

Consistent temperature and field dependence in weak localization

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The temperature and magnetic field dependence of the resistivity of thin Mg and Au films are measured. The parameters of weak localization such as the inelastic lifetime $\tau_i(T)$ and the spin-orbit (so) coupling time τ_{so} are evaluated. The Mg film which has a small spin-orbit coupling can be changed into a strong spin-orbit coupler by covering it with 0.25 monolayers of Au. Since the inelastic lifetime is not affected by the small amount of Au only one parameter is changed. This does not only alter the magnetoresistance but also the temperature dependence of the film resistance. The whole set of magnetoresistance curves is well described by the theory. The change of the spin-orbit coupling essentially allows one to separate the temperature-dependent resistance caused by weak localization from other temperature-dependent contributions. It is consistent with the theory.

I. INTRODUCTION

At low temperature the conduction electrons of a metal possess a long inelastic lifetime (in a fixed-energy state). In a disordered thin film this yields quantum interferences in a spatially extended region which were first introduced by Abrahams *et al.*¹ and are generally called "weak localization." In this state the resistance shows logarithmic anomaly with respect to temperature. Since the calculations of its sensitivity to a magnetic field by Altshuler *et al.*² and Hikami *et al.*,³ this state has become a unique probe used to determine characteristic times in metals.

Physically weak localization represents an interference experiment with conduction electrons split into pairs of waves interfering in the backscattering direction.⁴ The intensity of the interference (integrated over time) can be easily measured by the resistance. A magnetic field introduces a magnetic phase shift in the electronic wave function and suppresses the interference after a flight time proportional to $1/H$. Therefore, the application of a magnetic field allows the observation of the electron's fate as a function of time.

Altshuler *et al.*⁵ predicted a similar anomaly of the resistance at low temperature in disordered two-dimensional electron systems which is caused by a modified electron-electron (or Coulomb) interaction. Subsequently, these first theoretical results were extended and modified by a number of authors.⁶⁻⁸ Weak localization is shown by Hikami *et al.*³ to change its temperature dependence completely by the presence of spin-orbit coupling and magnetic scattering. In addition, the contributions of weak localization and the Coulomb interaction cannot be simply added. A mixed contribution exists which modifies the first results.⁶⁻⁸ In addition, the electron-phonon interaction may alter the results. At the present time the various theoretical results do not yield a coherent picture for this mixed term but partially contradict each other.

Experimentally, the temperature dependence alone does not allow a test of the theory. Too many theoretical parameters are necessary which are not provided by the temperature dependence of the resistance. In particular, not even the sign of weak localization is generally known.

Many two-dimensional systems such as thin films of Pd, Pt, or Au alloys, whose logarithmic increase of the resistance with decreasing temperature has been interpreted as weak localization, may have a strong spin-orbit coupling and therefore the contribution of weak localization should have a reversed sign (which is hidden by the Coulomb interaction).

On the other hand, the magnetoresistance is generally dominated by the contribution of weak localization. The magnetoresistance of the Coulomb anomaly is either negligible or only present in large fields whereas the characteristic properties of weak localization can be determined in rather small fields. The characteristic field for the Coulomb interaction below which the magnetoresistance is negligible is, by a factor of $\hbar\tau_i/(2\pi k_B T)$, larger than that for weak localization. (For Mg at 4 K this factor is about 100.) Therefore, at the present time the magnetoresistance of two-dimensional systems is much better understood than the temperature dependence of the resistance and experimentally proved for electron inversion layers⁹⁻¹¹ and in thin films.¹²⁻¹⁷ The agreement with the theory is sometimes nearly perfect.

In the present paper we want to examine whether the temperature dependence is consistent with the magnetoresistance. Since the experimental temperature dependence consists of the contributions of weak localization and Coulomb interaction one has to separate them. We proceed in the following manner:

(1) Magnetoresistance measurements in small fields yield the inelastic lifetime $\tau_i(T) \approx T^{-p}$, and the spin-orbit (so) coupling time τ_{so} . Therefore, one knows the relevant parameters of weak localization and their temperature dependence. This allows one to calculate the temperature-dependent contribution of weak localization to the resistance. If one considers a two-dimensional system in which the spin-orbit coupling is changed from 0 to infinity while the temperature-dependent inelastic lifetime is not altered, then the temperature-dependent correction of the conductance ΔL_{wl} changes sign and reduces by a factor of $\frac{1}{2}$. Experimentally, this manipulation of the spin-orbit coupling can be achieved by superimposing Mg with a fraction of a monolayer of Au. This experiment al-

allows one to extract the contribution of weak localization to the conductance and to compare it with the theory. We will demonstrate that the magnetoresistance and the temperature dependence of the resistance are consistent.

(2) The application of a large magnetic field suppresses the resistance anomaly of weak localization. Therefore, the remaining temperature dependence is essentially caused by the Coulomb anomaly (and the thermal part of the resistance).

