Spin-glass dynamics with conserved magnetization

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The dynamical response of a spin-glass is studied at the mean-field level for models in which the total magnetization is conserved. The first model considered is totally dissipative, like the conventional relaxational ones, but the Langevin equation is diffusive. In the $k \to 0$ limit, the diffusion constant is unaffected by the proximity of the spin-glass transition, but the region of k space in which hydrodynamics is valid shrinks to the origin as $T \to T_g$. In the rest of k space, the dynamics are effectively the same as in the relaxational model, with a relaxation rate $\propto T - T_g$. At T_g, spin correlations have the $t^{-1/2}$ behavior of the relaxational model. Mode coupling is added in the second model, and a self-consistent calculation gives a transport coefficient $\propto [\ln(T-T_g)]^{1/2}$ [and $\propto (\ln\omega)^{1/2}$ at T_g]. Sound propagation is also examined, and the sound-damping rate is found to have similar logarithmic behavior. The sound speed varies smoothly through T_g. Below T_g we find new $\omega^{-1/3}$ singularities in the transport coefficient, spin-correlation function, and sound damping.

I. INTRODUCTION AND MODELS

An understanding of the dynamical behavior of spin-glasses seems not only to be essential to the interpretation of a wide variety of experiments, but also to be rather helpful in clarifying the mysterious nature of the equilibrium spin-glass state.[1-4] To date, however, most of the dynamical theories along this line have dealt with models in which the spin dynamics are purely relaxational. For metallic Ruderman-Kittel-Kasuya-Yosida (RKKY) spin-glasses, Korringa relaxation of the impurity spins makes these models natural. Even for insulating spin-glasses, it would appear that such models are appropriate for very long times, since the presence of random anisotropy or dipolar forces breaks the rotational invariance of the Heisenberg model and eventually makes the total magnetization relax.[5] However, these symmetry-breaking terms may be quite small, thus many experiments may not be sensitive to this feature. It is therefore relevant to examine the dynamics of rotationally invariant models in which the total spin is conserved.

We have, in fact, studied such models a few years ago.[6,7] In the present paper we examine systematically the questions raised there in the light of progress made in the intervening time in understanding the dynamics of relaxational models. We also have to revise and generalize some of the conclusions we reached then.

As in that work, we use Langevin soft-spin models to facilitate the formal perturbation theory. The effective Hamiltonian is

$$H_{\text{eff}} = \frac{1}{2} \sum_{i,j} (r_0 \delta_{ij} - J_{ij}) \vec{S}_i \cdot \vec{S}_j + \frac{1}{4} u \sum_i (\vec{S}_i^{\,2})^2 , \tag{1.1}$$

where the exchange bonds J_{ij} are independent Gaussian variables with zero mean and variance Δ_{ij}. It is convenient to define $\Delta = \sum_j \Delta_{ij}$; Δ occurs frequently in our formulas below.

We consider two kinds of dynamics. In the first model, the Langevin equation

$$\frac{\partial \vec{S}_i}{\partial t} = \Gamma_0 \nabla^2 \frac{\delta H}{\delta \vec{S}_i} + \vec{\eta}_i(t) \tag{1.2}$$

is purely diffusive. The notation ∇^2 means a lattice second derivative; its Fourier transform is

$$-K^2(k) \equiv -2 \sum_{\mu=1}^{d} (1 - \cos k_\mu) \to -k^2 \quad \text{as } k \to 0 . \tag{1.3}$$

The appearance of $\Gamma_0 \nabla^2$ rather than the simple constant $-\gamma_0$ of relaxational models reflects the conservation of spin.

In the second model, $\partial \vec{S}_i / \partial t$ acquires another term, representing the nondissipative motion of the magnetic moments in the exchange fields of their neighbors,

$$\frac{\partial \vec{S}_i}{\partial t} = \Gamma_0 \nabla^2 \frac{\delta H}{\delta \vec{S}_i} + \lambda \vec{S}_i \times \sum_j J_{ij} \vec{S}_j + \vec{\eta}_i(t) . \tag{1.4}$$

Following the conventional terminology of the critical dynamics literature, we call this a "mode-coupling" model. In either model the noise $\vec{\eta}_i(t)$ satisfies

$$\sum_j e^{-i \vec{k} \cdot \vec{r}_{ij}} \langle \eta_{i\alpha}(t) \eta_{j\beta}(t') \rangle = 2T\Gamma_0 K^2(k) \delta(t-t') \delta_{\alpha\beta} . \tag{1.5}$$

For formal purposes, it is convenient to generalize all the foregoing to $3n$-component spins, with the equations of motion (1.2) and (1.4) valid for each of the n triplets of components $S^{(l)}$ $(1 \le l \le n)$. This permits an expansion in

$1/n$, in analogy to the strategy of De Dominicis and Peliti[8] in a different problem.

The diagrammatic perturbation theory for these models (in both the lattice coordinates and k space) was described in Ref. 7, to which the reader is referred for details. Here we just mention that a solid line stands for the susceptibility, a small square for the interaction u, and a closed circle for the mode-coupling vertex λJ_{ij}. The bonds J_{ij} in the equation of motion are represented by wavy lines, and the averaging of these random factors is given by linking them together in pairs, the resulting linked pair giving the variance Δ_{ij}. Correlation functions $C_{ij}(t)$ are indicated by "circled lines" in the standard fashion.[9,10] Equivalently they can be thought of as made up of two susceptibility lines joined by a noise vertex $\Lambda_0(k)=2/\Gamma_0 K^2(k)$. In general, we can write

$$C(k,\omega)=G(k,\omega)\Lambda(k,\omega)G(k,-\omega)\ , \tag{1.6}$$

where $\Lambda(k,\omega)$ is a dressed noise vertex, as in Refs. 3 and 4.

Incidentally, in Ref. 6 it was incorrectly stated that the fluctuation-dissipation theorem (FDT)

$$C(k,\omega)=\frac{2T}{\omega}\,\mathrm{Im}G(k,\omega) \tag{1.7}$$

was violated in the presence of the randomness. What is true is that one cannot use $\Lambda=\Lambda_0$ in Eq. (1.6),

$$\mathrm{Im}G(k,\omega)\neq |G(k,\omega)|^2\frac{\omega}{T\Gamma_0 K^2(k)}\ , \tag{1.8}$$

but this is not the FDT. Fortunately, no incorrect conclusions were drawn from this statement; on the contrary, the necessary vertex corrections replacing Λ_0 by Λ were in fact taken into explicit account.

