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Electron inelastic lifetime and electron-electron attraction strength in Al films

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Magnetoresistance measurements on 100-Å films of granular Al from 3 to 20 K are reported and evaluated using the localization and interaction theories. The electron inelastic scattering rate is well fitted by a sum of terms due to electron-electron and electron-phonon scattering. The electron-electron attraction strength, $\beta(T)$, is in good agreement with Larkin's theory.

In disordered superconducting films the combination of localization and Coulomb interaction effects (reviewed by Fukuyama[1]) gives a resistance which is a complicated function of temperature and which is almost impossible, in practice, to separate into its components.[2] An approach which offers a number of advantages is the measurement of the magnetoconductance (MC). Although the theories for the MC in disordered metals[3-5] are, at first sight, even more intricate than the theories for the temperature dependence, many of the terms in the interaction theory can be neglected in the small-field limit considered here. We have separated the localization and interaction terms which contribute to the MC in our Al films and have determined the inelastic scattering time, τ_i,[6] the spin-orbit scattering time, τ_{so}, and the electron-electron attraction strength, β.

Our films were prepared by evaporating 99.999% pure Al onto room-temperature sapphire substrates in an oxygen atmosphere of 8×10^{-5} torr. The sample dimensions, 7.40×0.52 mm^2, were defined with a brass mask held just above the substrate. The resistance measurements were all carried out at fixed temperature by varying the magnetic field. Each point was measured at least once with an HP3456A digital voltmeter in its four terminal resistance mode, giving six steady digits of accuracy. When the sample resistance was low or the temperature sufficiently near to T_c, however, the relatively high bias current (1 mA if $R<1000\ \Omega$) depressed the measured magnetoresistance. In these cases we made low bias current (1–10 μA) bridge measurements using a PAR124 lock-in amplifier. At higher temperatures and in samples with resistances greater than 1000 Ω the two methods always gave the same results.

The diffusion constant, D, was obtained from measurements of the upper critical field just below T_c.[7] Measured values of T_c, D, the resistance ratio $\Gamma\equiv R(300\text{ K})/R(4\text{ K})$, and the sheet resistance $R_\square$ can be found in Table I.

The most important contribution to the MC at low fields comes from *localization* effects. Field dependence in the localization theory was first considered by Al'tshuler *et al.*[3] and later extended to include spin-orbit coupling and magnetic scattering by Hikami, Larkin, and Nagaoka[5] and Maekawa and Fukuyama.[8] The full result, neglecting magnetic scattering, as discussed in Ref. 9, is

$$\frac{\Delta G}{G_{00}}\equiv\frac{G(B,T)-G(0,T)}{e^2/2\pi^2\hbar}=\left[\ln\left(\frac{B_0}{B_i}\right)-\tfrac{3}{2}\ln(a+1)\right]-\left[\psi\left(\frac{1}{2}+\frac{B_0}{B}\right)-\psi\left(\frac{1}{2}+\frac{B_i}{B}(a+1)\right)+\tfrac{1}{2}\psi\left(\frac{1}{2}+\frac{B_i}{B}\right)-\tfrac{1}{2}\psi\left(\frac{1}{2}+\frac{B_i}{B}(a+1)\right)\right]$$
$$\equiv Y(B;B_i,B_{so},B_0)\ . \tag{1}$$

TABLE I. Relevant sample parameters. The values $v_F=1.3\times10^8$ cm/sec and $k_F=1.1\times10^8$ cm^{-1} are used.

Sample	A(○)	B(□)	C(△)	G(▲)	H(■)
$R_\square$ (Ω)	96	193	126	52	15
D (cm^2/sec)	4.1	3.2	3.4	6.5	20
Γ	1.052	1.030	1.044	1.073	1.220
T_c (K)	2.26	2.24	2.25	2.35	2.05
d (Å)	150	100	100	100	100
l (Å)	9.5	7.3	7.8	15	46
B_{so} (G)	1100	600	150	130	40
d_0^{expt} (Å)	54	40	32	36	66
$\theta_{e\text{-ph}}^{expt}$ (K)	16	19	18	13	10
$\theta_{e\text{-ph}}^{theory}$ (K)	16	20	18	12	8