II. EXPERIMENTAL RESULTS

For the experimental investigation Mg and Au films are used. Mg has a small spin-orbit coupling according to its small nuclear charge while Au has a strong spin-orbit coupling. For the investigation of weak localization and electron-electron interaction we need thin films with high resistance. Since the method of quench condensation is very favorable for preparing rather homogeneous and continuous films with high resistance it is very suitable for the present task. In particular, the classical magnetoresistance is negligible. The apparatus in which the experiments are performed has been described in Refs. 18 and 19. In an ultrahigh vacuum of at least 10^{-11} Torr the evaporation rate of Mg is first adjusted to about 10–20 atomic layers per minute. After that the film is quench condensed at 4.5 K onto a substrate of crystalline quartz. The conductance is registered during the evaporation and the evaporation is stopped when the required resistance per square is observed. Films with resistances between 250 and 80 Ω per square are condensed. The thickness of the films has been determined with a quartz oscillator which has a sensitivity of $\frac{1}{10}$ atomic layers of Mg and of about $\frac{1}{100}$ atomic layers of Au (because Mg and Au differ in their densities by roughly a factor of 10). After the evaporation the film is annealed at 40 K for several minutes. We present here as a typical example the experimental results for a Mg film with a square resistance of about 80 Ω . The thickness of the film is 8.4 nm. Figure 1 shows the resistance of the film as a function of $\ln T$ (upper curve). Since in a two-dimensional system the conductance is the more appropriate quantity [see, for example, Eq. (1)] we have added on the right side the conductance scale in units of $L_{00} = e^2 / (2\pi^2 \hbar) \approx (80 \text{ k}\Omega)^{-1}$.

This is the physically essential scale which is drawn in all figures. This plot has the advantage that it combines the physically relevant scale with the resistance plot which is more familiar. Of course the conductance scale increases in a downward direction.

Furthermore, the magnetoresistance is measured at five temperatures in the field range between -7 and $+7$ T. These results are shown below.

In a second evaporation step the Mg film is covered with a small fraction of an atomic layer of Au. The lower curve in Fig. 1 shows the temperature dependence of the same Mg film after a coverage with 0.25 atomic layers of Au. The temperature dependence of the resistance changed clearly. Again the magnetoresistance is measured at the same temperatures as before.

The magnetoresistance is strongly temperature dependent. For a comparison with the theory it is not useful to draw the magnetoresistance as a function of the applied field in tesla. Therefore, we have chosen the magnetic

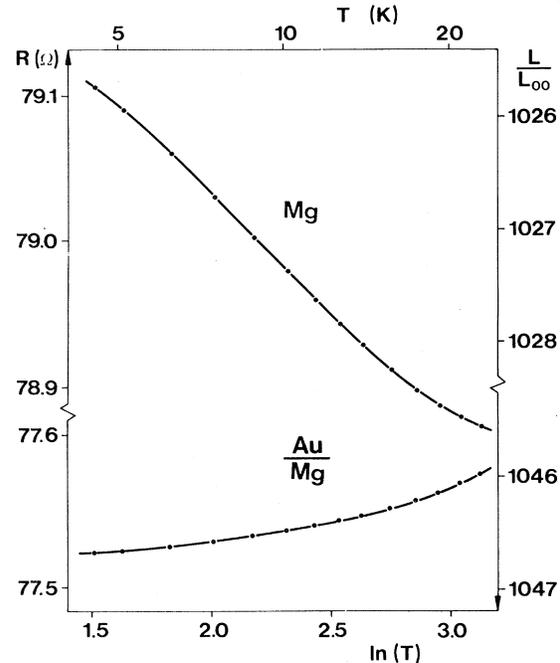


FIG. 1. Resistance [i.e., the normalized conductance $L/L_{00} \approx (80 \text{ k}\Omega)/R$ using the right scale] as a function of $\ln T$ for a Mg film with a thickness of 8.4 nm. The same measurement after a superposition with 0.25 atomic layers of Au and another annealing at 40 K.

field scale for each temperature in such a way that the magnetoresistance curves are optimally represented. The units of the field are shown on the right side of each magnetoresistance curve. In Fig. 2(a) the magnetoresistance curves are plotted for the pure Mg film. The points represent the experimental results. In Fig. 2(b) the magnetoresistance is plotted for the MgAu system. The field scale for each temperature is the same as in Fig. 2(a).

Since pure Mg has a small spin-orbit coupling we investigate also the temperature dependence of a strong spin-orbit coupler, i.e., thin Au films. We present here as a typical example the experimental results for a Au film with a square resistance of about 86 Ω . The thickness of the film is 6.9 nm. Figure 3 shows the corresponding resistance as a function of $\ln T$ in a magnetic field of 0 and 7 T. As before the magnetoresistance is measured. In Fig. 4 the magnetoresistance is plotted at different temperatures. The points represent the experimental results.