We will also want to examine sound propagation in these models.[11–14] We do this by adding a dynamical piece to the bonds J_{ij}:

$$J_{i,i+e_\mu}\to J'_{i,i+e_\mu}=J_{i,i+e_\mu}(1+g\nabla_\mu\psi_{i\mu})\ , \tag{1.9}$$

where ∇_μ is a lattice gradient and $\psi_{i\mu}$ is a lattice displacement field. The phonons have a zeroth-order Hamiltonian

$$H_p=\tfrac{1}{2}\sum_{i,\mu}[C_0^2(\nabla_\mu\psi_{i\mu})^2+\pi_{i\mu}^2]\ \text{ with }\ \pi_{i\mu}=\dot\psi_{i\mu} \tag{1.10}$$

and a coupling to the spins, by virtue of (1.9), of

$$H_{\rm sp}=g\sum_{i,\mu}J_{i,i+e_\mu}\nabla_\mu\psi_{i\mu}\vec S_i\cdot\vec S_{i+e_\mu}\ . \tag{1.11}$$

The Langevin equation for the lattice is simply that for coupled damped oscillators

$$\frac{\partial\pi_{i\mu}}{\partial t}=-\frac{\partial H}{\partial\psi_{i\mu}}+\gamma_0\nabla^2\frac{\delta H}{\delta\pi_{i\mu}}+\xi_{i\mu} \tag{1.12}$$

with the driving noise subject to

$$\sum_j e^{i\vec k\cdot\vec r_{ij}}\langle\xi_{i\mu}(t)\xi_{j\nu}(t')\rangle=2T\gamma_0K^2(k)\delta_{\mu\nu}\delta(t-t')\ . \tag{1.13}$$

In this model the local exchange is changed by an amount proportional to the local strain of the lattice; this change is of the same sign as the original bond strength, regardless of whether the bond is ferro- or antiferromagnetic. This is supposed to represent the situation in, e.g., $Eu_xSr_{1-x}S_2$, where, for example, moving two magnetic ions closer together increases wave-function overlap and, thus, the magnitude of the exchange, whatever its sign. This model is slightly different from those studied earlier by us and Khurana,[11,12] where the strain was coupled effectively to the spin magnitude $\vec S_i^{\,2}$ rather than to $\vec S_i\cdot\vec S_{i+e_\mu}$. For our purposes here it is effectively the same as that considered recently by Fischer.[14]

In diagrams we indicate the phonon propagator by a wavy line. Having now run out of diagrammatic symbols, we cannot consider any further variants of these models in this paper.

II. PURELY DISSIPATIVE MODEL; SUPPRESSION OF DIFFUSION

A. $T>T_g$ $(\omega\to 0)$

At the level of approximation we adopt throughout this paper (leading order in $1/z$, where z is the coordination number of the lattice and spin dimensionality $n=\infty$), the spin susceptibility is dressed by the self-consistent self-energy insertions of Fig. 1. Thus we effectively consider a spherical model, whose statics were solved by Kosterlitz *et al.*[15] The loop diagram is frequency independent, so we just incorporate it into the static inverse susceptibility r along with the zero-frequency part of the term proportional to Δ:

$$G^{-1}(k,\omega)=\frac{-i\omega}{\Gamma_0K^2(k)}+r+\frac{\Delta}{N}\sum_k[G(k,0)-G(k,\omega)]\ . \tag{2.1}$$

We observe immediately that since $\sum$ is k independent, the transport coefficient, defined as the inverse of the coefficient of $-i\omega/k^2$ in $G^{-1}(k,\omega)$, is unchanged from Γ_0. However, if we define a k-dependent generalized transport coefficient $\tilde\Gamma(k,\omega)$ by

$$G^{-1}(k,\omega)=\frac{-i\omega}{\tilde\Gamma(k,\omega)K^2(k)}+r\ , \tag{2.2}$$

the result $\tilde\Gamma(k,0)\to\Gamma_0$ holds only for $k\to 0$. To see this, just expand the last term in Eq. (2.1) in ω. For $\omega\to 0$ we let

$$\frac{1}{\tilde\Gamma(k,0)K^2(k)}\equiv\frac{1}{\tilde\Gamma_0K^2(k)}+b \tag{2.3}$$

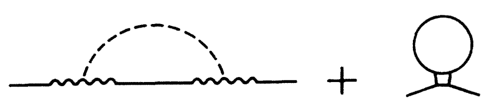

FIG. 1. Self-energy diagrams for the purely dissipative model.

and solve for b. We obtain simply

$$b=\frac{\Delta}{N}\sum_k\left[\frac{1}{\Gamma_0 K^2(k)}+b\right]G^2(k,0) \tag{2.4}$$

so that

$$b=\frac{\Delta}{r^2 N\Gamma_0(1-\Delta\Pi_0)}\sum_k\left[\frac{1}{K^2(k)}\right], \tag{2.5}$$

where

$$\Pi_0=\frac{1}{N}\sum_k G^2(k,0)=\frac{1}{r^2}\ . \tag{2.6}$$

Since $\Delta\Pi_0\to 1$ as $T\to T_g$, b diverges as $(T-T_g)^{-1}$, and $\tilde\Gamma^{-1}$ can be written as

$$\frac{1}{\tilde\Gamma(k,0)}=\frac{1}{\Gamma_0}\left[1+\frac{K^2(k)a_0^2}{1-\Delta\Pi_0}\right] \tag{2.7a}$$

$$\equiv\frac{1}{\Gamma_0}[1+K^2(k)\xi^2] \tag{2.7b}$$

in terms of a spin-glass correlation length ξ. [The parameter a_0^2 in Eq. (2.7a) is the average of $1/K^2(k)$ over the Brillouin zone that appears in (2.5). It is finite for $d>2$.] Thus whenever $\xi^2k^2\gg 1$, the effective static kinetic coefficient $\tilde\Gamma(k,0)K^2(k)$ is approximately k independent rather than proportional to k^2,

$$\frac{1}{\tilde\Gamma(k,0)K^2(k)}\to\frac{\xi^2}{\Gamma_0}\propto\frac{1}{T-T_g}\ . \tag{2.8}$$

Hence when T is near T_g, the $\omega\to 0$ dynamics in most of k space are relaxational rather than diffusive. There is a small region ($k\xi\ll 1$) where hydrodynamic behavior is still present, but it shrinks to nothing as $T\to T_g$. The behavior of the effective kinetic coefficient in the relaxational region is the same as that for the previously studied relaxational models,[7,16–18] namely, a critical slowing down. From now on we set $r=T$, so the susceptibility follows a Curie law above T_g.

B. $T=T_g$ (and $\omega\gg T\Gamma_0 a_0^2/\xi^4$)

The above static calculation has a frequency region of validity that shrinks to nothing as $T\to T_g$. We proceed to treat the situation at T_g as we did in Sec. II A for $T>T_g$, but we include the frequency dependence of the effective transport coefficient $\tilde\Gamma(k,\omega)$, or, equivalently, of the parameter b in (2.3). Now, to find a nontrivial result, the right-hand side of (2.1) must be expanded to second order in $b(\omega)$, since the linear terms which led to (2.4) cancel when $\Delta\Pi_0=1$. Truncating the expansion at the first opportunity, we obtain

$$-i\omega b(\omega)(1-\Delta\Pi_0)=\frac{-i\omega a_0^2}{\Gamma_0}+\frac{\omega^2 b^2(\omega)}{T}\ , \tag{2.9}$$

where we have approximated $\Delta\Pi_0\approx 1$ except on the left-hand side. At T_g, the left-hand side vanishes, so $b(\omega)\propto\omega^{-1/2}$. Equivalently, the effective kinetic coefficient is

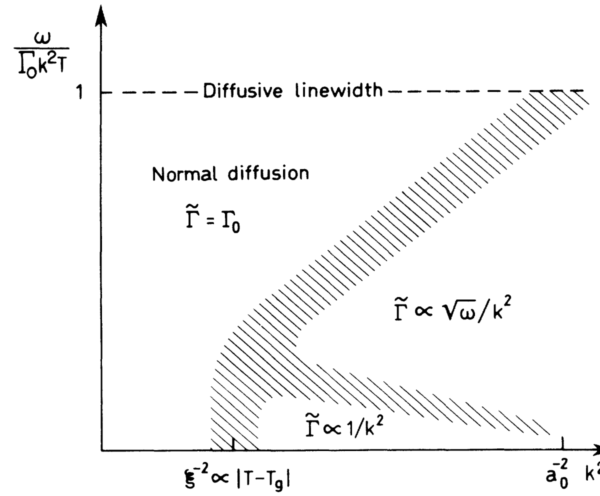

FIG. 2. Regions of (k,ω) space with different effective dynamics for the purely dissipative model of Sec. II.

