

Memory-function approach to ellipticity and Faraday rotation in a metal-oxide-semiconductor system

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We use the memory-function approach to calculate the ellipticity and Faraday rotation due to the two-dimensional electron gas at the oxide-semiconductor interface of a metal-oxide-semiconductor system. Experimentally determined memory-function values of Allen *et al.*, as a function of the photon frequency ω , are used in our analysis. For the corresponding results with varying external magnetic field B we have used the theoretical results of Ting *et al.* for the memory function. Comparison is also made with the recently reported experimental results of Piller and Wagner.

Recently we calculated¹ the ellipticity and Faraday rotation due to the two-dimensional electron gas (2D EG) at the oxide-semiconductor interface of a metal-oxide-semiconductor (MOS) system using the single-particle relaxation- or collision-time approximation (two-dimensional Drude model), in the case where the directions of both the incident radiation of frequency ω and the external dc magnetic field $B > 0$ are oriented normal to the 2D EG. In this work we investigate the ellipticity and Faraday rotation using the memory function or current-relaxation kernel $M(\omega, B)$ developed by Götze and Wölfe.² Cyclotron resonance has recently been under investigation both experimentally^{3,4} and theoretically⁵⁻¹¹ in the 2D EG in Si inversion layers of a MOS system using the memory-function approach to explain shifts away from the cyclotron-resonance frequency.

In terms of the memory function, the zero-temperature conductivity of the 2D EG may be written⁵⁻⁷ as

$$\sigma_{\pm} = \frac{iNe^2/m}{\omega \pm \omega_c + M(\omega, B)}, \quad (1)$$

where

$$M(\omega, B) = M'(\omega, B) + iM''(\omega, B), \quad (2)$$

the prime and double prime denote real and imaginary parts, respectively, N is the electron surface concentration, m is the effective mass, $\omega_c = eB/mc$ is the cyclotron frequency, and the \pm denotes left or right circular polarization of the incident radiation. $M'(\omega, B)$ and $M''(\omega, B)$ are related by the Kramers-Kronig relation and $M''(\omega, B) \geq 0$.

Introducing a frequency- and magnetic-field-dependent mass $m^*(\omega, B)$ and relaxation time $\tau^*(\omega, B)$ [$= \nu^{*-1}(\omega, B)$ where $\nu^*(\omega, B)$ is the collision frequency] defined by

$$m^*(\omega, B) = m[1 + M'(\omega, B)/\omega] \quad (3)$$

and

$$\begin{aligned} \tau^{*-1}(\omega, B) &= \nu^*(\omega, B) \\ &= M''(\omega, B)/[1 + M'(\omega, B)/\omega], \end{aligned} \quad (4)$$

respectively, the conductivity of the 2D EG can be written in the more familiar Drude form as^{5,6}

$$\sigma_{\pm} = \frac{iNe^2/m^*}{\omega \pm \omega_c^* + i\nu^*}, \quad (5)$$

where $\omega_c^* \equiv eB/m^*c$ and we have suppressed the ω and B dependence of $m^*(\omega, B)$, $\tau^*(\omega, B)$, and $\nu^*(\omega, B)$.

The transmission coefficients of the left and right circularly polarized components of a linearly polarized wave for the 2D EG at the oxide-semiconductor interface are¹²

$$t_{\pm} = \frac{2n_0}{n_0 + n_s + (4\pi/c)\sigma_{\pm}}, \quad (6)$$

where n_0 and n_s are the indices of refraction of the oxide and semiconductor, respectively. Analogous to our approach in Ref. 1, we rewrite Eq. (6) as

$$t_{\pm} = |t_{\pm}| e^{i\xi_{\pm}}, \quad (7)$$

with

$$|t_{\pm}| = \frac{2n_0}{n_0 + n_s} \left[\frac{(\omega \pm \omega_c^*)^2 + (\nu^*)^2}{(\omega \pm \omega_c^*)^2 + (\nu^* + \omega_{ps}^*)^2} \right]^{1/2}, \quad (8)$$

$$\xi_{\pm} = -\tan^{-1} \left[\frac{\omega_{ps}^*(\omega \pm \omega_c^*)}{(\omega \pm \omega_c^*)^2 + \nu^*(\nu^* + \omega_{ps}^*)} \right], \quad (9)$$

and where

$$\omega_{ps}^* \equiv 4\pi Ne^2/m^*c(n_0 + n_s). \quad (10)$$

Furthermore, we have already shown^{1,13} that the Faraday rotation θ_M and the ellipticity δ_M (the subscript M refers to the fact that we are using the memory-function approach) are related to the transmission coefficients by

$$\frac{t_-}{t_+} = \left[\frac{1 - \delta_M}{1 + \delta_M} \right] e^{-2i\theta_M}. \quad (11)$$

