Flow properties of ^{3}He moving through ^{4}He II at temperatures below 150 mK

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Measurements were performed at temperatures below 150 mK concerning the flow properties of ^{3}He moving through ^{4}He in tubes, while the ^{4}He component is macroscopically at rest. Empirical relations have been found for the distributions of the pressure, flow-rate density, concentration, and temperature in the tubes, as functions of the external conditions.

In the hydrodynamics of ^{3}He moving through superfluid ^{4}He at very low temperatures it is usually assumed that the ^{3}He and ^{4}He components are moving without mutual friction[1]: the "mechanical-vacuum" (MV) model. According to this model, in the steady state, forces on the ^{4}He component due to pressure gradients are balanced by gradients in the osmotic pressure, while pressure forces on the ^{3}He are supposed to be balanced by viscous forces. In the laminar regime the ^{3}He–flow-rate density will then form a Poiseuille profile. Within the framework of this model, the temperature and pressure distributions in cylindrical tubes are calculated from the enthalpy balance, taking viscous heating and thermal conduction into account.[2] In previous papers it has been reported that some basic consequences of this model are in contradiction to experimental results obtained in our laboratory.[3,4] Deviations of a similar kind were also reported by Niinikoski[5] and by Frossati et al.[6]

In this Rapid Communication new results are reported concerning the flow properties of ^{3}He moving through ^{4}He in tubes while the ^{4}He is macroscopically at rest. A set of empirical relations is found, determining the distributions of pressure (p), ^{3}He molar flow-rate density (j_3), ^{3}He molar concentration (x), and the temperature (T) from the external condidtions. All these relations are qualitatively different from the predictions based on the MV model. This agrees with our earlier observations that a mutual friction plays an important role in the hydrodynamics of ^{3}He-^{4}He mixtures.

In Fig. 1 the experimental configuration is depicted. The flow resistance Z is usually a cylindrical tube (length L, diameter D). It is mounted at the entrance of the dilute-exit channel of the mixing chamber of a dilution refrigerator. The temperature of the mixing chamber T_m can be varied above a certain minimum value by supplying a heating power $\dot{Q}$ to the ^{3}He entering the mixing chamber. Temperatures are measured by calibrated Speer carbon resistors with an accuracy of 1 mK. The ^{3}He concentration x_m in the mixing chamber is determined from the equation for the solubility curve,[7]

$$x_m = 0.066(1 + 10T_m^2) \ . \tag{1}$$

At the exit of Z the temperature T_e is measured. The concentration x_e is determined with an accuracy of 0.1%, with use of a capacitive technique. Changes in the pressure difference Δp across Z are detected with a capacitive pressure gauge P with an estimated resolution of 0.2 Pa. In principle, the ^{3}He circulating through the system with flow rate $\dot{n}_3$ contains a certain amount of ^{4}He. In the experiments described in this Rapid Communication the ^{4}He frac-

tion in the total flow rate $\dot{n}$ is smaller than 5%. However, in certain cases (high flow resistance and a high flow rate) the ^{4}He contribution amounted to 50%. Since in these cases the influence of the circulating ^{4}He on the phenomena to be described below was small, the influence of a 5% admixture of ^{4}He is negligible. In order to study the effect of the dimensions of the tubes more than 20 different tubes have been investigated. Some relevant parameters varied between the following limits: 0.3 mm $\leq D \leq$ 2.3 mm; 5 mm $\leq L \leq$ 1.4 m; 12 mK $\leq T_m \leq$ 120 mK; 12 mK $\leq T_e$ $\leq$ 150 mK; 1.3% $\leq x_e \leq$ 7.6%; and 0.13 mmol/s $\leq \dot{n} \leq$ 1.4 mmol/s.