$$\tilde\Gamma(k,\omega)K^2(k)=\Gamma_0 K^2(k)1+\frac{a_0^2\Gamma_0K^2(k)}{(-i\omega\Gamma_0 a_0^2/T)^{1/2}}\ . \tag{2.10}$$

Thus at every k there is a characteristic frequency $\omega_k\approx T(a_0k)^2\Gamma_0k^2$ below which the spin transport becomes singular. In this region

$$\tilde\Gamma(k,\omega)K^2(k)\approx\left[\frac{-i\Gamma_0\omega}{a_0^2T}\right]^{1/2}, \tag{2.11}$$

again in correspondence with the result for relaxational models.[7,16–18]

This same analysis also applies above T_g whenever the $\omega^2b^2(\omega)$ term in (2.9) is much larger than the linear term on the left-hand side. This amounts to the condition $\omega\gg T\Gamma_0a_0^2/\xi^4$.

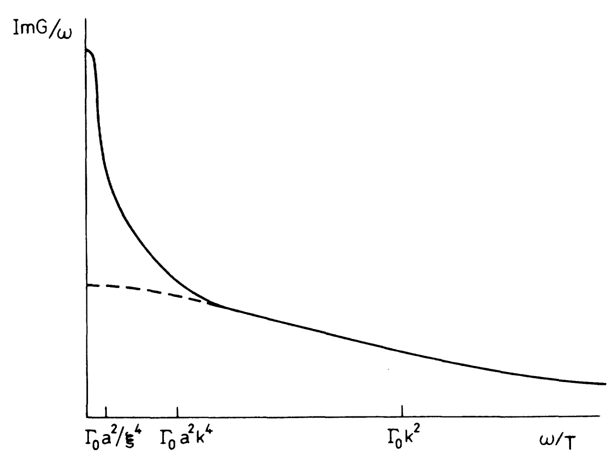

FIG. 3. Schematic picture of the neutron line shape expected in the model of Sec. II. The dotted line is the continuation of the normal diffusive Lorentzian.

C. Line-shape analysis

The resulting shape of the spectral density ImG (or of the neutron scattering line shape $\propto \text{Im}G/\omega$) can be quite interesting. Figure 2 shows the regions in (k,ω) space where the anomalous kinds of behavior we have discussed occur. There is a large area where the usual hydrodynamics is valid, but below the line $\omega_0/\Gamma_0 k^2 T=(a_0 k)^2$ and for $k\xi > 1$ the diffusion becomes effectively relaxational ($\tilde{\Gamma}\sim 1/k^2$). Within this latter region there is a further dividing line $\omega/\Gamma_0 k^2 T = a_0^2/k^2\xi^4$, well below which $\tilde{\Gamma}$ becomes approximately frequency independent, but above which one finds the singular frequency dependence (2.11).

We note that for $ka_0 \ll 1$, *both* crossover frequencies $T(ka_0)^2\Gamma_0 k^2$ and $T\Gamma_0 a_0^2/\xi^2$ are small compared to the diffusive linewidth $T\Gamma_0 k^2$. Thus what one expects to observe is a narrow peak *on top of* the normal diffusive Lorentzian line (Fig. 3). The width of the extra peak is roughly $T\Gamma_0 a^2/\xi^4$ and its height ($\propto \xi^2/\Gamma_0$) is a factor $(\xi k)^2$ greater than that of the underlying diffusive line. The extra peak has $\omega^{-1/2}$ wings extending out to roughly $\omega = T\Gamma_0 a^2 k^4$. Note that the width of the extra peak is *not* the static limit of the effective kinetic coefficient calculated in Sec. II A. That quantity was proportional to ξ^{-2} or $T-T_g$, while the linewidth was proportional to ξ^{-4} or $(T-T_g)^2$.

III. MODE-COUPLING MODEL

We now add the spin precession term to the equation of motion, as in Eq. (1.4). Now the transport coefficient can be changed from Γ_0, even at $k=0$. We take as our starting point the corrections calculated above and proceed to do perturbation theory in the (formal) mode-coupling parameter λ. We start with the equation of motion (1.4), which we write out explicitly as

$$\sum_p \left[\left(\frac{-i\omega}{\Gamma_0 K^2(p)}+r_0\right)\delta_{kp}-J(k,p)\right]\vec{S}_p(\omega) = -\frac{u}{N}\sum_{p,p'}\int\frac{d\omega_1 d\omega_2}{(2\pi)^2}[\vec{S}_p(\omega_1)\cdot\vec{S}_{p'}(\omega_2)]\vec{S}_{k-p-p'}(\omega-\omega_1-\omega_2)$$
$$+\frac{\lambda}{\Gamma_0 K^2(k)\sqrt{4N}}\sum_{p,p'}[J(k-p,p')-J(k-p',p)]$$
$$\times\int\frac{d\omega_1}{2\pi}\vec{S}_p(\omega_1)\times\vec{S}_{p'}(\omega-\omega_1)+\frac{\vec{\eta}_k(\omega)}{\Gamma_0 K^2(k)} . \tag{3.1}$$

Perturbation theory proceeds by iteration and averaging over the distribution of J's. We have seen that for $\lambda=0$ this averaging leads to an effective kinetic coefficient $\tilde{\Gamma}(k,\omega)K^2$, so we can sum up the effect of all the terms of Fig. 1 by replacing the left-hand side of (3.1) by

$$G_1^{-1}(k,\omega)\vec{S}_k(\omega) \equiv \left[\frac{-i\omega}{\tilde{\Gamma}(k,\omega)K^2(k)}+r\right]\vec{S}_k(\omega) . \tag{3.2}$$

What about the right-hand side, in particular, the λ term? We argue that the correct procedure is to replace the $\Gamma_0 K^2$ in the denominator there by $\tilde{\Gamma}(k,\omega)K^2$ as well. This makes the effective equation of motion

$$\frac{\partial\vec{S}_k(t)}{\partial t} = -\int_{-\infty}^{t} dt'\tilde{\Gamma}(k,t-t')K^2(k) r\vec{S}_k(t') + \frac{\lambda}{\sqrt{4N}}\sum_{p,p'}\{[J(k-p,p')-J(k-p',p)]\vec{S}_p(t)\times\vec{S}_{p'}(t)\} + \vec{\eta}_k(t) . \tag{3.3}$$

That is, the corrections we have made so far just replace Γ_0 by $\tilde{\Gamma}$ (which has nonlocal space and time dependence) but do not change the λ term in the equation of motion. This is physically reasonable—a change in the friction coefficient should not change the undamped (inertial) term in the equation of motion. [There is no explicit term proportional to u on the right-hand side of Eq. (3.3) because at the present one-loop order (sufficient to first order in $T-T_g$) effects of the quartic interaction have been included in the definition of r (Fig. 1).]