Thus

$$\theta_M = \frac{1}{2}(\xi_+ - \xi_-) \quad (12)$$

and

$$\delta_M = \frac{|t_+| - |t_-|}{|t_+| + |t_-|}. \quad (13)$$

Hence

$$\theta_M = \frac{1}{2} \tan^{-1} \frac{2\omega_{ps}^* \omega_c^* [\omega^2 - (\omega_c^*)^2 - \nu^* (\nu^* + \omega_{ps}^*)]}{[(\omega + \omega_c^*)^2 + \nu^* (\nu^* + \omega_{ps}^*)][(\omega - \omega_c^*)^2 + \nu^* (\nu^* + \omega_{ps}^*)] + (\omega_{ps}^*)^2 [\omega^2 - (\omega_c^*)^2]} \quad (14)$$

and

$$\delta_M = \frac{4\omega \omega_c^* \omega_{ps}^* (2\nu^* + \omega_{ps}^*)}{\{[(\omega + \omega_c^*)^2 + (\nu^*)^2]^{1/2} [(\omega - \omega_c^*)^2 + (\nu^* + \omega_{ps}^*)^2]^{1/2} + [(\omega - \omega_c^*)^2 + (\nu^*)^2]^{1/2} [(\omega + \omega_c^*)^2 + (\nu^* + \omega_{ps}^*)^2]^{1/2}\}^2} \quad (15)$$

The corresponding rotation and ellipticity when $M(\omega, B)$ is replaced with a constant (Drude model) are denoted as θ and δ , respectively, and were investigated in Ref. 1.

From Eq. (14), the condition for null Faraday rotation¹⁴ is

$$\omega^2 = (\omega_c^*)^2 + \nu^* (\nu^* + \omega_{ps}^*), \quad (16)$$

which, using Eqs. (3) and (4), may be rewritten as

$$[\omega + M'(\omega, B)]^2 = \omega_c^2 + M''(\omega, B)[M''(\omega, B) + \omega_{ps}], \quad (17)$$

where $\omega_{ps} \equiv 4\pi N e^2 / mc(n_0 + n_s)$.

Using the experimentally determined values of the memory function as a function of the photon frequency ω [Fig. 1(b) of Ref. 4] and the theoretical values of $M(\omega, B)$ as a function of the external magnetic field B (Fig. 3 of Ref. 5), we present, in Figs. 1 and 2, plots of θ_M vs ω and B , respectively. We also include the corresponding plots of θ (Ref. 1) for comparison. Figure 1 shows that with the

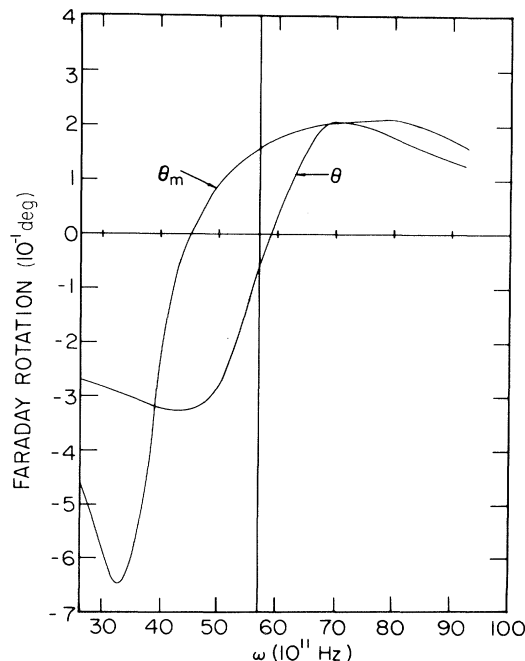


FIG. 1. Plot of the Faraday rotation θ_M and θ vs the photon frequency ω using the parameters $N=2.3 \times 10^{11} \text{ cm}^{-2}$, $B=6.15 \times 10^4 \text{ G}$, $n_0=1.95$, $n_s=3.44$, and $m=0.19m_e$, where m_e is the electron rest mass, and a constant collision time $\tau=7.7 \times 10^{-13} \text{ sec}^{-1}$ for the two-dimensional Drude model. The vertical line corresponds to the value $\omega = \omega_c$.

memory-function approach we may now achieve null Faraday rotation for a photon frequency $\omega < \omega_c$ and that $\theta_M|_{\omega=\omega_c} > 0$, in marked contrast to the two-dimensional Drude model investigated in Ref. 1. In Fig. 2 we used Eq. (4) for low external magnetic field (i.e., $\omega_c < \omega$), where the oscillations of the real and imaginary parts of the memory function are small, to calculate the constant collision time τ used in the two-dimensional Drude model. The oscillations in the θ_M -vs- B plot for large external magnetic field ($\omega_c > \omega$) are due to the oscillations of the memory function (see Fig. 3 of Ref. 5).

