

Experimental evidence versus exchange theory of resistivity saturation

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In a recently published paper Laughlin proposes an exchange-interaction theory of resistivity saturation based on the Altshuler-Aronov depression of the density of states at the Fermi level. In my view this contradicts the many experimental data on disordered metals in which the density of states does not decrease relative to the ordered phase, yet resistivity saturation takes place. Exchange-correlation (and localization) effects become important in the regime in which $k_F L \sim 1$ and $\sigma \sim \sigma_{\min}$. This is not the case in typical saturated metals, where $k_F L \sim 3-6$ and $\sigma_{\text{sat}} \sim (10-30)\sigma_{\min}$.

I. INTRODUCTION

In a recently published paper¹ Laughlin proposes that the phenomenon of resistivity saturation^{2,3} results from an interplay between the effects of disorder (static or thermal) and the exchange interaction. Specifically it is claimed that the saturation results from the universal Altshuler-Aronov⁴ depression of the density of states at the Fermi surface and the corresponding increase in the Fermi velocity with increasing disorder. The parallel-resistor model^{5,2,3}

$$\frac{1}{\rho} = \frac{1}{\rho_{\text{ideal}}} + \frac{1}{\rho_{\text{sat}}}, \quad (1)$$

which was shown to describe the effect of saturation fairly well, according to Ref. 1, is an approximate relation which happens to give values of ρ that are close to those predicted by the exchange interaction theory. To the extent that (1) agrees with Laughlin's formula for ρ he writes

$$\frac{N(0)}{N_0(0)} \simeq \left[1 - \frac{\rho}{\rho_{\text{sat}}} \right]^{1/2}, \quad (2)$$

where $N_0(0)$ is the initial unsuppressed density of states, $N(0)$ is the density of states in a disordered material with resistivity ρ , and ρ_{sat} is the quantity which enters the parallel resistor model and can be determined experimentally. Let us examine formula (2).

First, we note that in the limit $\rho = \rho_{\text{sat}}$ formula (2) predicts $N(0) \simeq 0$. This is contrary to the fact that many completely disordered and amorphous metals possess a finite, sometimes rather high, $N(0)$. One can cite here examples of amorphous Mo-Re alloys and disordered NbN, i.e., superconductors with high transition temperatures and flat $\rho(T)$. Allowing for the approximate nature of (2), we shall not base our argument on this limiting case, and will instead consider that in a highly disordered metal ρ can easily be as high as, say $(0.7-0.9)\rho_{\text{sat}}$. Then (2) predicts $N(0)/N_0(0) \sim 0.3-0.5$. Indeed, a reduction of $N(0)$ upon disordering takes place in a number of materials which, as a rule, possess high $N(0)$ in the ordered state: for example, in high- T_c A15 compounds Nb₃Sn and V₃Si. The decrease of $N(0)$ with increasing ρ agrees with the relation (2), at least in the trend. It may be worth noting that this decrease was previously ascribed to the electron lifetime smearing of the density-of-states structure

near the Fermi energy: an explanation first proposed by Crow *et al.*⁶ for disordered Nb and subsequently much discussed in the literature on A15 compounds.⁷ This viewpoint finds strong support in calculations of Testardi and Mattheiss,⁸ in which the theoretical band structures are combined with electron lifetime effects, resulting in substantial (factor of 2-3) $N(0)$ decrease with disorder in V₃Si, Nb₃Ge, Nb₃Sn, and Nb.

However, this is not the main point of this paper. The main point is to show that in a number of disordered metals with saturated or nearly saturated resistivity no reduction of $N(0)$ is observed compared to the value of $N(0)$ in the ordered state. In other words, while some metals show $N(0)$ decrease upon disordering, others have $N(0)$ increased or unchanged; however, all of them show typical saturation behavior. If this is the case, relation (2) is not universal, and the exchange theory of Ref. 1 is not a common cause of saturation in various metals. Yet a good deal of evidence suggests that resistivity saturation is a universal effect manifesting itself in all metals with sufficiently short electronic mean free path. Laughlin apparently subscribes to the same view: In Fig. 1 of his paper examples of the saturation behavior in different metallic systems are presented, and his arguments based on Fermi liquid theory are, of course, of a very general nature.

In what follows we present several cases which, although not equally unambiguous, support the fact that relation (2) is not universal. We do not attempt to present all the existing evidence.

Ideally we would like to have simultaneously specific heat data, photoemission data, tunneling data, T_c , and data on resistivity as a function of temperature for both ordered and disordered phases of the same material. Unfortunately, such comprehensive sets of data do not seem to exist. We shall try to make the best of what is available.