We now proceed to calculate the further corrections to the effective transport coefficient when $\lambda\neq 0$. The first correction comes from Fig. 4. Following Ma and Mazenko,[9] who worked out the corresponding problem for a ferromagnet, we identify the bubble in the diagram as $G^{-1}(k,0)\delta\Gamma/\tilde{\Gamma}$, where $\tilde{\Gamma}$ is the effective kinetic coefficient in Eq. (3.3):

$$T\frac{\delta\Gamma}{\tilde{\Gamma}(k,\omega)} = \frac{\lambda^2}{N\tilde{\Gamma}(k,\omega)K^2(k)}\sum_{p,p'}\langle[J(k-p,p')-J(k-p',p)][J(p-k,-p')-J(p+p',k)]\rangle_{\text{av}}$$
$$\times\int\frac{d\omega'}{2\pi}C(p',\omega-\omega')\frac{G(p,\omega')}{\tilde{\Gamma}(p,\omega')K^2(p)} . \tag{3.4}$$

As we noted above, in most of reciprocal space, the correlation and response functions, as well as $\tilde{\Gamma}K^2$, are independent of momentum. Thus we replace them on the right-hand side by local p-independent quantities. Then the average over the distribution of the J's and the sum on p and p' is straightforward. Using a nearest-neighbor model, for which[7]

$$\langle J(k_1,k_2)J(k_3,k_4)\rangle_{\text{av}} = \frac{\Delta}{Nd}\sum_\mu[\cos(k_{1\mu}+k_{3\mu})+\cos(k_{1\mu}-k_{4\mu})]\delta_{k_1+k_3-k_2-k_4} , \tag{3.5}$$

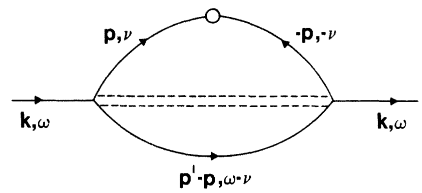

FIG. 4. Lowest-order mode-coupling correction to the response function.

we find a k-independent result:

$$\delta\Gamma(\omega)=\frac{\lambda^2\Delta}{2Td}\int\frac{d\omega'}{2\pi}C(\omega-\omega')G(\omega')\left[\frac{a_0^2T}{-i\Gamma_0\omega}\right]^{1/2}. \tag{3.6}$$

As far as the singular behavior is concerned, we can take the $G(\omega')$ in the integral to be independent of frequency. Since C also has inverse-square-root behavior, the integral is logarithmically divergent at $\omega=0$ and $T-T_g$. There is a natural upper cutoff for the integration at $\omega'\approx T\Gamma_0 a_0^{-2}$; the lower cutoff depends on $T-T_g$ and ω. For $\omega=0$ and $T>T_g$, the lower cutoff is $T\Gamma_0 a_0^2/\xi^4$, leading to

$$\delta\Gamma(0)=\frac{2\lambda^2a_0^2}{\pi d\Gamma_0}\ln\left[\frac{\xi}{a_0}\right]. \tag{3.7}$$

In the other limit $(T-T_g)$, the lower cutoff is ω itself:

$$\delta\Gamma(\omega)=\frac{\lambda^2a_0^2}{2\pi\Gamma_0 d}\left[\ln\left[\frac{T\Gamma_0}{a_0^2|\omega|}\right]+\tfrac{1}{2}i\pi\,\mathrm{sgn}\omega\right], \tag{3.8}$$

again to logarithmic accuracy. We note that $\delta\Gamma$ has an imaginary part (finite in the $\omega\to 0$ limit at T_g), so there is a hint of a tendency to propagate waves in the spin-glass phase. These corrections are manifestly the leadings ones in a $1/d$ expansion.

In Ref. 7 the result $\delta\Gamma\propto\xi$ was obtained by a combination of two errors. The first was that a bad approximation for $C(\omega-\omega')$ was used: It was taken to be a Lorentzian of width $T\Gamma(k,0)K^2(k)$. As we have seen, the true line shape has inverse-square-root wings and a central peak which is much narrower than $T\Gamma(k,0)K^2$ $[O((T-T_g)^2)$ instead of $O(T-T_g)]$. The second was that the replacement of Γ_0 by $\tilde{\Gamma}(p,\omega')$ at the right-hand vertex of the diagram was ignored. This is a very serious error in most of reciprocal space $(p\xi\gg 1)$. [A technical note: whether to call this correction a vertex or a self-energy correction is a matter of convention. Here we followed this formalism outlined by Ma and Mazenko,[9,10] where G is the dynamical susceptibility. In the alternative formalism[19] where the factor $(\Gamma K^2)^{-1}$ is absorbed into G instead of appearing in the vertex, these corrections are propagator (self-energy) corrections rather than vertex corrections.]

We now make the calculation self-consistent by feeding these corrections back into the calculations of Sec. II. We recalculate $\tilde{\Gamma}(k,\omega)$ as we did there, but with Γ_0 replaced by $\hat{\Gamma}(\omega)=\Gamma_0+\delta\Gamma(\omega)$, the result of including the mode-coupling corrections just described. Then Eq. (2.11) becomes

$$\tilde{\Gamma}(k,\omega)K^2(k)\approx\left[\frac{i\hat{\Gamma}(\omega)\omega}{a_0^2T}\right]^{1/2} \tag{3.9}$$

in most of k space, and thus

$$C(k,\omega)\approx\left[\frac{2a_0^2}{T\hat{\Gamma}(\omega)\omega}\right]^{1/2}. \tag{3.10}$$

These modified forms must then be put into the calculation of $\delta\Gamma(\omega)$. Equation (3.6) then becomes

$$\delta\Gamma(\omega)=\frac{\lambda^2a_0^2\Delta}{2T^2d}\int\frac{d\omega'}{2\pi}\left[\frac{1}{\omega'\hat{\Gamma}(\omega')}\right]\approx\frac{\lambda^2a_0^2}{2\pi d}\int\frac{d\ln\omega'}{\hat{\Gamma}'(\omega')}. \tag{3.11}$$

The upper cutoff in the integral is at $\omega'\approx T\hat{\Gamma}(\omega')a_0^{-2}$ and the lower cutoff is at the larger of $T\hat{\Gamma}(\omega')a_0^2/\xi^4$ and ω. In the high-frequency region $[\omega\gg T\hat{\Gamma}(\omega)a_0^2/\xi^4]$ we set $\delta\Gamma(\omega)\gg\Gamma_0$, so $\delta\Gamma(\omega)\approx\tilde{\Gamma}(\omega)$. Then it is clear that a $\hat{\Gamma}(\omega)$ proportional to $(\ln\omega^{-1})^{1/2}$ will solve (3.11). Keeping only the leading singular term, we get

$$\hat{\Gamma}(\omega)=\left[\frac{\lambda^2a_0^2}{\pi d}\ln\frac{T\Gamma_0}{a_0^2|\omega|}\right]^{1/2} \tag{3.12}$$

in this region [and for all $\omega\ll T\hat{\Gamma}(\omega)a_0^{-2}$ at T_g]. We can fix the imaginary part by analyticity, i.e., by letting $\ln|\omega|^{-1}\to\ln|\omega|^{-1}+\frac{1}{2}i\pi\,\mathrm{sgn}\omega$, so we have

$$\hat{\Gamma}(\omega)\approx\left[\frac{\lambda^2a_0^2}{\pi d}\right]^{1/2}\left[\ln^{1/2}\left[\frac{T\Gamma_0}{a_0^2|\omega|}\right]+\frac{i\pi\,\mathrm{sgn}\omega}{4\ln^{1/2}T\Gamma_0/a_0^2|\omega|}\right] \tag{3.13}$$

at small ω. The imaginary part of $\hat{\Gamma}(\omega)$ thus goes to zero at $\omega=0$, in contrast to the finite limit found [Eq. (3.8)] in the non-self-consistent calculation above. In both the real and the imaginary parts of $\hat{\Gamma}$ the anomalous behavior is weakened by the self-consistency effects. As $\omega\to 0$ our result goes over to

$$\hat{\Gamma}(0)=\left[\frac{4\lambda^2a_0^2}{\pi d}\ln\left[\frac{\xi}{a_0}\right]\right]^{1/2} \tag{3.14}$$