In Figs. 3 and 4 we present plots of the ellipticity δ_M and δ vs ω and B , respectively. Figure 3, for varying photon frequency ω , shows a substantial shift, due to memory effects, in the maximum of the ellipticity away from the resonance position $\omega = \omega_c$. The plot of δ_M vs B gives the remarkable prediction of a plateau of constant ellipticity in a neighborhood of the resonance position. We also obtain a relative maximum and a relative minimum of the

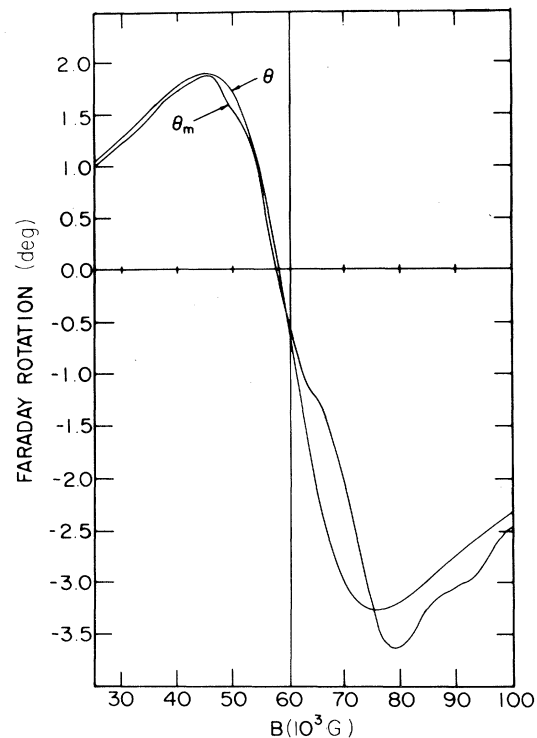


FIG. 2. Plot of the Faraday rotation θ_M and θ vs the magnetic field B using the parameters $N=2.6 \times 10^{12} \text{ cm}^{-2}$, $\omega=5.59 \times 10^{12} \text{ sec}^{-1}$, and n_0, n_s, m , and τ as in Fig. 1. The vertical line corresponds to the value $\omega = \omega_c$.

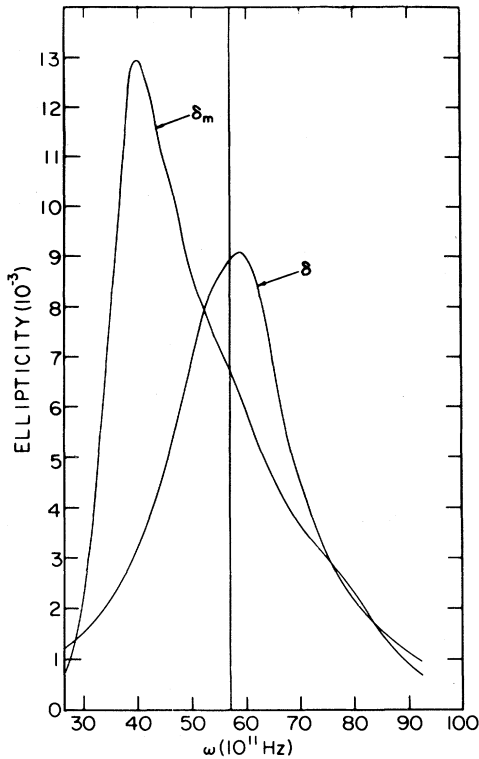


FIG. 3. Plot of the ellipticity δ_M and δ vs the photon frequency ω using the same parameters as in Fig. 1.

ellipticity for high magnetic fields ($\omega_c > \omega$) due to the oscillations of the memory function for varying external magnetic field.