Compiling the list of materials which show no depression of $N(0)$ upon disordering it is natural to start with metals showing an enhancement of the superconducting transition temperature T_c in the amorphous (disordered) state. Such are most of the simple superconductors (Sn, In, Bi, Ga, Be, Zn, Al) and many of their alloys, as well as some transition metals (Mo, W) and their alloys. An enhancement of T_c upon disordering almost always implies an increase of the electron-phonon coupling constant λ . This is evident from the tunneling data, where avail-

able, as well as from the McMillan relation⁹ between T_c and λ :

$$T_c = \frac{\langle \omega \rangle}{1.2} \exp \left[-\frac{1.04(1+\lambda)}{\lambda - 0.62\lambda\mu^* - \mu^*} \right], \quad (3)$$

where μ^* is the Coulomb pseudopotential and $\langle \omega \rangle$ is the suitably averaged¹⁰ phonon frequency. With the use of McMillan's⁹ factorization of λ , it can be written as

$$\lambda = \frac{N(0)\langle I^2 \rangle}{M\langle \omega^2 \rangle}, \quad (4)$$

where $\langle I^2 \rangle$ is the average over the Fermi surface of the electron-phonon matrix element and M is the atomic mass. While the increase in λ , if observed, does not directly imply an increase in $N(0)$, the relation (4) allows setting of a reasonable lower limit for the possible decrease of $N(0)$ consistent with the data. This is made possible by the existence of certain empirical observations concerning the parameters entering (4). Finally, specific-heat data coupled with T_c measurements, as well as photoemission data, allow rather unambiguous conclusions about the behavior of $N(0)$.

II. EXPERIMENTAL DATA

A. Mo and Be data

Let us illustrate some of these points using data on Mo (Table I). We take parameters for crystalline Mo from the work of Butler,¹¹ which should be consulted for further references. For amorphous Mo we use the tunneling data of Kimhi and Geballe.¹² We have employed the relation $\langle \omega \rangle \simeq \frac{1}{2}(\omega_{\log} + \langle \omega^2 \rangle^{1/2})$ (Ref. 13) to obtain $\langle \omega \rangle$ in a crystalline state, since $\langle \omega \rangle$ rather than ω_{\log} is given in Ref. 12 for amorphous Mo. We use the value¹² of $\mu^* = 0.08$ in both crystalline and amorphous cases; it is consistent with the empirical value $\mu^* = 0.09$ given for crystalline Mo by McMillan.⁹ The use of a conventional $\mu^* = 0.13$, as was done by Butler¹¹ introduces a relatively small change in λ . From (4) and Table I we find

$$\frac{N_a(0)\langle I_a^2 \rangle}{N_c(0)\langle I_c^2 \rangle} = \frac{\lambda_a \langle \omega_a^2 \rangle}{\lambda_c \langle \omega_c^2 \rangle} \simeq 0.9. \quad (5)$$

Here the subscripts a and c refer to the amorphous and crystalline materials, respectively. Resistivity of the amorphous Mo ($200 \mu\Omega \text{ cm}$) (Ref. 12) is typical of a saturated transition metal. A free-electron estimate based on the assumption that the mean free path is equal to the nearest-neighbor distance in the lattice agrees with this value of ρ_{sat} for Mo if the number of free electrons per atom is taken to be between 1 and 2. Hence formula (2) predicts a

strong decrease in $N(0)$ between crystalline and amorphous Mo. However, from (5) we see that in order for $N(0)$ to decrease, for example, by a factor of 2, $\langle I^2 \rangle$ must increase by a factor of 1.8. Such a large increase of $\langle I^2 \rangle$ is unrealistic; indeed, $\langle I^2 \rangle$ is expected to decrease in the amorphous state due to the increased atomic volume: $\langle I^2 \rangle \propto V_a^{-2}$ (Ref. 11). Furthermore, empirical correlations of Varma and Dynes¹⁴ suggest that for a given transition-metal series the ratio $\langle I^2 \rangle / M\langle \omega^2 \rangle$ is approximately constant. If we assume this correlation to be valid in crystalline-amorphous Mo, then $N_a(0)$ must be considerably higher than $N_c(0)$. The increase in $N_a(0)$ compared to $N_c(0)$ was indeed observed¹⁵ in photoemission experiments [ultraviolet photoelectron spectroscopy (UPS)].