(again keeping only the leading log). These results could have been obtained, within factors of $\sqrt{2}$, by simply replacing both $\delta\Gamma(\omega)$ and Γ_0 by $\hat{\Gamma}(\omega)$ in (3.7) and (3.8); that is, the fact that the $\hat{\Gamma}^{-1}(\omega')$ appears inside the integral on the right-hand side of Eq. (3.11) rather than as an external $\hat{\Gamma}^{-1}(\omega)$ factor makes only a quantitative difference. The same fractional logarithm is obtained either way because $\hat{\Gamma}^{-1}(\omega')$ varies so slowly relative to $1/\omega'$ itself. The condition for the validity of the results (3.12)–(3.14) is clearly that the singular correction $\delta\Gamma(\omega)$ must be much larger than Γ_0. In the opposite limit, Eqs. (3.7) and (3.8) apply.

We do not go explicitly through a line-shape analysis as in Sec. II D again, since the only change is that wherever

Γ_0 appears in that discussion it should be replaced by $\hat{\Gamma}(\omega)$. With this slightly nonlinear distortion of the frequency axis in Fig. 2, the previous description remains valid. We expect these corrections to be very hard to detect experimentally, since they involve only logarithmic corrections to power-law behavior. The exception to this statement is the long-wavelength region $k\xi \ll 1$, where there is no power law to mask the logarithm.

Though we have only considered the case of infinite spin dimensionality so far, the critical properties above T_g are not expected to change (as long as we are above the upper critical dimensionality) for finite n. This was shown explicitly for the relaxational model in Ref. 18. The point is simply that the multiple-loop corrections to Eq. (2.1) all remain finite as $T \to T_g$, so they just lead to a corrected r. The generalization to the diffusive case and $\lambda \neq 0$ are trivial.

IV. SOUND PROPAGATION

We now couple sound to the spin fluctuations via the model of Eqs. (1.9)–(1.13). To leading order in the reciprocal of the lattice coordination number, we then just have to compute the polarization bubble to lowest order (g^2). It is simplest to compute it in real (lattice) space, as shown in Fig. 5(a):

$$\Pi_{ij}(\omega) = \frac{3g^2\Delta}{d}(2\delta_{ij} - \delta_{i,j+\hat{e}_1} - \delta_{i,j-\hat{e}_1}) \times \int \frac{d\omega'}{2\pi} C(\omega')G(\omega-\omega') , \quad (4.1)$$

where we have taken the sound to be propagating in the $\hat{e}_1$ direction and have taken the response and correlation functions to be independent of momentum, as we have found them to be near T_g in most of the Brillouin zone ($k\xi \gg 1$). We now Fourier transform and take the long-wavelength limit, since sound wavelengths of interest are many lattice spacings:

$$\Pi(k,\omega) = \frac{3k^2g^2\Delta}{d} \int \frac{d\omega'}{2\pi} C(\omega')G(\omega-\omega') . \quad (4.2)$$

We note that for this model, vertex corrections of the types we consider in Ref. 11—both those involving the ladders of random bonds [Fig. 5(b)] and those from the $u(\vec{S}_1)^4$ interaction [Fig. 5(c)]—do not contribute at the present level of approximation. (They do not vanish identically [see, e.g., Fig. 5(d)], but they are beyond mean-field theory.)

We first note that $\Pi(k,\omega)$ is finite everywhere, and so the sound speed and attenuation remain finite. Now consider the imaginary part

$$\mathrm{Im}\Pi(k,\omega) = \frac{3\omega k^2g^2\Delta}{4Td} \int \frac{d\omega'}{2\pi} C(\omega')C(\omega-\omega') , \quad (4.3)$$

using the FDT on (4.2). The integral is the same type of logarithmically divergent quantity we encountered in Sec. III. Thus the damping function

$$\gamma(\omega) \equiv \frac{\mathrm{Im}\Pi(k,\omega)}{k^2\omega} = \frac{3g^2a_0^2}{2d\pi\Gamma_0} \ln \min\left[\left[\frac{\xi}{a_0}\right]^4, \frac{T\Gamma_0}{a_0^2|\omega|} \right] \quad (4.4)$$

without mode coupling, and

$$\gamma(\omega) = \frac{3g^2a_0}{\lambda\sqrt{\pi d}} \left\{ \ln \min\left[\left[\frac{\xi}{a_0}\right]^4, \frac{T\Gamma_0}{a_0^2|\omega|} \right] \right\}^{1/2} \quad (4.5)$$

for $\lambda \neq 0$ (to logarithmic accuracy). These divergences are much weaker than those found for the model of Ref. 11.

We can obtain the change in sound speed from

$$\delta c^2(T) = c^2(T) - c^2(\infty) = \frac{1}{k^2}\Pi(k,0) < 0 . \quad (4.6)$$

This is quite simple to evaluate exactly for the present model. We apply the Kramers-Kronig method to Eq. (4.3):

$$\mathrm{Re}\Pi(k,0) = \int \frac{d\omega}{\pi} \frac{\mathrm{Im}\Pi(k,\omega)}{\omega} = \frac{3k^2g^2\Delta}{2Td} \int \frac{d\omega}{2\pi}\frac{d\omega'}{2\pi} C(\omega')C(\omega-\omega') . \quad (4.7)$$

Since $C(t)=1$ at $t=0$, we have simply

$$\delta c^2(T) = -\frac{3g^2\Delta}{2Td}, \quad T > T_g . \quad (4.8)$$

Fischer[13] found this behavior for high T, but from (4.7) we can see that it holds everywhere above T_g.

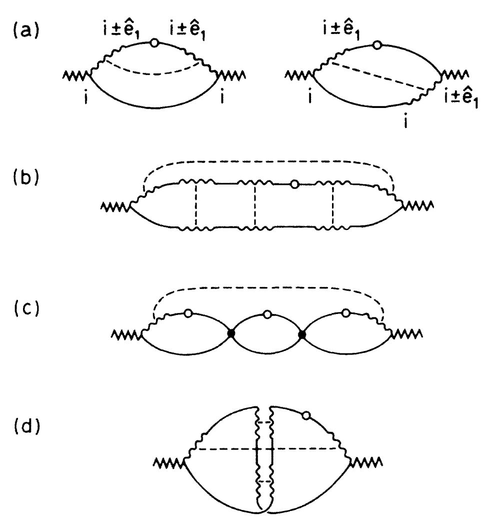

FIG. 5. (a) Diagrams for Π_{ij}. The wavy lines represent the factors J_{ij} in H_{sp}. (b) Example of a vertex correction involving a ladder of pairs of random bonds. This class of corrections vanishes. (c) Example of a vertex correction involving strings of bubbles. These also vanish. (d) Nonvanishing kind of vertex correction involving "maximally crossed" ladders.