III. THEORETICAL BACKGROUND

A. Weak localization

Anderson *et al.*²⁰ and Gorkov *et al.*²¹ predicted for the temperature-dependent conductance of weak localization

$$\Delta L = -\Delta R / R^2 = -L_{00} \ln(\tau_i / \tau_0) = p L_{00} \ln T + \text{const.} \quad (1)$$

There are, however, complications because of the influence of spin-orbit coupling on weak localization which causes a

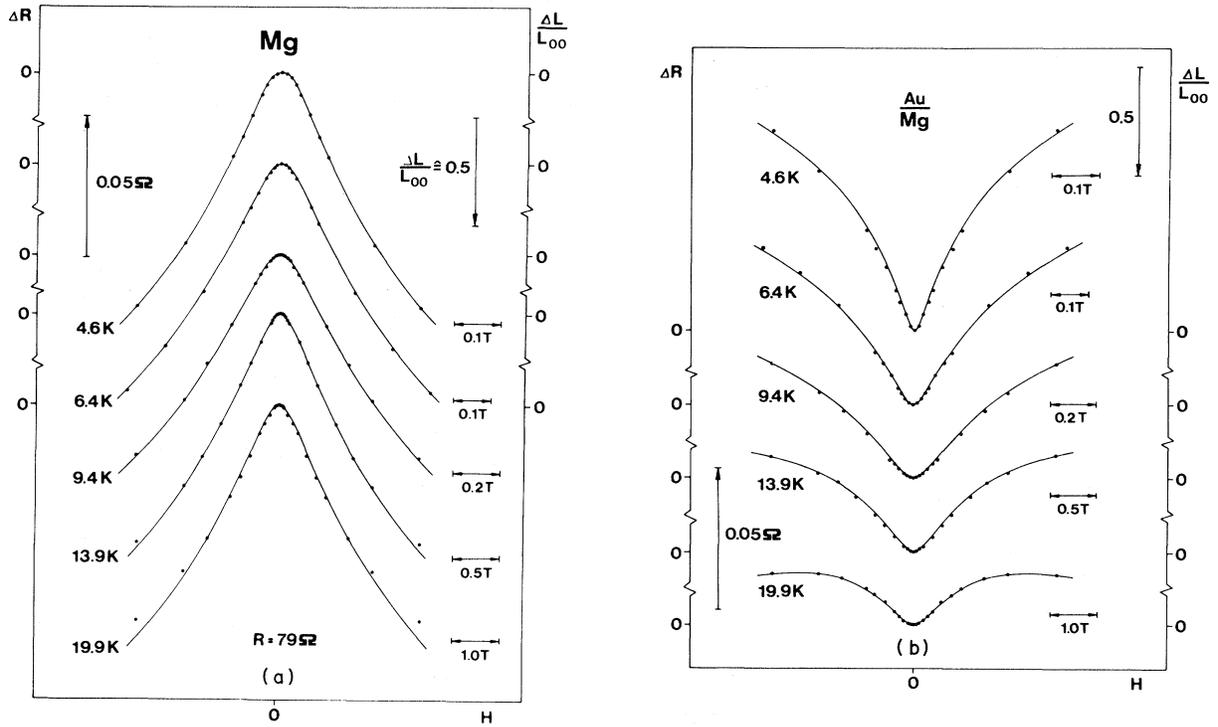


FIG. 2. (a) Magnetoresistance (i.e., $[L(H) - L(0)]/L_{00}$ using the right scale) of a Mg film ($d = 8.4$ nm) as a function of the field H . The units of the field are shown beside each magnetoresistance curve. The points represent the experimental results. The solid curves are calculated using the characteristic fields $H_i(T)$ plotted in Fig. 5 and $H_{so} = 0.0046$ T. (b) Magnetoresistance of the same Mg film after superposition with 0.25 atomic layers of Au. The units of the magnetic field are the same as in (a). The points represent the experimental results. The solid curves are calculated with the same set of $H_i(T)$ as in (a) and the new value $H_{so} = 0.54$ T.

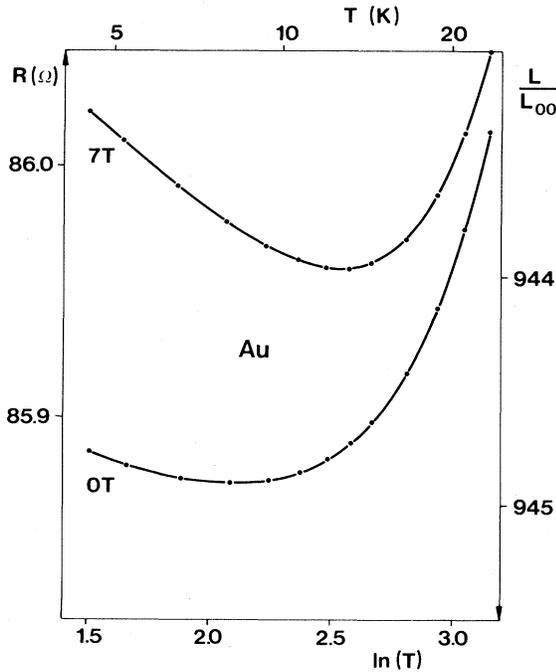


FIG. 3. Resistance [i.e., the normalized conductance $L/L_{00} \approx (80 \text{ k}\Omega)/R$ using the right scale] as function of $\ln T$ for a Au film with a thickness of 6.9 nm. The measurements are taken in magnetic fields $H = 0$ and 7 T.

