Random-field distributions and tricritical points

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Explicit conditions on the random-field distribution function $p(\vec{H})$ are given in order to obtain a tricritical point within mean-field theory. At zero field, a minimum $[p^{(2)}(0)>0]$ implies a first-order transition at low temperature. A maximum $[p^{(2)}(0)<0]$ will also induce a first-order transition, provided that $p^{(2)}(0)p^{(6)}(0)<\frac{7}{15}[p^{(4)}(0)]^2$ for $p^{(4)}(0)>0$ and $p^{(6)}(0)<0$. Otherwise, the transition is second order and there is no tricritical point.

In the study of Ising systems in a random field, Aharony[1] suggested that at low temperature the associated phase transition should be first order (second order) whenever the symmetric distribution function of the random field $p(\vec{H})$ has a minimum (maximum) at zero field. Following that work, Andelman[2] presented an extension of that criterion based on a qualitative analysis of the distribution function maxima. Recently, we carried out a detailed analysis of the phase diagram corresponding to the free-energy expansion of a fully isotropic n-vector model carried up to degree eight.[3] Our results can be implemented for the Ising $(n=1)$ random model to give explicit conditions on $p(\vec{H})$ in order to obtain a first-order transition at low temperature, i.e., a tricritical point (at sufficiently high temperature the transition is second order).

Starting with the free-energy expansion

$$F=a|\vec{S}|^2+b|\vec{S}|^4+c|\vec{S}|^6+d|\vec{S}|^8 \ , \tag{1}$$

where $\vec{S}$ is the order parameter, we found[3] that for $b>0$, $c>0$, and $d>0$, the corresponding transition is always second order. On the other hand, for $b>0$, $c<0$, and $d>0$, the transition is first order in the range $b<\frac{1}{4}c^2/d$ and second order for $b>\frac{1}{4}c^2/d$.

Generalizing Aharony's calculations[1] (to degree eight in the expansion) we find that the free-energy expansion of an Ising system in a random field is given by (1) where a, b, c, and d are functions of βH and βzJ. The variables are, respectively, $\beta=1/k_BT$ where k_B is the Boltzmann constant, T is the temperature, H is the random-field intensity, z is the coordination number, and J is the ferromagnetic coupling.

Focusing on low temperature, we make an expansion of a, b, c, and d in powers of β^{-1} which gives, respectively, at order 2,

$$\begin{aligned} a&=1-2zJp(0)+O(\beta^{-2}) \ ,\\ b&=-\tfrac{1}{6}(zJ)^3p^{(2)}(0)+O(\beta^{-2}) \ ,\\ c&=-\tfrac{1}{180}(zJ)^5p^{(4)}(0)+O(\beta^{-2}) \ ,\\ d&=-\tfrac{1}{10080}(zJ)^7p^{(6)}(0)+O(\beta^{-2}) \ , \end{aligned} \tag{2}$$

where

$$p^{(n)}(0)=\left.\frac{d^np(H)}{d|\vec{H}|^n}\right|_{|\vec{H}|=0} \ .$$

Using our results we find that: (i) for $p^{(2)}(0)>0$ (minimum at $|\vec{H}|=0$) the transition is first order as suggested by Aharony[1], (ii) when $p^{(2)}(0)<0$ (maximum at $|\vec{H}|=0$) a first-order transition still occurs if

$$p^{(4)}(0)>0,\ p^{(6)}(0)<0 \ ,$$

and

$$p^{(2)}(0)>\tfrac{7}{15}[p^{(4)}(0)]^2/p^{(6)}(0) \ , \tag{3}$$

otherwise, (iii) the transition is second order. The application of the above results if the distribution is Gaussian,[4] i.e.,

$$p_G(|\vec{H}|)=\frac{1}{\sqrt{2\pi\lambda}}e^{-|\vec{H}|^2/2\lambda} \ , \tag{4}$$

yields a second-order transition. Note that this conclusion does not follow from the fact that $p_G(0)$ is a maximum[1] but rather from the nonfulfillment of the last inequality in (3).

We have shown that Ising systems in a random field will exhibit tricritical points in two cases: (i) The symmetric distribution function $p(|\vec{H}|)$ has a minimum at zero field, and (ii) $p(|\vec{H}|)$ has a maximum at $|\vec{H}|=0$, and its higher moments fulfill Eq. (3).

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