The results of the measurements can be summarized as follows:

(1) The pressure drop Δp across Z is for all tubes independent of T_m or $\dot{n}$ within the resolution of the pressure gauge. Together with the observation that the level difference between the phase boundaries in the two mixing chambers of a double mixing chamber is zero within a few mm,[2,3] it follows that
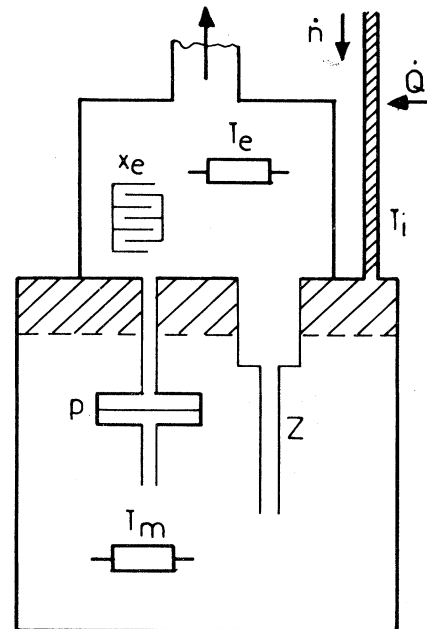

FIG. 1. Schematic drawing of the experimental arrangement. The flow resistance Z constitutes the entrance of the dilute-return line of the mixing chamber of a dilution refrigerator.
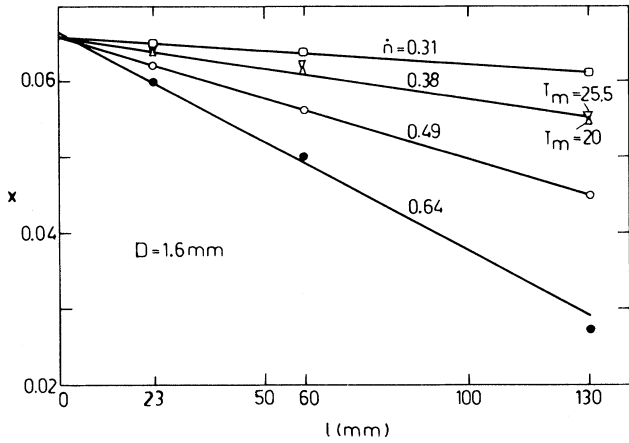
FIG. 2. Measures x-l dependences for four different flow rates which are given in the figure. The values of x and T are measured at $l = 0$, 23, 60, and 130 mm, respectively. The measured x-l dependences can be represented by straight lines. For $\dot{n} = 0.38$ mmol/s, points are given for $T_m = 20$ mK ($\triangle$) and $T_m = 25.5$ mK (∇). The measured T^2-l dependences (not shown here) also are straight lines in agreement with Eqs. (6) and (8).

$$\Delta p = 0 \ . \tag{2}$$

From the MV model pressure changes in the order of 10–1000 Pa would be expected.

(2) The concentration x in Z as a function of the distance l from the tube entrance can be written as

$$x(l) = x_m + \beta_1 l \ , \tag{3}$$

where β_1 depends on $\dot{n}_3$ and D, but not on T_m. This relation is determined by the use of a tube which is cut in pieces. The pieces are connected in series by experimental spaces where x and T are measured. The results are given in Fig. 2. The x-l dependences are also determined in a series of experiments where the concentrations x_e at the end of Z have been measured for different values of L. The x_e-L relationships obtained in this way are consistent with Eq. (3).

(3) The ^{3}He flux j_3 is constant in a tube, not only in the axial direction, but also in the radial direction. This follows from a series of experiments in which tubes of equal length and varying diameter D are used. For each tube the concentration difference $\Delta x = x_m - x_e$ has been measured as a function of $\dot{n}$ as depicted in Fig. 3. From these results the $\dot{n}$-D dependence for constant Δx can be determined. The result is given in Fig. 4. It shows that $\dot{n}$ is proportional to D^2. This suggests that j_3 is homogeneous in the tube and given by

$$j_3 = \dot{n}_3/A \ , \tag{4}$$

where A is the cross-sectional area of the tube. An experiment where Z consisted of four parallel tubes of 0.6 mm diameter gave identical x_e and T_e variations as one single tube with $D = 1.2$ mm and equal length. Apparently only the area of the cross section and not its shape is important. This is clearly at variance with a Poiseuille flow profile as obtained in the MV model.