decrease of the resistance with decreasing temperature and changes weak localization into weak antilocalization.²² In addition, magnetic impurities block weak localization. Hikami *et al.*³ included spin-orbit coupling and magnetic scattering in their analysis of quantum interferences. Instead of using the characteristic times such as τ_i , τ_{so} , etc., we introduce the corresponding characteristic fields H_i , H_{so} , etc. These fields can be directly determined in magnetoresistance measurements. The relation between τ_n and H_n is

$$H_n \tau_n = \hbar / (4eD) = \hbar e R d N / 4, \quad (2)$$

where $D = 1 / (e^2 N R d)$ is the diffusion constant, R the resistance per square, d the thickness of the film, and N the density of states. Expressed in terms of the characteristic fields the result by Hikami *et al.* for the temperature dependence of the conductance is

$$\Delta L_{wl} = -\Delta L / R^2 = -L_{00} [\ln(H_1/H_2) - \ln(H_3/H_4) / 2]. \quad (3a)$$

The H_n are defined in the following manner:

$$\begin{aligned} H_1 &= H_0 + H_{so} + H_s, \\ H_2 &= \frac{4}{3} H_{so} + \frac{2}{3} H_s + H_i, \\ H_3 &= 2H_s + H_i, \\ H_4 &= \frac{4}{3} H_{so} + \frac{2}{3} H_s + H_i, \end{aligned} \quad (3b)$$

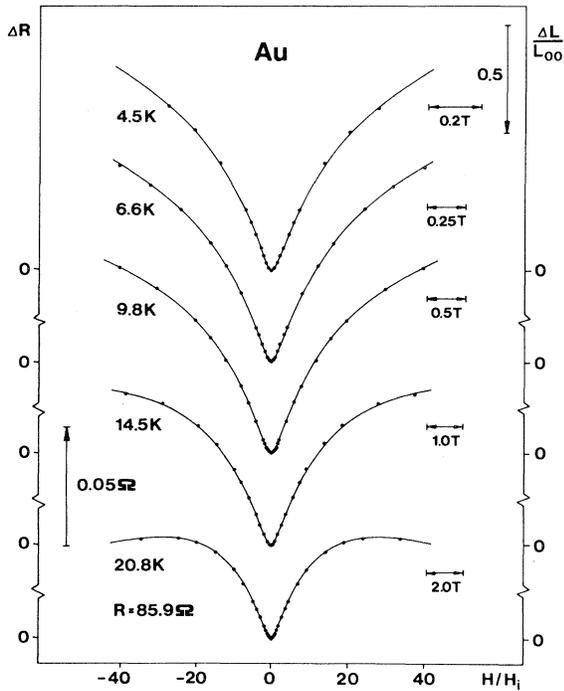


FIG. 4. Magnetoresistance (or $[L(H) - L(0)]/L_{00}$) of a Au film as a function of the field H measured in units of the inelastic field $H_i(T)$. The points represent the experimental results. The solid curves are calculated using the characteristic fields plotted in Fig. 5.

where H_0 corresponds to the elastic lifetime τ_0 , H_i to the inelastic lifetime τ_i , H_{so} to the spin-orbit coupling time τ_{so} , and H_s to the magnetic scattering time τ_s . (The distinction between the x and the z components of the scattering times is neglected because it hardly affects the evaluation.)

A detailed analysis of weak localization and a quantitative determination of the characteristic times can be achieved by magnetoresistance measurements. The field dependence of the conductance was first calculated by Altshuler *et al.*² in the absence of spin-orbit coupling. Hikami *et al.*³ and Maekawa and Fukuyama²³ extended the calculation by including spin-orbit coupling and magnetic scattering. The formula by Hikami *et al.* is given by Eq. (4),

$$\begin{aligned} \Delta L_{wl}/L_{00} = - \{ & [\Psi(0.5 + H_1/H) - \Psi(0.5 + H_2/H)] \\ & + [\Psi(0.5 + H_3/H) \\ & - \Psi(0.5 + H_4/H)]/2 \}, \end{aligned} \quad (4)$$

where Ψ is the digamma function and H the applied field.

B. The Coulomb interaction

Altshuler *et al.*⁵ calculated an additional contribution to the conductance at low temperature caused by the Coulomb interaction which is modified by impurity scattering in two-dimensional and quasi-two-dimensional systems. They obtained a similar correction for the con-

duction of a thin film as in the case of weak localization:

$$\Delta L_C = -\Delta R/R^2 = -L_{00}(1-F)\ln(\tau_T/\tau_0), \quad (5)$$

where F is a screening factor which has for a thin film (which is three dimensional with respect to its electronic properties) the form

$$F = [K/(2k_F)]^2 \ln[1 + (2k_F/K)^2], \quad (6)$$

where K^{-1} is the screening length in three dimensions: $K^2 = Ne^2/\epsilon_0$. As in the case of weak localization also the Coulomb interaction is determined by the diffusion of the conduction electrons. However, the characteristic diffusion time is τ_T and is inversely proportional to the temperature:

$$\tau_T = \hbar/(2\pi k_B T). \quad (7)$$

As a matter of fact the definition of \hbar/τ_T varies in the literature between $k_B T$ and $4\pi k_B T$. This difference does not matter if one considers only Eq. (5) but is important if we have to compare τ_T with other times, for example, with the spin-orbit coupling time τ_{so} . In particular, the criterion for two dimensionality $D\tau_T \gg d^2$ (d is the film thickness) depends on the correct value of τ_T . I choose the above definition since at finite temperature the Green function decays as $\exp(-\pi k_B T/\hbar)$ and in the evaluation of the Kubo graph the product of two Green functions occurs.

