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Universal dynamical scaling in the clock model

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The development of order in the clock (Z_N) model is studied following a quench from a disordered phase to a low-temperature nonequilibrium state below the "ferromagnetic" critical point. The structure factor is shown to obey dynamic scaling for the two cases studied ($N=6$ and 26) and to be remarkably insensitive to the underlying morphological structure. The scaling functions for both cases are essentially identical to that of the Ising antiferromagnet. The rate of growth of domains is also studied.

The dynamics of pattern formation following the quench of a system from a disordered state to a nonequilibrium state below a transition temperature has recently received much attention.[1,2] In particular, the structure function $S(k,t)$ has been found to satisfy dynamic scaling to a good first approximation after an initial transient period t_0, i.e.,

$$S(k,t)=\kappa^{-d}(t)F(k/\kappa(t)),\quad t\geqslant t_0\ . \tag{1}$$

Here $\vec{k}$ is the wave vector measured relative to the Bragg positions of the ordered structure, $\kappa(t)$ is a characteristic time-dependent wave number [such as an inverse domain size $\bar{R}^{-1}(t)$], d is the dimensionality and $F(x)$ is a scaling function. An important issue is to identify possible dynamical universality classes for which the exponent n which is assumed to characterize the time dependence of $\kappa(t)$ (i.e., $\kappa(t)\simeq At^{-n}$ in some time interval) and the scaling function $F(x)$ are the same. Our current theoretical understanding of universality classes is rather limited, but it is clear that as in critical phenomena the degeneracy p of the order parameter, corresponding to a symmetry which is broken at the transition, is a relevant variable. For example, after a quench small domains of the ordered phase form which are separated by interfaces. Since there are p different types of order phases, a complicated morphological structure may form, particularly for large p, which can depend on the underlying lattice structure. Recent studies of the Q-state Potts model[2] suggest that, for $Q\geqslant 3$, vertices at which three or more interfaces meet play an important role in determining the domain growth rate. Although the exponent $n(Q)$ was estimated in these studies for several values of Q, no determination of $S(k,t)$ was made.

To gain further insight into the universality classes, we study in this Rapid Communication the two-dimensional Z_N vector Potts or clock model which has been the subject of considerable theoretical study lately.[3,4] For sufficiently large N (thought to be $N\gtrsim 5$) there are three phases. At low temperature (which is the region of our current study) there is an Ising-type ordered phase. At higher temperatures there is an intermediate "discrete vortex" phase similar to the low-temperature phase of the XY model (Kosterlitz-Thouless transition). At high temperatures, there is a disordered phase with exponentially decaying correlations. In this model the two-dimensional spin vector is described by a complex phase, $e^{i(2\pi/N)p}$, with $p=1,2,\ldots,N$, at each site of a square lattice. The Hamiltonian is

$$H=-J\sum_{\langle ij\rangle}\cos(\theta_i-\theta_j),\quad \theta_i=\frac{2\pi p}{N},\quad p=1,2,\ldots,N\ , \tag{2}$$

where the sum is over nearest-neighbor pairs. In this dynamical study of an $M=m\times m$ system with periodic boundary conditions we have used Glauber dynamics, in which a given spin can flip from a given orientation to another allowed orientation. In our Monte Carlo simulation a trial flip consisted of randomly turning a spin one unit ($2\pi/N$) either to the left or to the right. Larger orientational changes are unlikely at the low temperatures which we study here, so we ignore them. (Similar restrictions in orientational changes have also been made in other studies.[4]) We have updated spins sequentially to allow spin-wave-type excitations to occur more easily at low temperatures. We chose to study the $N=6$ and $N=26$ cases to see the role of the degeneracy of the order parameter on the growth rate of domains and the structure function and compare our results with those for the Q-state scalar Potts model.[2] In this study the exponent n decreased continuously from $n=0.5$ to $n=0.41$ for Q varying from 2 to 25 and apparently remained constant at $n=0.41$ for $Q>25$. Vertices played a dominant role for large Q.