Here we use $G=1/R_{\square}$, B is the applied field, ψ is the digamma function, and $a=4B_{so}/3B_i$. The characteristic fields are defined by the relation $B_x \equiv \phi_0/4\pi D\tau_x$, where $\phi_0=hc/2e$ is the superconducting flux quantum and x is "0" for elastic scattering, "i" for inelastic scattering, and "so" for spin-orbit scattering. In fields such that $B \ll B_i$, Y may be approximated by the quadratic function $Y(B;B_i,B_{so},B_0)=m_0B^2$, where

$$m_0=(1/24B_i^2)[(1-a-a^2/2)/(1+a)^2] \ .$$

It is important to notice that m_0 varies from $1/24B_i^2$ for weak spin-orbit scattering ($B_i \gg B_{so}$) to $-1/48B_i^2$ in the strong spin-orbit scattering limit. For typical inelastic times of about 10^{-10} sec, the characteristic field B_i will be on the order of 10 G.

The *interaction* theory supplies three additive terms to the general expression for the MC in two dimensions (2D). Corrections to the conductance from the particle-hole channel[10] and the particle-particle channel[4] are orders of magnitude smaller than the localization term in the small fields that we have used and may be neglected. The third correction to the conductance, due to superconducting fluctuations above T_c, comes from the Maki-Thompson (MT) diagram[11,12]

$$\Delta G/G_{00}=-\beta(T)\,Y(B;B_i,B_{so}=0,B_0) \ , \qquad (2)$$

where $\beta(T)$, which diverges near T_c as $(\ln T/T_c)^{-1}$, is a measure of the electron-electron attraction strength. Since Y is the same function that appears in (1), the MT term in the MC will be of the same order of magnitude as the localization term and we must include it in our analysis.

Keeping both the localization and MT contributions, the total expression for the MC becomes

$$\Delta G/G_{00}=Y(B;B_i,B_{so},B_0) - \beta(T)\,Y(B;B_i,B_{so}=0,B_0) \ , \qquad (3)$$

with Y given in (1). For fields much smaller than B_i, the quadratic approximation

$$\Delta G/G_{00}=mB^2=[m_0-\beta(T)/24B_i^2]B^2$$

may be used, with m_0 given above.

Typical MC data are shown in Fig. 1. For $T \gtrsim 15$ K the MC is always positive because of the small value of the Larkin parameter $\beta(T)$ far above T_c and large values of the inelastic field B_i at high temperatures. At intermediate temperatures the slope of the MC changes from negative to positive at about 1 kG. This may be attributed to the fact that B_i has fallen to a value of order of the spin-orbit field B_{so}, and to the saturation of the MT term in high fields. As T gets nearer to T_c the slope of the MC becomes everywhere negative, which is consistent with our expectations for $B_i < B_{so}$ and $\beta > 1$.

A correct analysis of our data requires the determination of three parameters: B_i, β, and B_{so}. Using the curvature at small fields, $m(T)$, and the MC at a fixed intermediate field [chosen to be much less than $H_{c2}(0)$], we determined two of the three parameters, B_i and β, in terms of the third, B_{so}, at each temperature.[13] By varying B_{so} we were then able to adjust the curve shape to achieve the best overall fit. Since this method only works well at higher temperatures ($T \gtrsim 10$ K) where $B_i \gtrsim B_{so}$, we determined B_{so} at 10 K and assumed that the spin-orbit scattering rate was independent of temperature.[6] Our fits of theory to experiment are shown as solid lines in Fig. 1. Comparably good fits could not be obtained if we assumed that $B_{so}=0$, as was done in Ref. 9. Moreover, the values of $\tau_i(T)$ and $\beta(T)$ obtained by assuming $B_{so}=0$ are substantially different from the results of the more thorough analysis.