A magnetic field hardly has an orbital effect on the Coulomb anomaly—in contrast to its effect on weak localization—and therefore causes no magnetoresistance. Lee and Ramakrishnan²⁴ showed, however, that there is an effect of the field on the spins of the electrons which causes a positive magnetoresistance. The asymptotic behavior is given by the following equation, but in the evaluation of the experimental results the complete formula is used:

$$[L(H) - L(0)]/L_{00} \approx \begin{cases} (F/2) \times 0.084h^2 & \text{for } h \ll 1 \\ (F/2) \ln(h/1.3) & \text{for } h \gg 1, \end{cases} \quad (8)$$

where $h = g\mu_B H/k_B T$.

As in weak localization the spin-orbit coupling also modifies the correction to the resistance, although much less dramatically. The Kubo graph which arises from the Hartree particle-hole graph and which yields the part proportional to F is reduced by a factor of 2 (Ref. 25) in the presence of dominating spin-orbit coupling. However, in a real system and for temperatures above 4.2 K the spin-orbit coupling is not dominating. Because of the complex formalism only the limits of vanishing and dominating spin-orbit coupling have been calculated as yet.

C. Coulomb interaction with particle-particle propagators

The contributions of weak localization and of Coulomb interaction cannot be simply added. A mixed contribution exists which modifies the first results severely.⁶⁻⁸ It arises from a class of Kubo graphs which one could classify as Fock and Hartree terms with particle-particle propagators including Coulomb interaction. In the absence of spin-orbit coupling Fukuyama obtained for the tempera-

ture dependence

$$\Delta L/L_{00} = -F \ln(\tau_T/\tau_0).$$

Altshuler *et al.*⁸ and Larkin⁷ included the electron-phonon interaction and repeated the Coulomb interaction and obtained for the temperature-dependent correction to the resistance (including the Maki-Thompson term^{26,27})

$$\Delta L/L_{00} = -g(T),$$

where

$$g(T) = \lambda / \{1 + \lambda \ln[\gamma\eta/(\pi k_B T)]\},$$

λ is the dimensionless bare interaction constant, $\gamma = \exp(0.577)$, and η is the cutoff parameter. For a superconductor with the transition temperature T_c , $g(T)$ is given by $-1/\ln(T/T_c)$. If one characterizes a normal conducting metal by a superconductor with a small superconducting transition temperature T_c then $g(T)$ is quite small and negligible in the evaluation.

In this case the magnetoresistance of the metal film should also be negligible. The magnetoresistance has, according to Altshuler *et al.*⁸ and Larkin,⁷ the same field dependence as weak localization. It is only multiplied by a factor $-\beta(T)$, which one can calculate from $g(T)$ and which should be negligible under the assumption that the normal metal is a low-temperature superconductor.

Fukuyama²⁵ obtained a different magnetoresistance for fields larger than H_T . This does not disturb the evaluation of the inelastic field which depends on the magnetoresistance at fields of the order of a few H_i , which is generally much less than H_T .

IV. EVALUATION OF THE EXPERIMENTAL RESULTS

The magnetoresistance measurements allow the determination of the important parameters of the electronic system, i.e., the inelastic lifetime and the spin-orbit coupling time or the corresponding fields H_i and H_{so} , respectively. These parameters are determined by adjustment so that the theoretical curve reproduces the experimental results. It is important that this adjustment is unique and this requires some caution and effort. Mg is a metal with a small spin-orbit coupling. When H_{so} is clearly less than H_i but not zero then it is not possible to determine the two parameters independently as we have demonstrated in Ref. 17. However, the superposition of the Mg with Au saturates the effect of the spin-orbit coupling. When H_{so} becomes much larger than H_i then the magnetoresistance depends only on H_i and yields the correct value of H_i . Therefore, we start with the evaluation of the magnetoresistance curve of MgAu at 4.6 K. Since we found in a former experiment (Ref. 16) that the superposition of Au on Mg does not change H_i we can use the same H_i for the magnetoresistance curve for pure Mg. With this knowledge one obtains indubitably the strength of the spin-orbit coupling H_{so} in pure Mg. With the use of this value for pure Mg at the highest temperature (18.9 K) one can determine the inelastic field H_i at this temperature. Finally, we can take the value H_i (18.9 K) of pure Mg and determine at the same temperature the strength of the spin-orbit coupling H_{so} in the MgAu film (this value

could hardly be determined at 4.6 K because here the ratio H_{so}/H_i was too large so that the spin-orbit coupling was saturated). With this value of H_{so} of MgAu the whole procedure is repeated and yields (after this second iteration) a consistent set of H_i and H_{so} . We obtain for H_{so} the following results: pure Mg $\Rightarrow H_{so} = 0.0046$ T and MgAu $\Rightarrow H_{so} = 0.54$ T. With these two values $H_i(T)$ can now be determined uniquely for each temperature. A common set of $H_i(T)$ reproduces the magnetoresistance curves for Mg and MgAu. $H_i(T)$ is plotted in Fig. 5 as a function of the temperature T in a double-logarithmic plot. We obtain a straight line with the slope of $p = 1.95$, which is quite close to our former results (Ref. 16) where we found the value 2.05. Figure 2 demonstrates the consistency of the evaluation. The two sets of theoretical curves in Figs. 2(a) and 2(b) are calculated for each temperature with the same value of $H_i(T)$, and each set is calculated with the same H_{so} . Only for the highest temperature we find—as always—a small deviation between experiment and theory. We checked whether this deviation might be due to the finite thickness of the film. However, the summation over $10k_z$ planes did not alter the theoretical magnetoresistance curves. We also examined a possible influence of the Coulomb interaction using (the complete form of) Eq. (8). There was no noticeable contribution in the plotted regime.