V. EFFECTS OF WEAK ANISOTROPY

In this section we consider briefly what happens to our results when the magnetization is not quite conserved. This could come about from dipolar forces or (random) anisotropy. We will assume that the effective Hamiltonian (1.1) remains rotationally invariant, but that the diffusive equation (1.2) is replaced by

$$\frac{\partial \vec{S}_i}{\partial t}=(-\gamma_0+\Gamma_0\nabla^2)\frac{\delta H}{\delta \vec{S}_i}+\vec{\eta}_i(t) \tag{5.1}$$

with a corresponding change $\Gamma_0 K^2 \to \Gamma_0 K^2+\gamma_0$ in the noise statistics (1.5). This means there is a characteristic length $l=(\Gamma_0/\gamma_0)^{1/2}$ beyond which the spin dynamics look relaxational rather than diffusive. Our results have to be modified for $q \lesssim l^{-1}$. (We will assume $l \gg a_0$; i.e., the magnetization is almost conserved.)

We start by repeating the calculations of Sec. II with these changes. This is straightforward. In place of (2.7) we find a general k-dependent relaxation rate $\tilde{\gamma}(k,0)$ given by

$$\tilde{\gamma}^{-1}(k,0)=\frac{1}{\gamma_0+\Gamma_0 K^2(k)}+\frac{a_0^2}{\Gamma_0(1-\Delta\Pi_0)} . \tag{5.2}$$

The parameter a_0^2 is now the average of $[\gamma_0/\Gamma_0 + K^2(k)]^{-1}$ over the Brillouin zone, but for large l/a_0 this is nearly the same as before. For $\xi/l \ll 1$, $\tilde{\gamma}(k,0)$ goes as

$$\tilde{\gamma}(k,0)\approx\begin{cases}\gamma_0, & k \ll l^{-1}\\ \Gamma_0 k^2, & l^{-1} \ll k \ll \xi^{-1} .\\ \Gamma_0/\xi^2, & k \gg \xi^{-1}\end{cases} \tag{5.3}$$

For $\xi \gg l^{-1}$ the diffusive region disappears, and we find $\gamma(k,0)\approx\Gamma_0/\xi^2$ for all k. Thus the asymptotic behavior is similar to that of the relaxation model—the relaxation time becomes enhanced,

$$\tau_{\text{eff}}=\tau_0\left[\frac{T_g}{T-T_g}\right] , \tag{5.4}$$

but the τ_0 is a_0^2/Γ_0 rather than γ_0^{-1}. Similarly, Eq. (2.9) for $b(\omega)$ at finite ω remains essentially unchanged. At T_g we find

$$\tilde{\gamma}(k,\omega)\approx\frac{\gamma_0+\Gamma_0 k^2}{1+\left[\dfrac{T a_0^2}{-i\omega\Gamma_0}\right]^{1/2}(\gamma_0+\Gamma_0 k^2)} \tag{5.5}$$

instead of (2.10). Thus the characteristic upper cutoff frequency on the $\omega^{-1/2}$ behavior of $\tilde{\gamma}(k,\omega)$ is now

$$\omega_k=\Gamma_0(l^{-2}+k^2)^2 T a_0^2 \tag{5.6}$$

and remains finite even at $k=0$.

Turning to the behavior of the spin-correlation function for general k and ω described previously by Fig. 2, the changes can be summarized as the following. (1) For $\xi \ll l$, there is no change, except that the "normal diffusion" region now has $\tilde{\Gamma}=\gamma_0/k^2+\Gamma_0$ instead of $\tilde{\Gamma}=\Gamma_0$. (2) For $\xi \gg l$, the normal diffusion region no longer extends down to $\omega=0$ for $k\xi \ll 1$. The upper shaded boundary is given by (5.6), which remains finite $[\approx T\gamma_0(a_0/l)^2]$ as $k\to 0$. This can be described pictorially by letting the horizontal axis be k^2-l^{-2} instead of k^2 (i.e., moving the vertical axis l^{-2} to the right).

Figure 3 is not changed, except that the normal diffusive linewidth $T\Gamma_0 k^2$ should now be $T\Gamma_0(k^2+l^{-2})$, and the crossover frequency ω_k where the $\omega^{-1/2}$ wings of the central peak begin to rise above the normal Lorentzian is $T\Gamma_0 a_0^2(k^2+l^{-2})^2$. The qualitative shape of the line is the same as before.

When mode coupling is added as in Sec. III, corrections to Γ_0, but not to γ_0 in (5.1), are generated. These corrections are still given by (3.12) –(3.14). One can see that this is true because the small region of k space where the finite γ_0 makes any difference $[k<\min(\xi^{-1},l^{-1})]$ is so small that its contribution to the integral for $\delta\Gamma(\omega)$ or $\hat{\Gamma}(\omega)$ is negligible. Thus we have in place of (5.3)

$$\tilde{\gamma}(k,\omega)\approx\begin{cases}\gamma_0, & k \ll l^{-1}[\Gamma_0/\hat{\Gamma}(0)]^{1/2}\\ \hat{\Gamma}(0)k^2, & [\Gamma_0/\hat{\Gamma}(0)]^{1/2}l^{-1} \ll k \ll \xi^{-1}\\ \hat{\Gamma}(0)/\xi^2, & k \gg \xi^{-1}\end{cases} \tag{5.7}$$

in the low-frequency region $\omega \ll T\hat{\Gamma}(\omega)a_0^2/\xi^4$, with $\hat{\Gamma}(0)$ given by (3.14). Similarly, instead of (5.5),

$$\tilde{\gamma}(k,\omega)\approx\frac{\gamma_0+\hat{\Gamma}(\omega)k^2}{1+\left[\dfrac{T a_0^2}{-i\omega\hat{\Gamma}(\omega)}\right]^{1/2}\gamma_0+\hat{\Gamma}(\omega)k^2} , \tag{5.8}$$

where $\hat{\Gamma}(\omega)$ is given by (3.12), for

$$T\hat{\Gamma}(\omega)a_0^2/\xi^4 \ll \omega \ll T\hat{\Gamma}(\omega)(k^2+l^{-2}) .$$

As in the case without mode coupling, the presence of a nonzero γ_0 in (5.8) makes very little difference except at very small k. At $k=0$, the boundary between $\omega^{-1/2}$ and normal Lorentzian behavior in $C(k,\omega)$ [the frequency above which $\tilde{\gamma}(k,\omega)\approx\gamma_0$] is now given by

$$\omega_0=T\gamma_0(a_0/l)^2\Gamma_0/\hat{\Gamma}(\omega_0) . \tag{5.9}$$

Thus one reaches the normal Lorentzian region of Fig. 3 at a slightly lower frequency than in the $\gamma=0$ case discussed above.

This discussion has been for the case $l \gg a_0$, where the nonconservation of total spin is a small effect. What is perhaps unexpected is that in the presence of mode coupling, the results (5.7) and (5.8) also apply in the opposite limit, where $\tilde{\gamma}_0(k)\equiv\gamma_0+\Gamma_0 K^2(k)$ is nearly k independent. We can see this by repeating the self-consistent calculation for $\Gamma_0=0$. The mode coupling still generates a term in $\tilde{\gamma}(k,\omega)$ proportional to $K^2(k)$, and the coefficient of K^2 is logarithmically divergent at T_g. Thus as one approaches the transition, this K^2 term dominates the γ_0, and the calculation of $\tilde{\Gamma}(k,\omega)$ is then just the same as in (3.9), leading to the result (5.8) again. The calculation of $\hat{\Gamma}(\omega)$ again follows (3.11)–(3.14), because in most of k space $\tilde{\gamma}(k,\omega)\approx(-i\omega\hat{\Gamma}(\omega)/a_0^2 T)^{1/2}$. [Actually, the Γ_0 inside the the logs in (3.12) and (3.13) must be replaced by $\hat{\Gamma}(\omega)$ itself. Indeed, it should be for self-consistency in the previous cases as well, but this was beyond the leading log order from which we were working. Here with $\Gamma_0=0$ we should use γ_0 in place of $\Gamma_0 a_0^{-2}$ at the corresponding level of approximation.] Thus the presence of the precessional terms in the equation of motion generates $\log^{1/2}$ corrections to the spin-correlation function power laws, regardless of whether the spin is conserved.