(4) The concentration difference Δx is given by
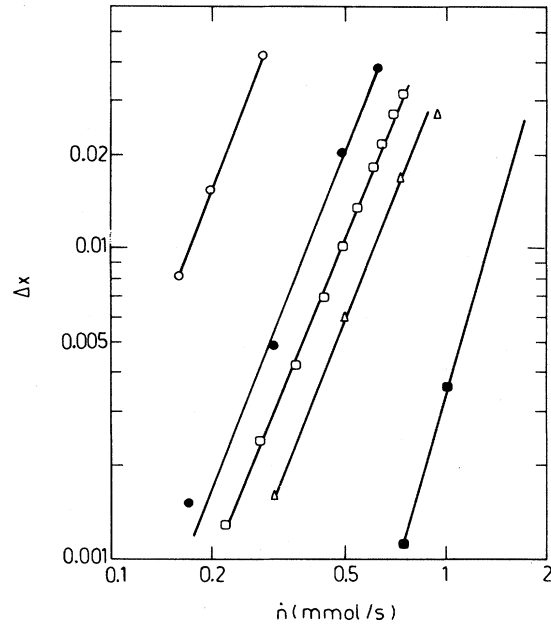
$$\Delta x = \beta_2 \dot{n}_3^\gamma \ , \tag{5}$$

FIG. 3. Measured Δx-$\dot{n}$ dependences for the following tube sizes (L and D given in mm): $L = 23$, $D = 0.8$ ($\bigcirc$); $L = 130$, $D = 1.6$ ($\bullet$); $L = 10.5$, $D = 1.2$ ($\square$); $L = 23$, $D = 1.6$ ($\triangle$); and $L = 23$, $D = 2.3$ ($\blacksquare$). The straight lines in the figure represent Δx-$\dot{n}$ dependences of the form $\dot{n}^\alpha \sim (\Delta x)$.
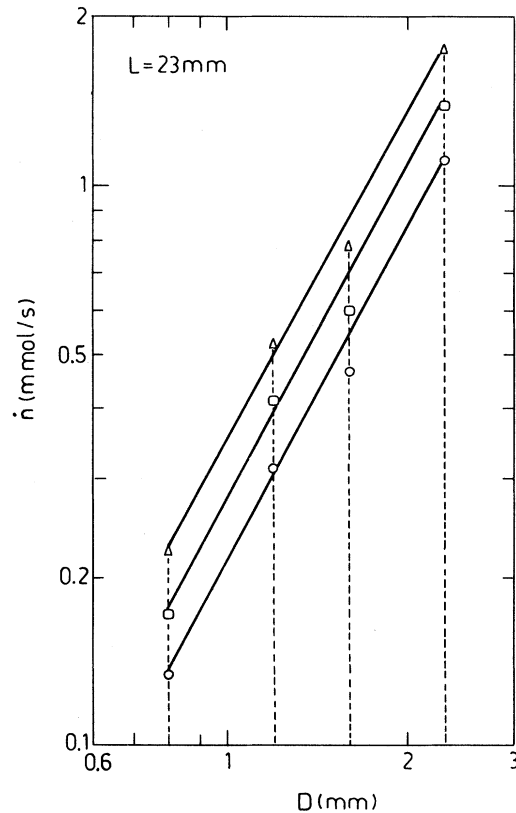

FIG. 4. Measured $\dot{n}$-D dependences for four tubes with $L = 23$ mm. Points for constant Δx can be represented by $\dot{n} \sim D^2$. Three different Δx values are shown here: 0.02 ($\triangle$), 0.01 ($\square$), and 0.005 ($\bigcirc$).

where α is a constant equal to 2.8 ± 0.4 (see Fig. 3). The factor β_2 depends on L and D but not on T_m.