With the values of H_{so} and H_i one can easily calculate the corresponding times τ_{so} and τ_i according to relation (2). For the product $H_n\tau_n$ we obtain from the data of the film (in the free-electron model) 3.2×10^{-13} T sec. Therefore, we find for τ_{so} of pure Mg the value 7.0×10^{-11} sec and for the MgAu 5.9×10^{-13} sec.

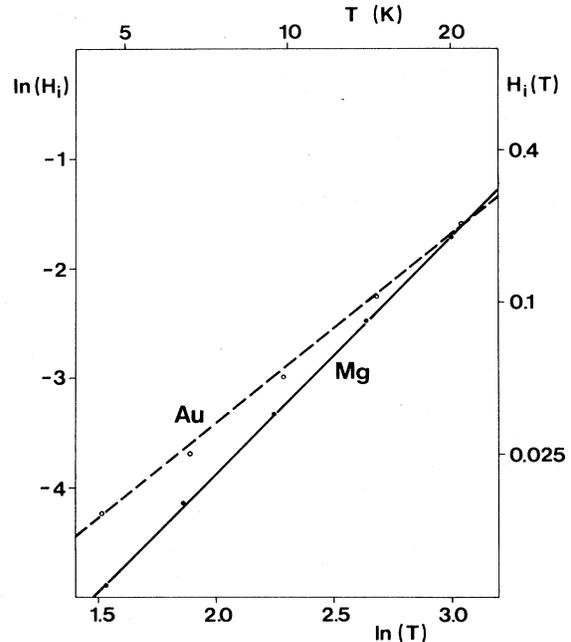


FIG. 5. Inelastic field H_i of the Mg and Au films as a function of temperature in a double-logarithmic plot. The inelastic lifetimes τ_i are obtained from the $H_i(T)$ by Eq. (2).

From the evaluation of the magnetoresistance measurements we know the temperature dependence of $H_i(T)$. Together with the two values of H_{so} we can calculate the temperature dependence of the resistance which is caused by weak localization. This is, of course, not the only contribution to the temperature dependence of the resistance. In addition, we have the Altshuler contribution caused by the electron-electron interaction and (at higher temperature) the thermal part. But when we take the difference in the resistance between the Mg and the MgAu films then the other contributions should cancel and we have a consistent check whether the temperature dependence of weak localization also obeys the theory. As we discussed above the electron-electron interaction can change if the spin-orbit coupling is changed. But in comparison with weak localization the limit of strong spin-orbit coupling is only reached when $1/\tau_{so}$ is much larger than $1/\tau_T$. τ_T varies in the temperature range from 4.5 to 18.9 K between 2.7×10^{-13} sec and 6.4×10^{-14} sec. These values are still considerably smaller than the spin-orbit coupling times. Therefore, the electron-electron interaction is even for the MgAu film with its strong spin-orbit coupling in the range (but not in the limit) of small spin-orbit coupling. The present state of the theory does not allow one to estimate the (small) remaining influence of the spin-orbit coupling on the contribution of electron-electron interaction. Even a strong spin-orbit coupling could only change the prefactor of $\ln T$ by $F/2$. [We calculate F in the free-electron approximation according to Eq. (6): $F \approx 0.52$.] The thermal part of the resistance by electron-phonon scattering should not be modified by the small superposition of Au. Therefore, we expect that the difference between the resistances of MgAu and Mg is essentially caused by weak localization. Both the experimental differences and the theoretical ones using $H_i(T)$ and the two H_{so} values are plotted in Fig. 6. (For a comparison with the theory the right scale for the conductance is appropriate.) We suppressed a constant shift of the experimental values against the theoretical ones. This shift is caused by the annealing of the MgAu film. Since one wants to be sure that the temperature dependence is reversible and that there are no annealing effects during the measurement the MgAu film has been annealed at 40 K, as the pure Mg film was. Since the healing of the lattice defects takes a very long time the Mg continued to heal during the annealing of the MgAu. Therefore, we can only compare the temperature dependence. There is good agreement between experiment and theory which shows a definite consistency between the magnetoresistance and the temperature dependence of the resistance. Only at lower temperatures does one find a deviation between the measured and the calculated resistance difference. The origin of this deviation might be due to neglected contributions of the Coulomb interaction in the particle-particle channel. The consistency has been checked for three different Mg films covered with a fraction of a monolayer of Au and the agreement was always of the same quality as in Fig. 6.