This means that even for RKKY spin-glasses, where Korringa relaxation is the dominant dynamical mechanism, these $\log^{1/2}$ corrections exist in principle. However, their observability is limited by the rapidity of the Korringa process—in our model, the fact that γ_0 is so large. In order to find the size of the true asymptotic region, we calculate the mode-coupling-induced $\delta\Gamma$ to $O(\lambda^2)$ and compare $a_0^{-2}\delta\Gamma$ with γ_0. The $\delta\Gamma$ we get is just (3.7), except that Γ_0 is replaced by $\gamma_0 a_0^2$. Thus we find the condition

$$T-T_g \ll T_g \exp\left[-\frac{\pi d a_0^2 \gamma_0^2}{\lambda^2}\right], \tag{5.10}$$

which exhibits explicitly the exponential difficulty one might encounter in trying to observe the asymptotic behavior.

Finally, what is the effect of a finite γ_0 on sound propagation? The answer is simple: Since the sound self-energy bubble (4.1) or (4.2) involves k sums over the whole Brillouin zone, γ_0 plays no important role (just as it failed to affect the mode-coupling bubble contribution above). The conclusions of Sec. IV therefore stand unchanged.

VI. $T < T_g$

For $n=\infty$, the low-T phase of the relaxational model is quite simple. It is a marginal state, with the same power law in the correlation function for all $T<T_g$ as described above at T_g.[17] (This happens because the equation that determines q turns out to be equivalent to the condition that the effective kinetic coefficient vanishes at $\omega=0$.)

In the absence of the precessional term in the equation of motion, the situation in our diffusive model is equally simple. The correlation and response functions are just the same as we found in Sec. II at T_g—the hydrodynamic region of k space has disappeared (for $\omega=0$) and the correlation function goes as $\omega^{-1/2}$.

When we include mode coupling, however, the situation changes qualitatively. Now there is a term in $\delta\Gamma(\omega)$ proportional to the E-A order parameter q, which comes from the part of $C(\omega-\omega')$ in (3.6) proportional to a δ function. Indeed, it is the dominant term for small ω in the spin-glass phase. Writing

$$C(\omega)=2\pi q\delta(\omega)+\tilde{C}(\omega), \tag{6.1}$$

we obtain from (3.6) the result

$$\delta\Gamma(\omega)\approx\frac{\lambda^2 q}{2d}\left[\frac{a_0^2 T}{-i\Gamma_0\omega}\right]^{1/2}, \tag{6.2}$$

which clearly dominates the old logarithmic term at low frequency. Making the calculation self-consistent as we did in Sec. III, we get

$$\hat{\Gamma}(\omega)=\frac{\lambda^2 q}{2d}\left[\frac{a_0^2 T}{-i\hat{\Gamma}(\omega)\omega}\right]^{1/2}. \tag{6.3}$$

Thus $\hat{\Gamma}(\omega)$ is now proportional to $(-i\omega)^{-1/3}$, and the correlation function is

$$C(\omega)\approx G^2(0)\left[2T\,\mathrm{Re}\frac{1}{\tilde{\Gamma}(k,\omega)K^2(k)}\right], \tag{6.4}$$

where $\tilde{\Gamma}$ is again given by (3.9). Thus

$$C(\omega)\approx\frac{2}{T}\mathrm{Re}\left[\frac{a_0^2 T}{-i\omega\hat{\Gamma}(\omega)}\right]^{1/2}=\frac{\sqrt{3}}{T}\left[\frac{2a_0^2 T d}{\lambda^2 q\omega}\right]^{1/3}. \tag{6.5}$$

The power law is thus modified by the precessional dynamics in the low-frequency region. For $\omega \gg (\lambda^2 q/2d)^2 a_0^2 T\Gamma_0^{-3}$, $\hat{\Gamma}(\omega)$ is smaller than Γ_0, and the correlation function goes back over to its critical $\omega^{-1/2}$ behavior. $C(\omega)$ obeys a kind of qualitative dynamical scaling shown in Fig. 6. Our result appears to disagree with that of Bray and Moore,[20] who find only a quantitative change as a consequence of precessional dynamics. They are of course dealing with a different situation, well below T_g and more like that described at the end of the preceding section, where precessional dynamics are added to a system dominated by fast relaxation ($\gamma_0 \gg \Gamma_0 a_0^{-2}$). Nevertheless, the arguments we used there apply again: The generation of a large $\delta\Gamma$ by the mode-coupling dynamics makes γ_0 effectively disappear from the calculation and the results (6.4) and (6.5) are recovered, independent of Γ_0 and γ_0. The method of Bray and Moore is very different from the present one, so we have not been able to trace the source of the discrepancy. It could just be that the $\omega^{-1/3}$ term found here goes away at low T (where their calculation is done), but there is nothing in the present theory to suggest that it does not persist to low T.

We turn now to the effect of the marginal spin dynamics below T_g on sound propagation, which is straightforward. From Eq. (4.3), we find a new piece of the imaginary part of $\Pi(k,\omega)$ proportional to $q\omega C(\omega)$. Thus the damping function $\gamma(\omega)$ acquires a piece proportional to $C(\omega)$ [in addition to (4.4)]:

$$\delta\gamma(\omega)=\frac{3g^2\Delta}{2Td}C(\omega). \tag{6.6}$$

Without mode coupling, this is just

$$\delta\gamma(\omega)\approx\frac{3g^2}{2d}\left[\frac{2a_0^2 T}{\Gamma_0\omega}\right]^{1/2}. \tag{6.7}$$

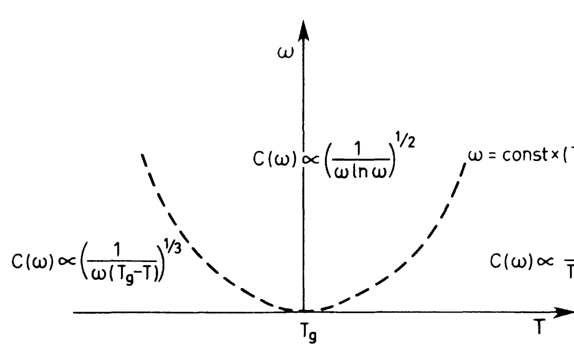

FIG. 6. Behavior of $C(\omega)$ with mode-coupling and marginal dynamics. The logarithmic term aside, $C(\omega)$ can be written as $\omega^{-1/2}f((T-T_g)^2/\omega)$.

Beton and Moore[21] also find both $\omega^{-1/2}$ and $\ln 1/|\omega|$ terms in $\gamma(\omega)$ in their low-T theory. The $\omega^{-1/2}$ term was also found by Fischer[13,14] and by Khurana[12] (for a different model). With the precessional dynamics, (6.6) gives

$$\delta\gamma(\omega)\approx\frac{3^{3/2}g^2}{2d}\left[\frac{2a_0^2Td}{\lambda^2 q\omega}\right]^{1/3} . \tag{6.8}$$

This is not in agreement with Beton and Moore because our $C(\omega)$ is different from theirs, as explained above.