It should be mentioned, however, that in the clock model the vertices at low temperatures are different from the vertices of the Potts model in the sense that the interfaces separate domains which differ by one orientational unit, due to the spin-wave–like nature of our Hamiltonian at low temperatures. Also at low temperatures for both $N=6$ and $N=26$ separate discrete vortices can appear (although not in pairs as in the Kosterlitz-Thouless phase). However, they appear to be statistically insignificant in our study and therefore do not substantially affect the dynamics. For $N=6$ and $m=60$ we studied several quench temperatures ($k_BT/J=0.05$, 0.20, 0.35, and 0.5) below the ferromagnetic ordering temperature[4] $k_BT_c/J\simeq 0.6$, in order to see the effect of temperature on domain growth. At $k_BT/J=0.2$ we also studied the linear size $m=100$, to check finite-size effects. There did not seem to be any significant effects. For $N=26$ and $m=60$ we studied one temperature, $k_B/J=0.02$. In all of these studies averages were taken over 25 runs. Following standard procedures we have calculated the structure function $S(k,t)$, which is the circular average of the Fourier transform of the nonequilibrium correlation function $\langle e^{i(\theta_i-\theta_j)}\rangle$. We have also computed the second moment $k_2(t)$. We thus have two possible choices for a scaling length $\kappa^{-1}(t)$ in Eq. (1), namely, $k_2^{-1/2}(t)$ or

$$L(t)\equiv[M^{-1}S(0,t)]^{1/2}/\psi(T)\ ,$$

where $\psi(T)$ denotes the equilibrium value of the order parameter. (This second choice has been suggested by Sa-

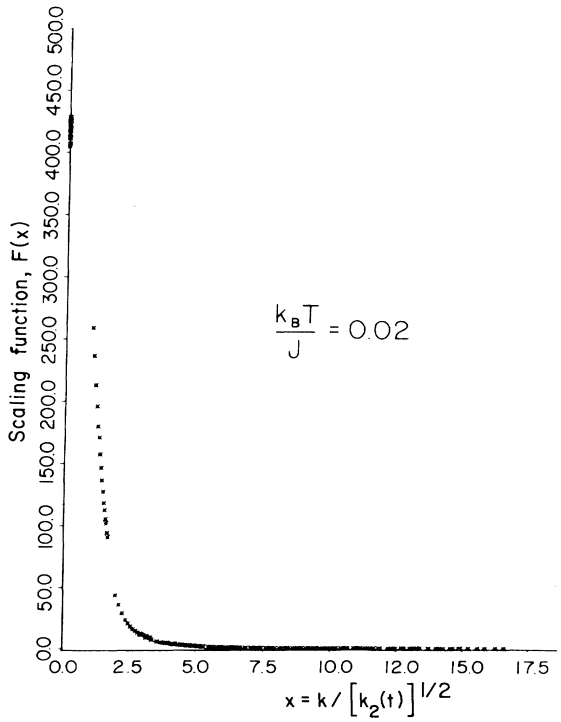


FIG. 1. The scaling function $F(x)$ of the dynamical structure factor for the $N=26$ state clock model. For $N=6$ the scaling function is identical with that of $N=26$. The scaling is performed with use of the second moment $k_2(t)$ as the scaling parameter, i.e., $F(x)=k_2(t)S(k,t)$, where $x=k/[k_2(t)]^{1/2}$.

diq and Binder.[1])