Before we evaluate the Au film we want to give another interesting confirmation of the theory. During the evaporation of the fraction of a monolayer of Au one observes an unexpected increase of the conductance. In Fig. 7 this change of the conductance is plotted (in units of L_{00}) as a

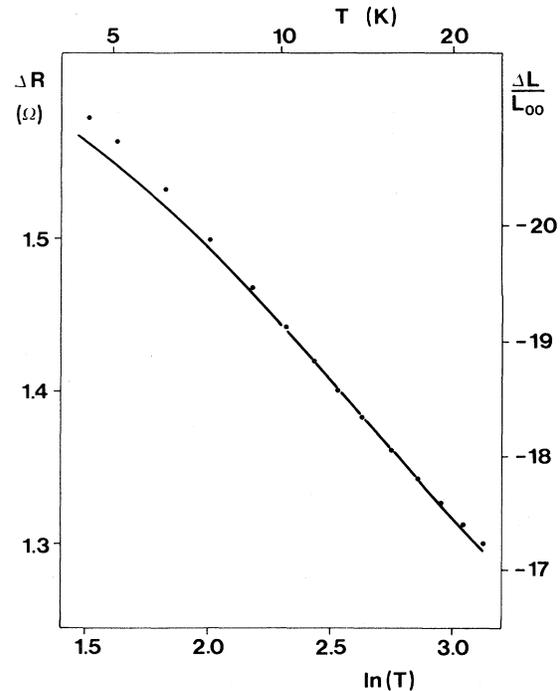


FIG. 6. Difference between the measured resistance of the Mg and the MgAu films as a function of temperature (solid circles). The solid curve gives the theoretical differences in the corresponding conductance (right scale) between the Mg and the MgAu films using the common set of $H_i(T)$ and the two different values of H_{so} which are evaluated from the magnetoresistance in Figs. 2(a) and 2(b).

function of the Au thickness during the evaporation of 0.2 atomic layers of Au on top of another Mg film. The increase of the conductance is essentially caused by the change of the spin-orbit coupling. Since we know H_{so} for the pure Mg and MgAu films we may linearly interpolate the value of H_{so} during the evaporation. Considering this change of H_{so} as a function of the Au thickness one ob-

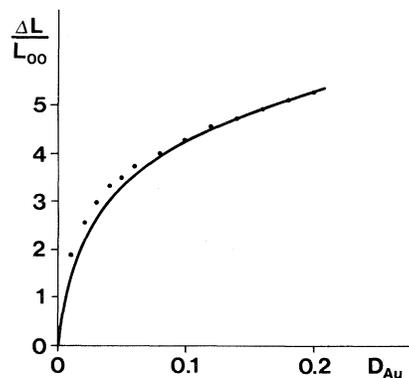


FIG. 7. Increase of the conductance $\Delta L/L_{00}$ during the evaporation of 0.2 atomic layers of Au. The experimental values are given by the closed circles. The solid curve is calculated under the assumption that the small amount of Au essentially only increases the spin-orbit coupling.

tains a theoretical change in the conductance which is given by the solid curve. Although there is almost a quantitative agreement with the observed change we only want to interpret it as a qualitative proof because we did not investigate the change in temperature and in the normal conductance increase which would be due to Au without a strong spin-orbit coupling. But even with these restrictions Fig. 7 demonstrates a high consistency between experiment and theory.

The analysis of the Au film is somewhat more difficult because the Au already has a strong spin-orbit coupling. At 4.5 K the evaluation of the magnetoresistance curves is unique and yields H_i since the effect of spin-orbit coupling is saturated. At about 20 K the magnetoresistance curve shows a maximum which allows one to adjust H_i and H_{so} at the same time. However, in this evaluation we assume a perfect description of the experiment by the theory even at 20 K. Since this agreement is not perfect for Mg one cannot exclude minor errors in the evaluation. The dependence of $H_i(T)$ is included in Fig. 5. It follows a T^p law with $p=1.7$. (We can, however, not exclude from the experimental data the fact that the field H_i has an exponent $p=1.85$ at high temperatures while at low temperature a small magnetic scattering caused by magnetic impurities in the concentration range of 1 ppm is superimposed.) The spin-orbit coupling field H_{so} has the value 1.3 T. Together with the product $H_n\tau_n = 2.78 \times 10^{-13}$ Tsec one obtains for the spin-orbit coupling time $\tau_{so} = 2.1 \times 10^{-13}$ sec. The parameters $H_i(T)$ and H_{so} allow one to calculate the temperature-dependent resistance of weak localization. Since the spin-orbit coupling of the Au is strong and cannot be reduced considerably one cannot extract the contribution of weak localization in the manner we did for Mg. Therefore, we checked the consistency of the temperature dependence in magnetic fields of 0 and 7 T. (This is almost an independent check because the magnetoresistance is essentially evaluated in fields up to $40H_i$.) In Fig. 8 we subtracted the calculated contribution of weak localization from the experimental resistance (conductance) and plotted the remaining part for 0 and 7 T as a function of $\ln T$. The two curves essentially agree but show noticeable deviations at low temperature. The (small) difference could be due to the Coulomb interaction in the particle-particle channel. We conclude that the change in the temperature dependence is essentially caused by weak localization. The result suggests that the $H=7$ T measurement shows the contribution of the Coulomb anomaly (in the particle-hole channel).