Compared to the case of Mo, substantially less data are available on crystalline and amorphous Be which has T_c changing from 0.026 K in the bulk crystalline material to 9.3 K in the amorphous one. Unlike the case of Mo, it is claimed¹⁶ that tunneling experiments have established the absence of a substantial Θ_D decrease in the amorphous phase. Hence the burden of responsibility for the enhanced λ lies entirely on $N(0)\langle I^2 \rangle$. If we use the pre-factor $\Theta_D/1.45$ instead of $\langle \omega \rangle/1.2$ in (3) and keep the same $\Theta_D = 1380$ (Ref. 16) in both phases, we find λ to be enhanced from 0.28 to 0.51. Allowing for some 15–20% softening of Θ_D in the amorphous phase does not change the value of λ_a appreciably. So we find

$$\frac{N_a(0)\langle I_a^2 \rangle}{N_c(0)\langle I_c^2 \rangle} \simeq 1.8.$$

We may argue as before that a strong increase in $\langle I^2 \rangle$ is unlikely, hence $N(0)$ must have increased in the amorphous phase. We will comment further on this point in the discussion of amorphous simple metals (Sec. II D).

B. Mo₃Ge data

Another example is provided by an $A15$ compound, Mo₃Ge. As was shown by Gurvitch *et al.*¹⁷ and by Lehmann *et al.*,¹⁸ T_c of this compound increases from 1.45 K in the ordered crystalline state to 5–6 K in the highly disordered, saturated state. In particular, $T_c = 4.25$ K corresponds to $\rho_0 \simeq 80 \mu\Omega \text{ cm}$. Careful resistivity data taken on samples progressively damaged by α particles when fitted to the parallel-resistor formula (1), show that $\rho_{\text{sat}} = 120 \mu\Omega \text{ cm}$.¹⁷ Hence at $\rho_0 \simeq 80 \mu\Omega \text{ cm}$, formula (2) predicts $N(0)/N_0(0) \simeq 0.58$. However, recent specific-heat measurements by Ghosh and Caton¹⁹ on neutron-irradiated Mo₃Ge samples indicate that $N(0)$ is unchanged within $\pm 5\%$ for the same sample with $T_c = 1.45$ and 4.25 K. The corresponding increase in λ from 0.4 to 0.54 [as deduced from (3)] can be accounted for by the observed decrease of the Debye temperature Θ_D : from 392 to 322 K.¹⁹ In fact, making use of the apparent constancy of $N(0)$ and assuming that $\langle \omega^2 \rangle \propto \Theta_D^2$, we find $\langle I^2 \rangle / \langle I_0^2 \rangle = 0.91$, where $\langle I_0^2 \rangle$ corresponds to the initial material with $T_c = 1.45$ K. This is in reasonable agreement with Butler's relation¹¹ $\langle I^2 \rangle \propto V_a^{-2}$ and the observed increase in the lattice parameter a_0 in the damaged samples^{17,18}. Using values of a_0 (Ref. 18) given for samples with $T_c = 1.45$ K and $T_c = 4.25$ K we find

TABLE I. Data on Mo.

	Crystalline Mo	Amorphous Mo
T_c (K)	0.92	8.8
$\langle \omega^2 \rangle^{1/2}$ (K)	268	163
$\langle \omega \rangle$ (K)	254	139
μ^*	0.08	0.08
λ	0.36	0.9
ρ ($\mu\Omega \text{ cm}$)	$\ll 1$ below 50 K	200

$$\langle I^2 \rangle / \langle I_0^2 \rangle = \left[\frac{a_0(T_c = 1.45 \text{ K})}{a_0(T_c = 4.25 \text{ K})} \right]^6 \approx 0.95.$$

In the work of Lehmann *et al.*¹⁸ the density of states was inferred from dH_{c_2}/dT and ρ_0 measurements. The density of states was found to initially increase and later to decrease with heavy-ion irradiations of Mo_3Ge . Hence there exists a discrepancy between the behavior of $N(0)$ in Refs. 19 and 18. Traditionally, one tends to favor specific-heat evidence. In any case, if we look at the data of Ref. 18, we find that for $\rho_0 < 110-115 \mu\Omega \text{ cm}$. $N(0)$ increases with damage. Of course, the prediction of (2) would be a strong (factor of 3) decrease, which is inconsistent with either set of data.