Our results for $S(k,t)$ show that dynamical scaling is satisfied at all the temperatures studied, for both $N=6$ and $N=26$. The scaling function $F(x)$ for these cases is the same and is shown in Fig. 1. In fact this scaling function for the Z_N model is essentially the same as that for the kinetic Ising ferromagnet ($p=2$), for which a reasonably accurate theory has recently been developed by Ohta, Jasnow, and Kawasaki.[5] (This theory is based on scattering from the random interfaces of the quenched anitferromagnet.) That $F(x)$ is essentially the same for these three cases is at first sight quite surprising, given the different morphological structures involved. In the antiferromagnet one has only two different types of domains, whereas for the $N=6$ and $N=26$ clock models one has, of course, 6 and 26 different types of domains, respectively. Thus in the clock model the morphology is much more complicated. This can be seen in Fig. 2 which shows a typical evolution for $N=26$. The morphology for $N=6$ is somewhat similar to that for $N=26$, except that the $N=6$ domains have a more "regular" shape. In addition the distribution of domain sizes is narrower than for the $N=26$ case. It is also clear from Fig. 2 that the interface structure is related to the underlying square lattice structure. It seems evident from Figs. 1 and 2 and from similar results for the antiferromagnet that the scaling function is determined by the interfaces rather than by vertices. The vertices only seem to affect the growth rate of domains and in this sense play a very minor role in determining the structure function [Eq. (1)]. It is possible, therefore, that for systems such as the Q-state Potts model and the Z_N model the scaling function is universal. This is a point which warrants further simulation and theoretical study.

We have also analyzed the domain growth law in the clock model but have been unable to give a definitive answer to the question of the growth law. In Figs. 3(a) and 3(b) we show the two quantities $k_2^{-1}(t)$ and $L^2(t)$ for $N=6$ and 26, respectively. As can be seen, $k_2^{-1}(t)$ and $L^2(t)$ have very similar behavior. We have fitted these to power-law forms of the type $A't^{2n}$ to determine a growth exponent, but our results depend somewhat on the region of time analyzed. As well, it is possible that there are correction terms to this power-law form, analogous to the situation in critical phenomena. We can, therefore, give only estimates of various "effective" exponents n_e which result from our power-law approximation. The exponents n_e for the interval $100 \leq t \leq 1000$ [where t is measured in Monte Carlo steps (MCS) per spin] are $n_e \cong 0.5 \pm 0.05$ and $n_e \cong 0.4 \pm 0.04$ for $N=6$ and 26, respectively. On the other hand, if we consider the $N=26$ model in the later stage of growth, say $350 \leq t \leq 1000$, the behavior is not inconsistent with an effective exponent of 0.5. Overall, however, the exponents quoted for the larger time interval seem more

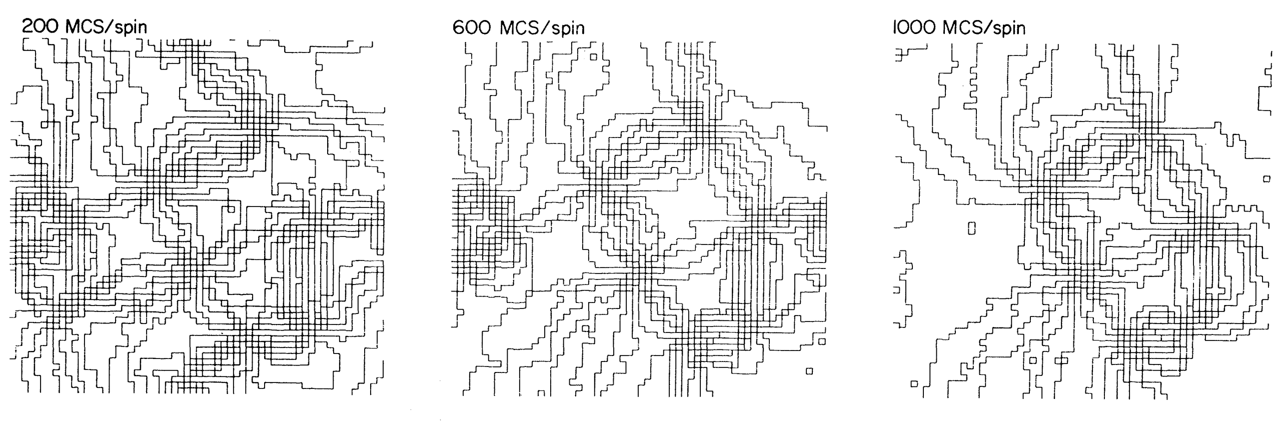
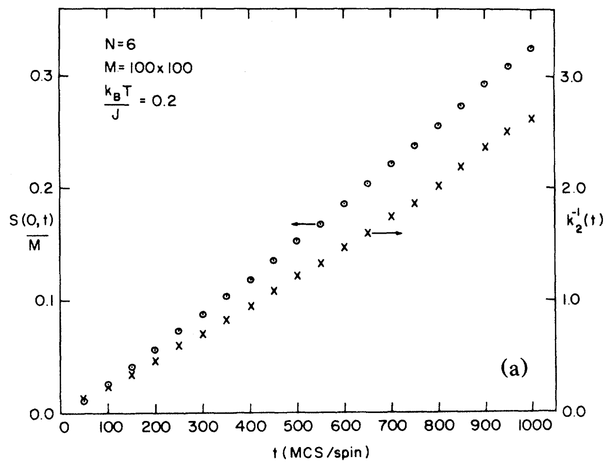