For the sound speed we proceed from (4.7), using the low-T form (6.1) for C. We find, by integrating (6.1) over all ω, that

$$\tilde{C}(t=0)=1-q . \tag{6.9}$$

Thus $\delta c^2(T)$ has a term similar to (4.8), multiplied by $(1-q)^2$, from the $\tilde{C}\tilde{C}$ term in (4.7). There are two (identical) cross terms in ReΠ, each equal to

$$\frac{3k^2g^2\Delta}{2Td}q(1-q) . \tag{6.10}$$

[The term proportional to q^2 is proportional to $\delta(\omega)$ and is therefore discarded, since what we want to compare a finite-frequency experiment with is the limit of $\Pi(k,\omega)$ as $\omega\rightarrow 0$. Physically, this means we assume the experiment is done during a time the spins remain frozen.] Thus we find

$$\delta c^2(T)=-\frac{3g^2\Delta}{2Td}1-q^2 . \tag{6.11}$$

Comparing (6.11) with (4.8), we see the result that c^2 *varies smoothly through* T_g (in contrast to previous results for different models). Only the second derivative changes discontinuously at T_g. The new result appears to be in agreement with the experiments of Hawkins and Thomas.[22] By expanding (4.12) around T_g, one easily finds that $\partial^2(\delta c^2)/\partial T^2=O((T_g-T)^2)$ below T_g, i.e., the curvature goes to zero as one approaches T_g from below.

The formula (6.11) also tells us that c^2 returns at $T=0$ to the value it would have in the absence of spin-glass effects ($\Delta=0$) if $1-q\propto T^2$ at low T. If $1-q$ is linear in T, c^2 remains less than $c^2(\infty)$ as $T\rightarrow 0$. These observations may be of some use in fitting data, as possible constraints on the separation of the magnetic and nonmagnetic contributions to $c^2(T)$.

This is of course based on the calculation of $\Pi(k,\omega)$ at $\omega=0$. When the imaginary part of Π has $\omega^x\,\mathrm{sgn}\omega$ behavior, we expect from analyticity that ReΠ also has an ω^x term. Thus for the mode-coupling case, δc^2 ought to have frequency dependence in this region $\propto|\omega|^{2/3}$. For $\lambda=0$, it would be $\omega^{1/2}$ behavior. This would appear as a deviation from linearity in the measured dispersion relation $\omega(k)$, with an effective $k^{2/3}$ or $k^{1/2}$ term, respectively. [Such a deviation from linearity would appear at T_g, or even above T_g for $\omega\gg T\Gamma_0 a_0^2/\xi^4$, but there the correction would only be logarithmic (a small $k\ln k$ term), and would be very difficult to detect.]

Finally, one can extend these results to finite n (again, as long as d is above the upper critical dimensionality) by taking advantage of the results of Sompolinsky and Zippelius.[18] For the relaxational model, they argue that, at least near T_g, one needs only to keep the simplest multiple-loop term (a single bubble). The result is that the exponent in the power law $C(\omega)\propto\omega^{-x}$ can be expanded as

$$x=\frac{1}{2}+\frac{(T_g-T)}{\pi n(3n+2)T_g}+O((T_g-T)^2) . \tag{6.12}$$

(Note that n as defined here is $\frac{1}{3}$ of their n.) It is then simple to generalize the arguments leading to (6.3) and (6.5). We find that $\hat{\Gamma}(\omega)$ and $C(\omega)$ are both proportional to $\omega^{-x/(1+x)}$. Similarly, the sound-damping results (6.7) and (6.8) become proportional to ω^{-x} and $\omega^{-x/(1+x)}$, respectively.

VII. CONCLUSIONS AND COMMENTS

The results of this paper can be summarized briefly.

(1) The introduction of a conserved total magnetization, via the $\Gamma_0\nabla^2$ term in the equation of motion (1.1), makes surprisingly small changes in the mean-field dynamics from what one finds approaching T_g in purely relaxational models. In most of k space, the dynamics exhibit the same critical slowing down as if $\vec{S}$ were not conserved. Only for a region of wave numbers $k\xi\ll 1$ in the center of the Brillouin zone is the normal diffusive behavior seen in the static limit. On the other hand, we found that for small k the ordinary hydrodynamic behavior is recovered at larger ω ($\gg\Gamma_0 Tk^4a_0^2$). The resulting line shape $C(k,\omega)$ is not simple for general k, but for small k it is approximately a narrow peak of width $\approx\Gamma_0 Ta_0^2/\xi^4$ with $\omega^{-1/2}$ wings, sitting on top of a usual diffusive Lorentzian.

(2) The further introduction of precessional terms (mode coupling) in the equation of motion leads to weakly divergent transport coefficients [$\propto(\ln\omega)^{1/2}$ or $(\ln\xi)^{1/2}$] and similar root-log corrections to the $\omega^{-1/2}$ power law in the correlation function and line shape.

(3) Our model for sound propagation leads to a logarithmically divergent sound-damping coefficient. The inclusion of precessional terms changes the log divergence to a root-log one similar to that in the transport coefficient.

(4) Below T_g, the results depend on which theory of the spin-glass phase we adopt. In the "short-time" theory of Ref. 3, the results turn out to be symmetric around T_g. For the longer-time situation of a marginally stable equilibrium or quasiequilibrium state, there are more profound changes—an $\omega^{-1/3}$ divergence in the transport coefficient in the precessional model and $\omega^{-1/3}$ behavior in $C(\omega)$ instead of the previous $\omega^{-1/2}$ form, and singular sound damping—$\gamma(\omega)\sim\omega^{-1/3}$ or $\omega^{-1/2}$ with or without precessional dynamics, respectively. In both theories, the sound speed varies smoothly through T_g.

(5) The inclusion of a spin-nonconserving term in the equation of motion does not affect these results at all (asymptotically close to T_g).

The applicability of our mean-field results, which are, formally speaking, based on a large dimensionality expansion [recall the factors of $1/d$ in, e.g., Eqs. (3.8) and (4.2)], to real materials is problematical. Neutron scattering[23] and dynamical susceptibility[24] measurements show a gradual slowing down, with the generation of a wide spectrum of relaxation rates over a range of temperatures well above the true transition temperature (defined, e.g., by the departure of the dc field-cooled susceptibility from its Curie high-T form. This important feature is absent from our high-d calculations. Nevertheless, we may hope that some qualitative features of our theory may persist (in less singular form) in $d=3$. We suggest that neutron spectra be examined in the light of our proposed line shape. In particular, both the suppression of spin diffusion for $k\xi \gg 1$ and the enhanced transport coefficient ($k\xi \ll 1$) are characteristic features to look for. While we do not expect the simple mean-field result $\xi \propto (T_g - T)^{-1/2}$ to be valid, it is possible that the dependence of the dynamical parameters on the true ξ could be reasonably given by the present theory. At the present level of theoretical understanding of these problems, ξ would have to be determined empirically in such an analysis.

We also hope that the present theory will be useful for future theoretical efforts to describe the true three-dimensional situation, in that they give a clue about how to add spin conservation and precession effects, once some understanding of the underlying gradual slowing down is achieved for simpler models.

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