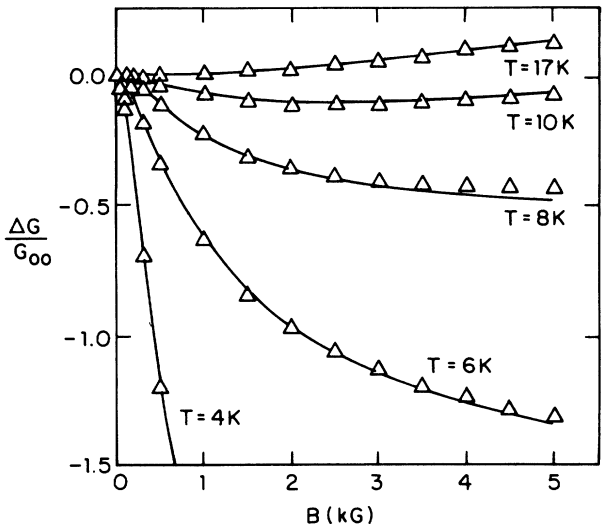

FIG. 1. The normalized magnetoconductance of sample C at four temperatures. The triangles are experimental data and the curves are the fits to Eq. (3).

The experimentally determined values of β are plotted as a function of reduced temperature $t=T/T_c$ in Fig. 2. Larkin's theoretical prediction for β,[12] which has no adjustable parameters, is shown as a solid line in the same fig-

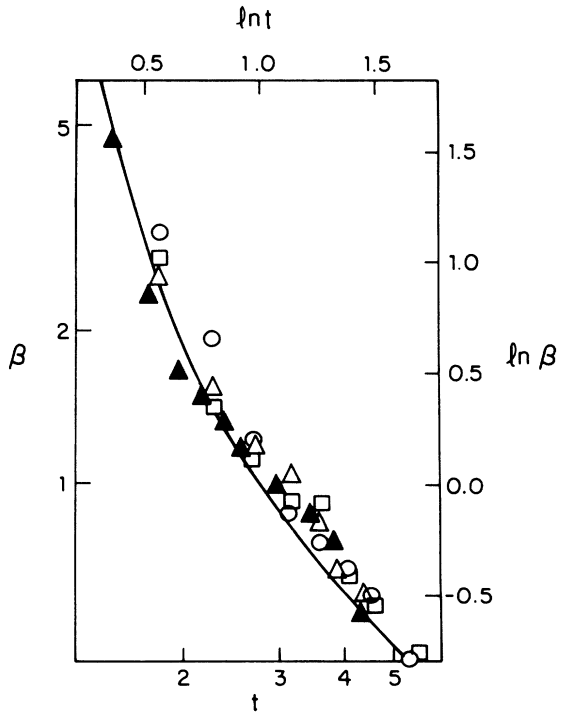

FIG. 2. A log-log plot of the electron-electron interaction strength, β, vs the normalized temperature $t=T/T_c$. The points are determined from experimental data as discussed in the text. The theoretical prediction for β from Ref. 12 is the smooth curve.

ure; it agrees very well with the data.

Theoretical predictions for the inelastic time in disordered metals vary with both the type of scattering and the sample dimensionality. For electron-electron scattering, the sample is two dimensional if $(\hbar D/k_B T)/d^2 \gg 1$, where d is the film thickness. At 5 K, this ratio exceeds 5 in our samples, indicating that they are into the 2D regime. For 2D systems thick enough that $k_F d > 1$, the inelastic rate for electron-electron (ee) scattering is[14]

$$\frac{1}{\tau_i^{ee}} = \frac{\pi}{k_F d} \frac{k_B T \ln(T_1/T)}{3Dm} , \tag{4}$$

where $k_B T_1 = \frac{32}{27}(me^4/2\hbar^2)(k_F l)^3$.

The most common phonon wave vector is roughly given by $q_T = k_B T/\hbar v_s$, which corresponds to a q_T^{-1} of about 50 Å at $T = 5$ K. This suggests that our samples are marginally into the 3D range for phonon scattering. The 3D nature of the phonons should, however, be enhanced by the intimate contact between the film and the Al_2O_3 substrate, despite the acoustic mismatch at the interface. In the dirty limit, $q_T l < 1$, the inelastic rate for 3D electron-phonon (e-ph) scattering has been determined by Thouless[15]

$$\frac{1}{\tau_i^{e\text{-ph}}} = 44(\Gamma - 1)\frac{300\text{ K}}{\Theta_D} q_T^4 \frac{v_F}{k_F^3} . \tag{5}$$

In the clean limit, $q_T l > 1$, this time must be multiplied by $2q_T l$.

