ellipsoidal dispersion law

$$\epsilon=\frac{1}{2m_0}(a_{11}p_x^2+a_{22}p_y^2+a_{33}p_z^2+2a_{12}p_xp_y+2a_{13}p_xp_z+2a_{23}p_yp_z)\ ,$$

and setting

$$A=\frac{k}{\Omega}(u_x\sin\theta\sin\alpha+u_xu_1\cos\theta)\ ,$$

$$B=\frac{k}{\Omega}(u_y\sin\theta\cos\alpha+u_yu_2\cos\theta)\ .$$

α is the angle between the principal planes of the constant-energy surface and the plane in which $\vec{B}_0$ rotates, given by

$$\tan\alpha=\frac{-(a_{11}-a_{22})\pm[(a_{11}-a_{22})^2+a_{12}^2]^{1/2}}{a_{12}}\ .$$

The quantities reported in the paper are given by the following expressions:

$$u_x=\frac{r(\bar{\alpha}_x)^{1/2}}{m_0},\quad u_y=\frac{r(\bar{\alpha}_y)^{1/2}}{m_0}\ ,$$

$$u_1=\gamma_x\cos\alpha+\gamma_y\sin\alpha,\quad u_2=\gamma_y\cos\alpha-\gamma_x\sin\alpha\ ,$$

$$\gamma_x=\frac{a_{22}a_{13}-a_{23}a_{12}}{a_{12}a_{22}-a_{12}^2},\quad \gamma_y=\frac{a_{11}a_{23}-a_{13}a_{12}}{a_{11}a_{22}-a_{12}^2}\ ,$$

$$r^2=2m_0\left[\epsilon-\frac{\bar{\alpha}_zp_z^2}{2m_z}\right]\ ,$$

$$\bar{\alpha}_x=a_{11}\cos^2\alpha+a_{22}\sin^2\alpha+a_{12}\sin\alpha\cos\alpha\ ,$$

$$\bar{\alpha}_y=a_{11}\sin^2\alpha+a_{22}\cos^2\alpha-a_{12}\sin\alpha\cos\alpha\ ,$$

$$\bar{\alpha}_z=a_{33}+a_{22}\gamma_y^2+a_{12}\gamma_x\gamma_y+a_{11}\gamma_x^2-a_{13}\gamma_x-a_{23}\gamma_y\ ,$$

$$v_x(\varphi)=g_1(r)\cos\varphi-g_2(r)\sin\varphi\ ,$$

$$v_y(\varphi)=g_3(r)\cos\varphi-g_4(r)\sin\varphi\ ,$$

$$v_z(\varphi)=g_5(r)\cos\varphi-g_6(r)\sin\varphi+\frac{\bar{\alpha}_zp_z}{m_0}\ ,$$

$$g_1(r)=u_x\cos\alpha,\quad g_2(r)=u_y\sin\alpha,\quad g_3(r)=u_x\sin\alpha\ ,$$

$$g_4(r)=u_y\cos\alpha,\quad g_5(r)=u_x u_1,\quad g_6(r)=u_y u_2\ ,$$

$$\Phi(\theta,\vec{\mathrm{E}})=\frac{\Omega}{krm_0}[\,\bar{\alpha}_x^{1/2}A\,(E_x\cos\alpha+E_y\sin\alpha+E_z u_1)$$

$$+\bar{\alpha}_y^{1/2}B(-E_x\sin\alpha+E_y\cos\alpha+E_z u_2)]\ ,$$

$$f^2(\theta)=m_0^{-2}[\bar{\alpha}_x^{1/2}\sin\theta\sin\alpha+\bar{\alpha}_x^{1/2}(\gamma_x\cos\alpha+\gamma_y\sin\alpha)\cos\theta]^2$$

$$+m_0^{-2}[\,\bar{\alpha}_y^{1/2}\cos\alpha\sin\theta+\bar{\alpha}_y^{1/2}$$

$$\times(\gamma_y\cos\alpha-\gamma_x\sin\alpha)\cos\theta]^2\ ,$$

$$c_1=\frac{a_{xx}+a_{yy}}{(b_{xx}+b_{yy})^{1/2}}\frac{(\pi/N)^{1/2}}{4\pi m_0^{3/2}k_B T}\frac{f^2(\theta)(\bar{\alpha}_x\bar{\alpha}_y)^{1/2}}{\bar{\alpha}_z\cos\theta}B_0^2\ ,$$

$$c_2=c_1^2\frac{a_{xx}a_{yy}}{(a_{xx}+a_{yy})^2},\quad a_{xx}=\frac{\bar{\alpha}_x\bar{\alpha}_y(\sin\theta+\gamma_y\cos\theta)^2}{\bar{\alpha}_x A'^2+\bar{\alpha}_y B'^2}\ ,$$

$$a_{yy}=\frac{\bar{\alpha}_x\bar{\alpha}_y\gamma_x^2}{\bar{\alpha}_x A'^2+\bar{\alpha}_y B'^2}\ ,$$

$$A'=A\frac{m_0\Omega}{k\Gamma\bar{\alpha}_x^{1/2}},\quad B'=B\frac{m_0\Omega}{k\Gamma\bar{\alpha}_y^{1/2}}\ ,$$

$$\delta=\left[\frac{2\bar{\alpha}_y\sin^2\theta+\bar{\alpha}_x}{\bar{\alpha}_x\bar{\alpha}_y}\right]_{\text{electrons}}+\cdots\ .$$

In the last equation the ellipsis represents an analogous term for the holes. N is the number of carriers.

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