One may take a view that Mo and its compounds provide a "pathological" example in a sense that the initial crystalline $N(0)$ is situated near the minimum in the density of states and lifetime smearing of the nearby peaks in $N(E)$ may enhance $N(0)$, as was often argued in the past.^{20,17} A similar situation is possible in Be.²¹ The decrease of $N(0)$ proposed in Ref. 1 may conceivably be masked by this effect. However, resistivities of α -Mo and irradiated Mo_3Ge exhibit the typical saturation behavior, while, according to Ref. 1, saturation should not take place unless $N(0)$ is decreasing.

C. Data on metallic glasses

Some other examples are provided by recent experiments on metallic glasses, in which samples are large enough to allow accurate electronic specific-heat measurements. For example, careful specific-heat work exists on $\text{Cu}_x\text{Zr}_{1-x}$ alloys²²⁻²⁴; in some cases both amorphous and crystalline (annealed) phase of the same composition were measured.^{22,23} Simultaneously there exist some resistivity data on these alloys.^{22,24-26} In the amorphous phases of $\text{Cu}_x\text{Zr}_{1-x}$ (in particular, for $x \approx 0.6$) resistivity $\rho_a(T)$ was reported to be practically temperature independent, with small negative $\rho_a^{-1}d\rho_a/dT$; values of $\rho_a = 200-350 \mu\Omega \text{ cm}$, typical of saturated metals, are quoted by different sources.^{22,24-26} The resistivity of the crystalline phase of $\text{Cu}_{0.57}\text{Zr}_{0.43}$ was found to have positive $\rho_c^{-1}d\rho_c/dT$, i.e., after the anneal $\rho_c(T)$ was found to fall below the nearly flat $\rho_a(T)$ curve with decreasing T .²² Unfortunately, no resistivity measurements in the crystalline phase have been reported below 500 K.²² However, one can deduce from the given slopes that the ratio $\rho_c(0)/\rho_a(0) \approx 0.8$ or small-

er.²⁷ Using this value and assuming arbitrarily that $\rho_a = 0.9\rho_{\text{sat}}$ [ρ_a really appears to but equal to ρ_{sat} , but we want to avoid an unphysical case of $N(0)=0$, as was discussed], we get from (2) $N_a(0)/N_c(0) \approx 0.6$, which represents only an upper bound for this ratio due to the lack of low-temperature ρ_c data and our assumption that $\rho_a = 0.9\rho_{\text{sat}}$. Contrary to this prediction, "dressed" densities of states $N^*(0) = N(0)(1+\lambda)$ measured by specific heat were found to be nearly equal in the two phases,^{22,23} being, if anything, slightly higher for the amorphous phase²³: $N_a^*(0) = 0.59 \pm 0.01$ and $N_c^*(0) = 0.56 \pm 0.01$ in states per eV, atom, and direction of spin. Since both amorphous and crystalline phases of $\text{Cu}_{0.6}\text{Zr}_{0.4}$ were not superconducting down to 0.3 K (Ref. 23) and had nearly the same $\Theta_D \approx 200 \text{ K}$ (Ref. 22 and 23), the renormalization factor $1+\lambda$ cannot be significantly different in the two phases and hence this conclusion stands also for the band-structure density of states $N(0) = N^*(0)/(1+\lambda)$.

Photoemission measurements of Cu-Zr alloys by Oelhafen *et al.*²⁸ and Amamou and Krill²⁹ show that $N(0)$ comes mainly from the Zr d band and scales with the amount of Zr in the alloy. No further reduction of $N(0)$ over and above this scaling is seen in the amorphous alloy compared to the crystalline Zr. In the words of Oelhafen *et al.*,²⁸ "there is no hint of a minimum in the density of states at E_F ."

Finally, specific-heat measurements of a rapidly quenched and subsequently annealed Pd-Si alloy³⁰ indicate $\sim 20\%$ decrease in the value of $N^*(0)$ in the annealed sample compared to $N^*(0)$ measured in the amorphous one. Unfortunately, simultaneous resistivity data are not available. It was found, however, that anneals caused the formation of crystalline phases.³⁰ Hence it is most probable that ρ is lower in the annealed sample. If this is so, our point is made again.

D. Amorphous simple metals and their alloys

A general argument can be given concerning the density of states in amorphous simple metals and their alloys. It turns out that in many cases $N(0)$ in the amorphous state can be very well estimated from the free-electron model.^{31,32} Resistivities at the same time are rather high and nearly temperature independent. Although values of ρ , when compared with free-electron estimates of ρ_{sat} , imply that the mean free path (MFP) $L > a$ in these simple metals, certainly these values are as high as $\sim 0.3-0.5\rho_{\text{sat}}$, i.e., appreciable decrease in $N(0)$ is expected from (2). In

TABLE II. Densities of states in amorphous and crystalline metals.