Combining (4) and (5) we find the total inelastic scattering rate obeys

$$\frac{\hbar}{\tau_i}\frac{k_F l}{k_B T} = \left(\frac{d_0}{d}\right) + \left(\frac{T}{\Theta_{e\text{-ph}}}\right)^P , \tag{6}$$

where P will be 3 in the dirty case and 2 in the clean case. Using (4) and (5), $k_F = 1.1 \times 10^8$ cm^{-1}, $v_s = 3.5 \times 10^5$ cm/sec, and $\Theta_D = 394$ K, the constants are $d_0 = (\pi/k_F) \times \ln(T_1/T) \approx 50$ Å, $\Theta_{e\text{-ph}}(\text{dirty}) = 9.3[(\Gamma - 1)D]^{-1/3}$ K, and $\Theta_{e\text{-ph}}(\text{clean}) = 3.7(\Gamma - 1)^{-1/2}$ K. The product $(\hbar/\tau_i)(k_F l/k_B T)$ vs T, for each sample, is plotted in Fig. 3 along with the least-squares fits to (6). The inset compares our measured values of d_0 with the theory of Ref. 14 while our measured values of $\Theta_{e\text{-ph}}$ are compared with Thouless's theory[15] in Table I. Samples A, B, C, and G are considered in the dirty limit for e-ph scattering, while H is taken to be in the clean limit.

For reasonable parameter values, our ee and e-ph scattering rates, as exhibited by d_0 and $\Theta_{e\text{-ph}}$, show remarkably good agreement with the theories outlined above. This is the first time that inelastic scattering times inferred from MC measurements in metallic films have been explained quantitatively by theory.

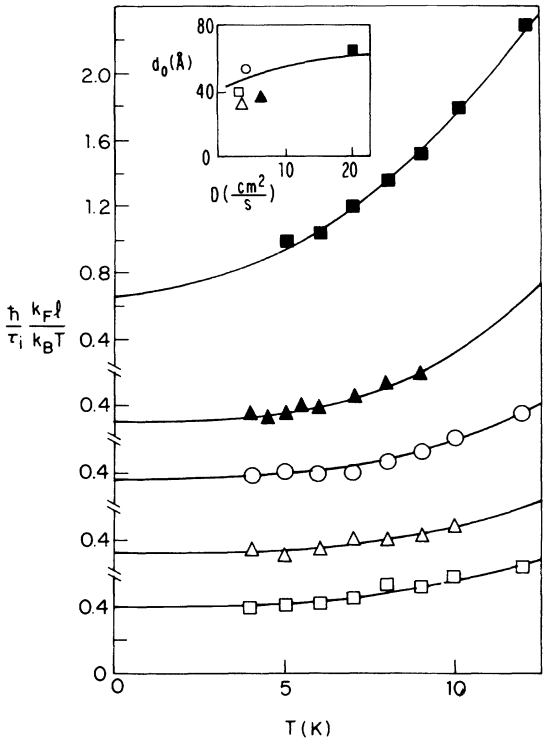

FIG. 3. Dependence of the product $(\hbar/\tau_i)(k_F l/k_B T)$ on temperature. The points represent the five samples described in Table I and the curves are fits to (6). The inset shows the dependence of the fitting parameter d_0 on D. The solid curve represents the theoretical value for d_0 from Ref. 14.

Using a tunnel injection method, Chi and Clarke[16] determined τ_i in relatively thick, clean Al films. All of their data are reasonably well fitted by the theory for τ_i outlined above. Inelastic times roughly ten times shorter were inferred from phase-slip center measurements[17] in films similar to Chi and Clarke's, but the authors question whether τ_i is actually measured in this experiment because current-induced relaxation mechanisms are also present.[17]

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