(5) The temperature-concentration relationship in a tube for given T_m can be represented by a straight line in the T^2-x diagram (Fig. 5) of the form

$$T^2 + \beta_3 x = T_m^2 + \beta_3 x_m \ . \tag{6}$$

The parameter β_3 is independent of l, D, or $\dot{n}_3$. Its value seems to vary slightly from 0.21 K^2 at low temperatures ($T_m \approx 12$ mK) to 0.19 K^2 for $T_m = 70$ mK.

Equation (6) has the nature of an enthalpy conservation law which, in general, relates the p, T, and x values at a certain point in the tube to the conditions at the entrance, independent of the flow rate or the tube dimensions. In our case the pressure dependence is absent because p is constant, hence the enthalpy conservation law must have the form $H_3(x,T) = H_3(x_m, T_m)$ as in Eq. (6). The enthalpy H_3 per mol ^{3}He can be calculated using $94.5 T_m^2$ for the low-temperature molar enthalpy of ^{3}He in the saturated dilute phase.[8] Equations (1) and (6) then give

$$H_3(x,T) = 83 T^2 + 17(x - 0.066) \ .$$

Equations (3), (4), and (5) can be summarized in the equation

$$x = x_m - \gamma l j_s^\alpha \ , \tag{7}$$

where γ is an experimentally determined constant equal to $(3.2 \pm 0.3) \times 10^{-8}$ when j and l are expressed in SI units. In combination with Eqs. (4) and (6), the relation

$$T^2 = T_m^2 + \gamma \beta_3 l (\dot{n}_3/A)^\alpha \tag{8}$$

is obtained. In our experimental setup T_e will always be smaller than the temperature T_i of the ^{3}He entering the mixing chamber. The T_e^2-x_e dependence in the limiting case is a straight line which is the limit of our experimental region in the T^2-x diagram (line 4 in Fig. 5).

In conclusion, we would like to note that Eqs. (2), (4), (6), and (7) constitute a complete set of equations determining the pressure, flow-rate density, concentration, and temperature profiles in tubes. Furthermore, the results confirm that the current model for describing ^{3}He flow in ^{4}He is invalid. The experimental determination of the relations between the various parameters can be regarded as a
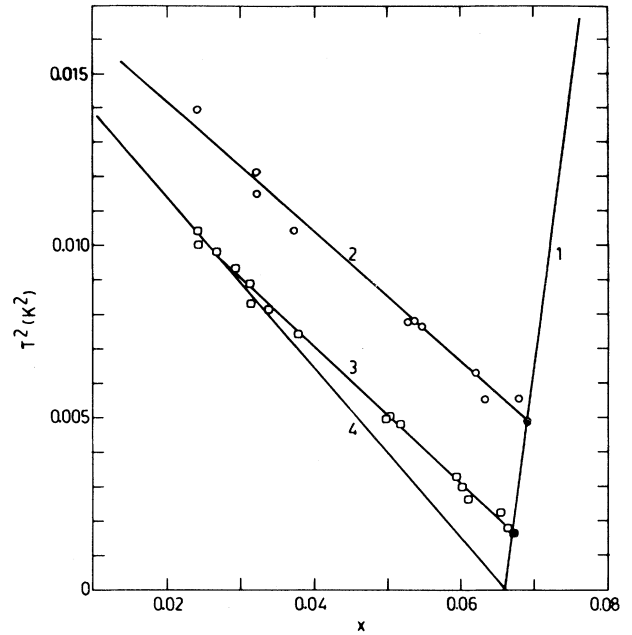

FIG. 5. T^2-x diagram. Lines 1–4 have the following meaning: 1: Phase separation line given by $T^2 = -0.10 + 1.5x$. 2: Least-squares fit to the measured T^2-x points ($\bigcirc$) corresponding to $T_m = 70$ mK for different flow rates and tube sizes. The line is given by $T^2 = 0.0180 - 0.19x$. 3: Least-squares fit to points ($\square$) with $T_m = 40$ mK. The line is given by $T^2 = 0.0151 - 0.20x$. 4: Limit of the experimental region corresponding to $T_e^2 = 0.0165 - 0.25x_e$.

first and necessary step for the development of a new and better theory.

ACKNOWLEDGMENTS

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