Element	Amorphous	Crystalline	$\frac{N_{\text{fe}}(0)}{N(0)}$	ρ_a ($\mu\Omega \text{ cm}$)
	Free-electron values ^a	Empirical values ^b		
	$N_{\text{fe}}(0)$ (states/eV atom spin)	$N(0)$ (states/eV atom spin)		
Pb	0.32	0.276	1.16	78
Sn	0.299	0.238	1.26	47
In	0.262	0.212	1.24	33
Ga	0.217	0.091	2.38	29
Be	0.106	0.032	3.31	
Zn	0.16	0.098	1.63	

^aReference 32.

^bReference 9.

the actual case the free-electron estimates $N_{fe}(0)$ are higher than values of $N(0)$ found experimentally in the crystalline case (Table II). The highest $N_{fe}(0)/N(0)$ is found for Be, for which in Sec. II A we deduced

$$\frac{N_a(0)\langle I_a^2 \rangle}{N_c(0)\langle I_c^2 \rangle} = 1.8.$$

If we substitute $N_{fe}(0)/N(0)$ from Table II in place of $N_a(0)/N_c(0)$, we obtain $\langle I_a^2 \rangle / \langle I_c^2 \rangle = 0.54$ in qualitative agreement with the expected decrease of $\langle I^2 \rangle$ in the amorphous phase.

Turning to the amorphous alloys of simple metals one finds that there is a nearly perfect agreement between $N_{fe}(0)$ estimates and measured $N(0)$ in, for example, Mg-Zn (Ref. 33) and $(Ag_{0.5}Cu_{0.5})_{1-x}Ge_x$ (Ref. 34) alloys. In case of $(Ag_{0.5}Cu_{0.5})_{1-x}Ge_x$ alloys resistivities are as high as $\sim 150 \mu\Omega \text{ cm}$.³⁵ Even though the crystalline values for $N(0)$ are not known, it is clear that this fine agreement with the free-electron model would be impossible if a depression in the density of states were indeed present.

E. A15 compound Nb_3Al

Apparently not all high- T_c A15 compounds show a sufficiently strong decrease of $N(0)$ with disordering to justify (2). As shown by Kwo *et al.*,³⁶ dH_{c2}/dT , ρ_0 measurements suggest that in Nb_3Al , $N(0)$ remains essentially constant upon radiation disordering of the material from $\rho_0 \approx 50 \mu\Omega \text{ cm}$ to $\rho_0 \approx 110 \mu\Omega \text{ cm}$. At the same time, resistivity behavior of Nb_3Al (Ref. 37) is no different from that of Nb_3Ge and other high- T_c A15 compounds in that it shows strong saturation with temperature and disorder. In contrast to the conclusions of Kwo *et al.*,³⁶ taking $\rho_s = 150 \mu\Omega \text{ cm}$ —a value typical of all high- T_c A15 compounds—we find from (2) $N(0)/N_0(0) \approx 0.5$ at $\rho_0 = 110 \mu\Omega \text{ cm}$.

III. DISCUSSION

As we have seen, in a number of metals and alloys disordering does not produce strong, or for that matter, any depression of the density of states at the Fermi level as suggested in Ref. 1. At the same time, this very disordering leads to strong resistivity saturation. Hence the effect of saturation must have an origin other than the exchange interaction.

Strong depression of $N(0)$ identified with exchange interactions was found experimentally in granular aluminum³⁸ and amorphous $Ge_{1-x}Au_x$ mixtures with $x < 0.2$.³⁹ I would like to point out that these effects are seen in a conduction regime which is different from the one in which resistivity saturation is observed. One way to identify the difference in the two regimes is to compare the typical values of conductivity at which these effects are observed; another is to compare the respective Ioffe-Regel parameters $k_F L$.

On the one hand, there are “true” metals such as elemental metals and their alloys, as well as alloys of metals with nonmetals with metal being the prevailing element. These are metallic systems with not too small a number of conduction electrons per atom (I will attempt to quantify this statement). These “true” metals may show saturation

effects in their resistivity as a function of static or thermal disordering. These effects, as was said, can be described rather well by the parallel-resistor formula (1), and, in all likelihood, they result from the MFP L approaching the interatomic separation a .^{2,3} In fact, Gurvitch³ derived (1) within the framework of Boltzmann theory from a well-known assumption⁴⁰ that a is the minimum possible value for a mfp. Typical values of $\rho_{sat} = \sigma_{sat}^{-1}$ in these “true” metallic systems lie between 100 and 300 $\mu\Omega \text{ cm}$. This is precisely the range of values predicted by the free-electron formula³ $\rho_{sat} = m/e^2 n \tau_0 = 1.29 \times 10^{18}/n^{2/3} a \mu\Omega \text{ cm}$ using typical metallic valencies of, for example, $Z = 1-5$. At the same time, the free-electron values of the Ioffe-Regel parameter $k_F L$, if we assume that $L = L_{min} = a$, fall between 3 and 6. All the examples of the materials cited in this paper and in Ref. 1 belong to this class.