Figure 9 shows the resistance versus $\ln T$ for the Mg and the MgAu films in an external field of 7 T. In these curves the thermal part of the resistance is not negligible and complicates the evaluation. We observe that the Mg and the MgAu films show two almost perfectly parallel resistance curves. The prefactor of the low-temperature slope (which corresponds to $1-F$) is 0.8 and 0.84, respectively. The same prefactor for the Au film in Fig. 8 is 0.9. We investigated another Au film with a resistance per square of 528 Ω in which the thermal part of the resistance (caused by electron-phonon interaction) was much smaller. It is shown in the upper part of Fig. 9. Its slope is 0.93.

A few points about the evaluation and interpretation should be pointed out: (i) The films are not perfectly two

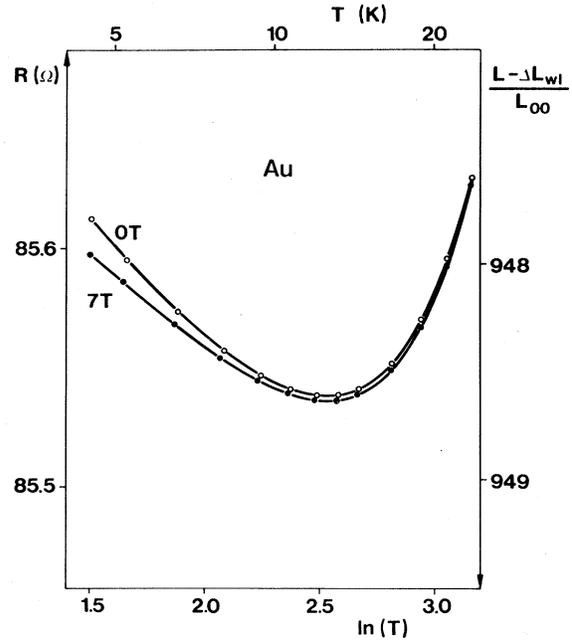


FIG. 8. Remaining resistance of the Au film as a function $\ln T$ in magnetic fields of 0 and 7 T. The (theoretical) temperature-dependent contribution of weak localization is subtracted.

dimensional with respect to Coulomb interaction. (It would be rather helpful if the theoretical papers would add the appropriate expressions for quasi-two-dimensional films with finite thicknesses.) (ii) Since the $1-F$ is close to 1 the magnetoresistance as calculated by Lee and Ramakrishnan should be small.

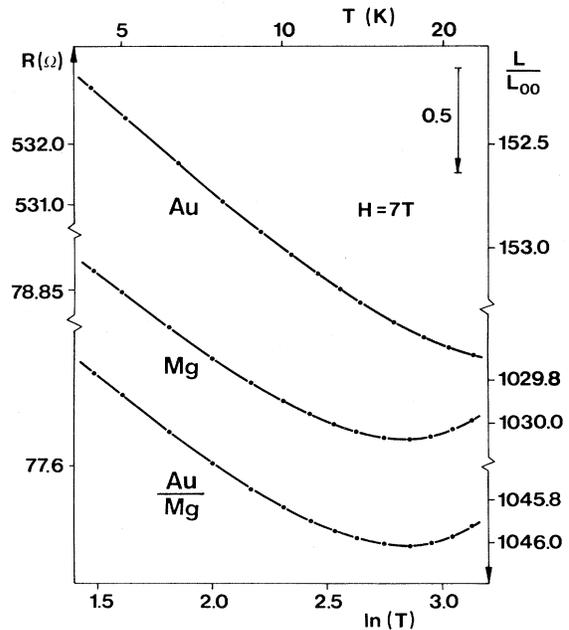


FIG. 9. Resistance of the Mg, MgAu, and Au films as a function $\ln T$. Weak localization is quenched by an external field of 7 T.

V. CONCLUSIONS

Thin Mg films with a resistance per square in the range between 80 and 250 Ω have been investigated. The small spin-orbit coupling of pure Mg can be strongly increased by a very small coverage with Au. Mg shows a negative magnetoresistance while MgAu has a positive one. Both sets of magnetoresistance curves can be well reproduced by the theory of weak localization. The adjusted parameters are the common temperature-dependent inelastic field $H_i(T)$ and the spin-orbit coupling field H_{so} , which is different for the two systems. The inelastic field $H_i(T)$ obeys essentially a T^2 law. One can calculate the contribution of weak localization to the temperature dependence of the resistance for both the Mg film and the MgAu film. This yields the theoretically expected change in the temperature dependence by the superposition of Mg with Au. It agrees with the experimentally observed change in the resistance.

Even the relatively strong increase of the conductance during the Au evaporation can be explained by the increase of the spin-orbit coupling and, as a consequence, the transition from weak localization to weak antilocalization. A large magnetic field suppresses the resistance anomaly of weak localization. In a field of 7 T the resistance of the Mg and the MgAu films shows the same temperature dependence. From the latter one obtains a slope of $\Delta L_C/L_{00}$ vs $\ln T$ which is about 0.8. For pure Au an analogous evaluation can be performed and yields corresponding results.

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