

### Anomalous shifting and narrowing of cyclotron resonance lines of two-dimensional electrons

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The dielectric function derived recently for  $\gamma < 1$ , where  $\gamma$  is the ratio of the field energy to the Fermi energy, is used to evaluate the memory function, width, and shift of the cyclotron resonance lines of two-dimensional electrons. The results are compared with the recent data of Wilson, Allen, and Tsui.

It has been revealed by Kennedy *et al.*,<sup>1</sup> and more recently by Wilson, Allen, and Tsui (hereafter, WAT), that cyclotron resonance in two-dimensional electron systems shows anomalous shifting and narrowing at low densities.<sup>2</sup> In particular, WAT have reported that it is the filling factor  $\nu = cnh/eH$  rather than the electron density  $n$  that is crucial for characterizing the cyclotron resonance data, and that anomalies occur when the filling factor is small. They have examined several theoretical models and concluded that, while all models suffer some drawbacks, the electrons form some kind of short-range ordered structure at low densities.

In the present article we make use of the dielectric function which we have derived recently for  $\gamma < 1$ , where  $\gamma$  represents the ratio of the field energy to the Fermi energy.<sup>3</sup> In the absence of magnetic field, the Fermi energy is  $(\hbar^2/2m)(2\pi n)$  so that  $\gamma$  reduces to  $\nu$ , the filling factor. In the presence of magnetic field, the Fermi energy depends on the field, causing field dependences of cyclotron parameters.<sup>4</sup>

We have evaluated a memory function from

$$M(\omega) = \frac{n_i}{4\pi nm} \int dq q^3 \frac{v(q)^2}{u(q)} \left[ \frac{1}{\epsilon(q, \omega)} - \frac{1}{\epsilon(q, 0)} \right],$$

where  $u(q) = 2\pi ne^2/q$ ,  $n_i$  is the impurity concentration, and  $v(q)$  is the impurity potential which is assumed to be

$$v(q) = v_0 \exp(-d^2 q^2/2),$$

where  $d = 17.7 \text{ \AA}$  is used in view of our previous analysis.<sup>4</sup> We have also chosen  $n_i v_0^2 = 1.2 \times 10^{-40} \text{ ergs cm}^2$ . The divergence in the dielectric function is avoided by replacing  $\omega$  by  $\omega + i\delta$ , where  $\delta$  is a small parameter.

Figure 1 represents the memory function which corresponds to  $\mu = 1.41$ ,  $H = 6.15 \text{ T}$ , and  $n = 2.1 \times 10^{11} \text{ cm}^{-2}$ .  $M_1$  and  $M_2$  are the real and imaginary parts (solid curves). The dotted curves represent the results of WAT. These two sets of curves are qualitatively similar, but the real parts are

somehow not very close.

The solid curve in Fig. 2 illustrates the FWHM (full width at half maximum) as a function of  $\nu$ . The data of WAT show that the FWHM falls into a single universal curve which has a maximum at  $\nu = 1$ . This is very interesting because, according to our theory,  $\nu \lesssim 1$  requires separate treatments. The reason why  $\nu = 1$  is important is simple. If  $\nu > 1$ , the (zero-field) Fermi energy is larger than the field energy so that the field does not destroy the quantum degeneracy of the system. The reason why  $\nu$  characterizes our theory is also simple: The actual Fermi energy is a function of  $\nu$ .

In Fig. 3 we have illustrated our theoretical shifts

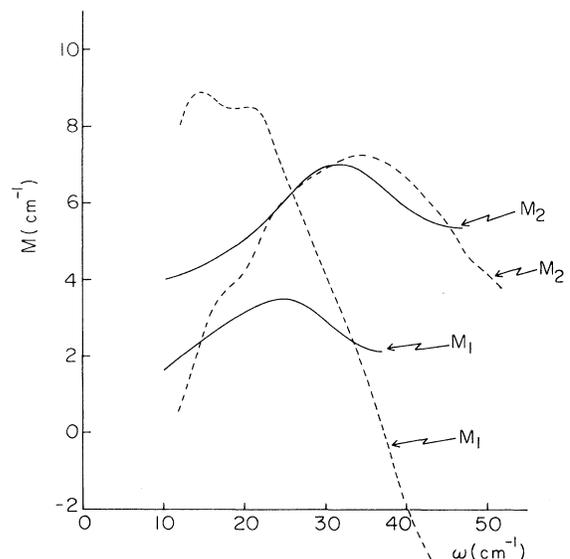


FIG. 1. Memory function vs frequency  $\omega$ .  $M_1$  and  $M_2$  represent, respectively, the real and imaginary parts of the memory function. Solid curve: present theory. Dotted curve: experimental results of Wilson, Allen, and Tsui (Ref. 2).