On the other hand, there are metallic systems which are much closer to the metal-insulator transition or at least to the regime in which such transitions have been predicted.⁴⁰ These are, typically, metal-oxide or metal-insulator mixtures (alloys) with a considerable amount of nonmetallic element present. It is these materials which exhibit “new physics” such as Anderson localization⁴¹ and/or Altshuler-Aronov exchange interaction effects.⁴ They have values of ρ which are close to Mott’s⁴⁰ $\rho_{max} = \sigma_{max}^{-1} = (0.026e^2/\hbar a)^{-1}$, the latter being 5000 $\mu\Omega \text{ cm}$ for $a = 3 \text{ \AA}$. Hence ρ_{max} is a factor of 10–30 higher than typical ρ_{sat} . While Mott himself always made a distinction between $\sigma(L = a)$ and σ_{min} ^{40,42} it is most common to see in the literature that this large difference is ignored and the two values are treated as one.

For materials with $\rho_{sat} < \rho < \rho_{max}$ one finds a “gray” region characterized by the negative temperature coefficient of resistivity,⁴³ probably due to the weak onset of localization and/or correlations.⁴⁴

Finally, one can see in a simple way why σ_{sat} is much larger than σ_{min} and how these values are related to $k_F L$. We write the Boltzmann conductivity in the following form:

$$\sigma = \frac{e^2}{3\pi^2 \hbar} k_F (k_F L) \quad (6)$$

and

$$k_F = (3\pi^2 n)^{1/3} = \frac{(3\pi^2 \beta Z)^{1/3}}{a}, \quad (7)$$

where β is a numerical coefficient of order unity which depends on the type of atomic arrangement; for example, $\beta = 1.0$ for a simple cubic lattice and $\beta = 1.3$ for a bcc lattice. In the saturation regime, $L = a$ and

$$\sigma_{sat} = (3\pi^2 \beta Z)^{2/3} \frac{e^2}{3\pi^2 \hbar a} = (3\pi^2 \beta Z)^{2/3} \left[0.034 \frac{e^2}{\hbar a} \right],$$

which is approximately equal to $(3\pi^2 \beta Z)^{2/3} \sigma_{min}$. For $Z = 1$ we find $\sigma_{sat} \approx 10 \sigma_{min}$. From (6) and (7) it is clear that $\sigma = \sigma_{min}$ corresponds to the lower limit of the Ioffe-Regel condition $k_F L > 1$, namely $k_F a = 1$. Further, from (7) we find that this is so for $Z = Z_{min} = (3\pi^2 \beta)^{-1}$. Hence in disordered materials with $L = a$, the metal-insulator

transition or any strong nonclassical effect is expected for $Z \leq 0.03$.⁴⁵ We can now define "true" metals as materials with $Z \gg Z_{\min}$. We venture to submit that in a homogeneous three-dimensional metal with $Z \gg Z_{\min}$, no matter how disordered, localization and exchange interaction effects should be relatively unimportant.

Note added in proof. The suggestion made in the Discussion that bulk disordered metals with saturated resistivity are still far from localization, as well as the tunneling data of Kimhi and Geballe (Ref. 12), both seem relevant to the recent theory of the "universal" T_c degradation in disordered $A15$ compounds by Anderson *et al.* [Phys. Rev. B **28**, 117 (1983)]. In that theory a large

localization-related increase in μ^* is predicted to occur with disordering. We note that the data of Ref. 12 on amorphous Mo appears to be inconsistent with large values of μ^* ; at the same time it is not clear why very disordered $A15$'s should be qualitatively different in this respect from disordered Mo. Details of this analysis are to be published.

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$$\langle \omega^n \rangle = \frac{\int \alpha^2 F(\omega) \omega^{n-1} d\omega}{\int \alpha^2 F(\omega) \omega^{-1} d\omega}$$

and $\omega_{\log} = \lim_{n \rightarrow 0} \langle \omega^n \rangle^{1/n}$.

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