FIG. 2. A typical evolution of domains in the $N=26$ state clock model (for a 60×60 system) following a quench to $k_BT/J=0.02$.



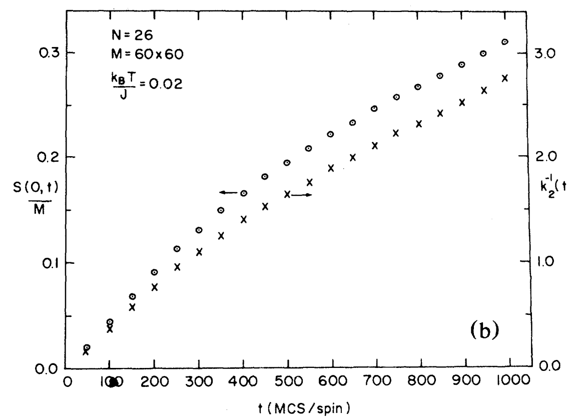


FIG. 3. The inverse second moment $k_2^{-1}(t)$ (crosses) and characteristic "area" $L^2(t)$ (circles) as a function of time. (a) $N=6$ state clock model (for a 100×100 square lattice) following a quench to $k_BT/J=0.2$. (b) $N=26$ state clock model (for a 60×60 square lattice) following a quench to $k_BT/J=0.02$. Arrows indicate the scales for $k_2^{-1}(t)$ and $L^2(t)$. Note that we have omitted the temperature-dependent normalization $\psi^2(T)$ in the plot of $L^2(t)$.

meaningful.

Finally, we compare some of our results with those obtained for the Q-state Potts model (on a square lattice) which has recently been used as a model of polycrystalline grain growth.[2] First, the exponents for $Q=6$ and $N=6$ seem quite different. We find an effective exponent of 0.5 for all the temperatures studied, whereas for the Potts model $n=n(T)$ and increases from $n\simeq0$ at $T=0$ to a value close to 0.5 at higher temperatures. Second, for $N=6$ our morphology reflects the underlying square lattice structure for all temperatures. Although this is true for the Potts model at very low temperatures, the morphology changes as the temperature increases to one quite similar to a Potts model on a triangular lattice, where at most three interfaces can meet at a vertex. [This change in morphology is believed responsible for the change in $n(T)$ for the $Q=6$ Potts model.] Third, our results for $N=26$ are rather similar to those for $Q=26$ although a definitive comparison has not yet been made. Fourth, we would expect that the structure function for the Potts model would exhibit a scaling behavior with an $F(x)$ very similar to that reported here.[6] Further work on these models, including quenches in the Kosterlitz-Thouless region, are currently in progress.

ACKNOWLEDGMENTS

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[6]Note that in this paper we have neglected the very weak temperature dependence in $F(x)$ reported by Kaski *et al.*, Ref. 1.