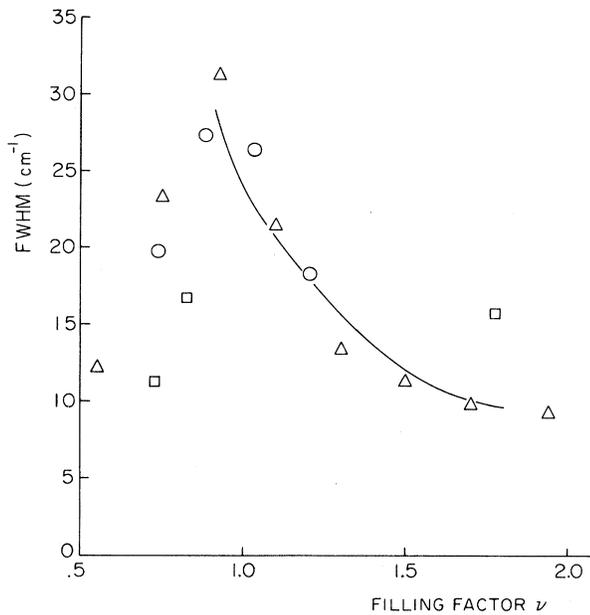


FIG. 2. FWHM as a function of filling factor  $\nu$ . Solid curve: present theory. Data: Wilson, Allen, and Tsui (Ref. 2).

(solid curve). The data of WAT show a minimum at around  $\nu = 1.3$  and the curve is concave upward. Although these two features are not reproduced, our theoretical curve tends to approach their data points for large  $\nu$ .

The reason why FWHM was reproduced better than resonance frequency is not very clear at this moment. We have observed a similar situation before<sup>4</sup> in which one of the two parameters, effective mass or relaxation time, could be fitted better than the other. Together with the small  $\nu$  region, a further extension of the theory is needed. On the other hand, the experimental difference between the maximum point of

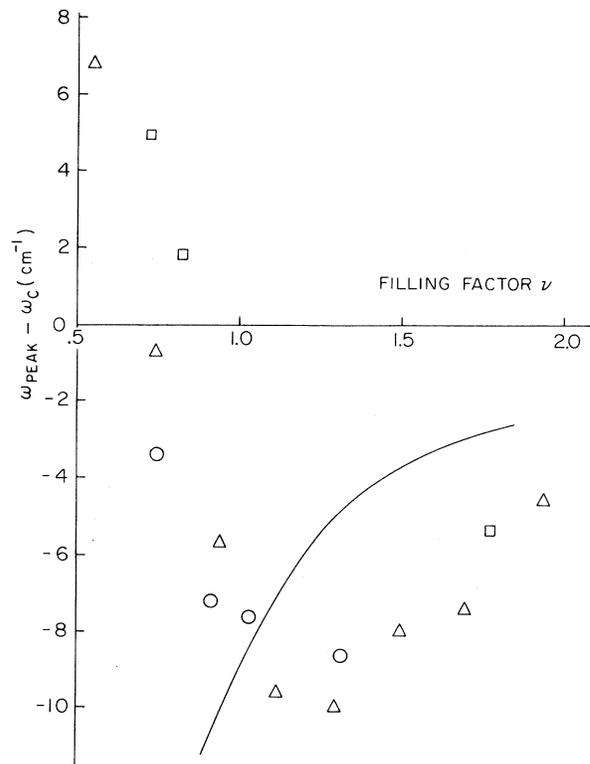


FIG. 3. Resonance frequency shift as a function of  $\nu$ . The data are taken from Ref. 2.

FWHM and minimum point of shift must also be explained theoretically.

#### ACKNOWLEDGMENT

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<sup>2</sup>B. A. Wilson, S. J. Allen, Jr., and D. C. Tsui, *Phys. Rev. B* **24**, 5887 (1981); S. J. Allen, Jr., B. A. Wilson, and D. C. Tsui, *ibid.* **26**, 5590 (1982).

<sup>3</sup>A. Isihara, L. Ioriatti, Jr., and D. Y. Kojima, *Z. Phys. B* **44**, 91 (1981).

<sup>4</sup>A. Isihara, M. Mukai, and S. J. Lee, *Phys. Rev. B* **26**, 3279 (1982); A. Isihara and M. Mukai, *ibid.* **24**, 7408 (1981).