We note also that from Eq. (15) we obtain a nontrivial condition for null ellipticity¹⁵

$$\omega_c^* \omega_{ps}^* (2\nu^* + \omega_{ps}^*) = 0, \tag{18}$$

which, using Eqs. (3) and (4) and recalling that $M''(\omega, B) > 0$, may be written as

$$[\omega + M'(\omega, B)]^{-1} = 0. \tag{19}$$

However, from the experimental⁴ and theoretical⁵ values of the memory function, it appears that this condition cannot be met experimentally for the inversion layer in a MOS system.

In conclusion, we have calculated the Faraday rotation and ellipticity produced by the inversion layer (2D EG) in a MOS system using the memory-function approach of Götze and Wölfe² for both experimentally⁴ determined and theoretically⁵ predicted values of the memory function and have found significant modifications to the two-dimensional Drude model of Ref. 1, which should be easily detectable experimentally.

Note added in proof. We have now completed an investigation of the effects of multiple reflections within the oxide layer and we find that they have a significant influence on the results for θ and δ , giving rise to an additional factor of $(n_0 + n_s)/(1 + n_s) = 1.22$. In addition, we find that

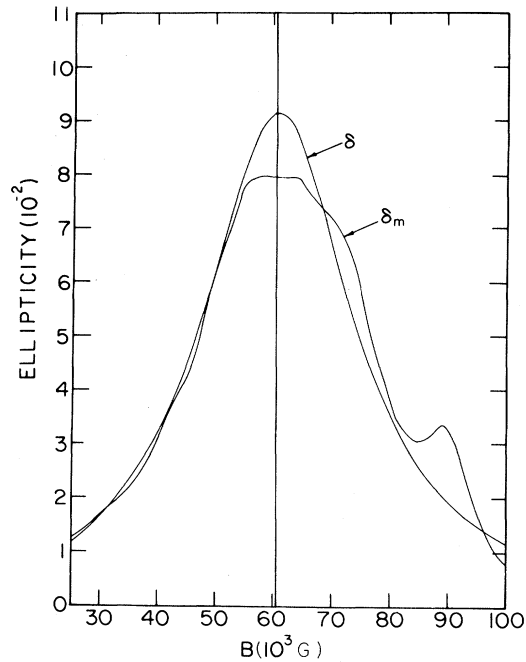


FIG. 4. Plot of the ellipticity δ_M and δ vs the magnetic field B using the same parameters as in Fig. 2.

the inclusion of contributions due to transmission through the metal has a negligible effect on our results. We conclude that all of our results for θ and δ given above should be multiplied by the factor of 1.22. A detailed discussion of these added results is presently under preparation.

In addition, the first observational results of Faraday rotation in a MOS system has been reported by Piller and Wagner.¹⁶ However, their theoretical analysis is confined

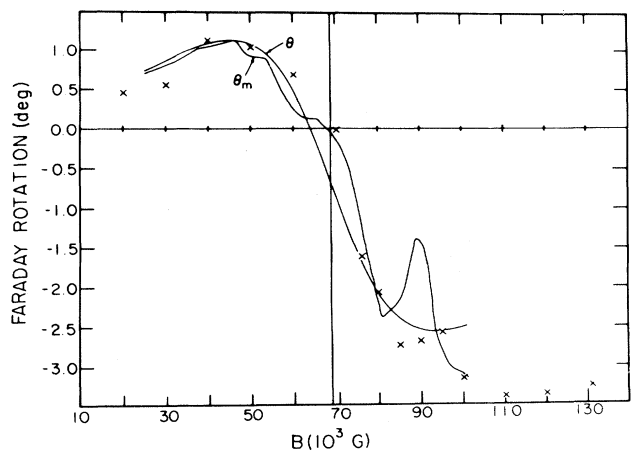


FIG. 5. "Best-fit" plot of the Faraday rotation θ_M and θ vs the external magnetic field B to the experimental results of Ref. 16. For θ a constant collision time $\tau = 4.5 \times 10^{-13}$ sec was used.

to the so-called "single-pass" situation, totally neglecting the important contributions which we have seen arise due to multiple reflections. Also, as we have previously shown,¹⁷ the single-pass results are strikingly different than the multiple-pass results, in that θ takes on both positive and negative values in the latter case, whereas θ is always negative in the single-pass case for the parameters of the present experiment. Thus we feel that the claim of agreement between theory and experiment¹⁶ is premature (in this context it should be noted that the solid and dotted curves in Fig. 2 of Ref. 16 do *not* represent theoretical re-

sults¹⁸).

In Fig. 5 we reproduce the Piller-Wagner experimental results. We also include our "best-fit" theoretical results (including the 1.22 factor discussed above) for θ and θ_M , corresponding to the absence and presence, respectively, of memory effects.

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