Hydrodynamics of ^{3}He-B in high magnetic fields

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We derive the hydrodynamic equations for ^{3}He-B in high magnetic fields. The order parameter is a nonunitary spin-orbit matrix and the orbit space is uniaxial. The influence of a strong magnetic field shows up in various features. There exists a third velocity which transforms only partially like a velocity under Galilean transformation. Many coupling terms between spin space and orbit space are found in the static as well as in the reversible and dissipative dynamical equations. The resulting main features are the mixing of first and second (and fourth) sound with longitudinal spin waves and the anisotropy of the material parameters. The value of the rotation angle θ_0, which minimizes the magnetic dipole energy, is lowered, the longitudinal NMR frequency is raised, and the transverse NMR frequency is shifted in ^{3}He-B in high magnetic fields.

I. INTRODUCTION

In recent years hydrodynamic theories of the various superfluid phases of ^{3}He were given,[1,2] both for the linear[3–11] and for the nonlinear domain.[12–18] The method of deriving hydrodynamic theories is well known and has been applied to various systems, ranging from fluids,[19] He II,[20,21] and magnets,[22,23] to crystals and liquid crystals.[24–35]

The hydrodynamics of the B phase has been considered up to now for the case of low magnetic fields only.[6,7,11,17] Thereby, the magnetic field $\vec{H}$ was treated perturbatively, i.e., symmetries and the equilibrium order parameter were left unchanged and only linear contributions of H were considered, i.e., only the free energy acquired an additional contribution due to the magnetic field (Zeeman term). In the present work we lift these restrictions. (A similar treatment of the A phase in high magnetic fields was recently given by the authors[36,37] and others.[38,39]) Thereby, the equilibrium order parameter turns out to be complex and nonunitary and the system becomes uniaxial in spin and orbit space due to the (strong) magnetic field (Sec. II). The implications of these new features on the hydrodynamic variables are discussed (Sec. II).

The static and dynamical linear equations for the true hydrodynamic variables are given in Secs. III and IV. The magnetic dipole interaction is taken into account in Sec. V. Thereby, we derive expressions for the field-dependent longitudinal NMR frequency and transverse NMR shift, which occur in the B phase in high magnetic fields. A normal mode analysis is performed (Sec. VI). We discuss some experiments by which the new features can be measured (Sec. VII). In Appendix A we make contact with microscopic descriptions by relating the phenomenological parameters with correlation functions, response functions, etc. The Galilean transformation behavior of the variables connected with broken symmetries is derived in Appendix B. For the nonlinear theory we refer to Ref. 40 and Appendix C.

II. EQUILIBRIUM ORDER PARAMETER AND BROKEN SYMMETRIES

The B phase of ^{3}He is characterized as a superfluid with spontaneously broken relative rotational symmetry of spin space against orbit space.[41] The Cooper pairs are in a spin and orbit triplet state ($\vec{S}=1$, $\vec{L}=1$). Thereby, the equilibrium order parameter has the structure $A^0_{\alpha i} \sim e^{i\varphi} n^0_{\alpha i}$ with

$$n^0_{\alpha i} = \tfrac{1}{2}\Delta_\uparrow(\hat{e}+i\hat{f})_\alpha(\hat{e}-i\hat{f})_i + \tfrac{1}{2}\Delta_\downarrow(\hat{e}-i\hat{f})_\alpha(\hat{e}+i\hat{f})_i + \Delta_3(\hat{e}\times\hat{f})_\alpha(\hat{e}\times\hat{f})_i \ , \tag{2.1}$$

where $\Delta_\uparrow$, $\Delta_\downarrow$, and Δ_3 refer to pairs with spin projection both up ($\uparrow\uparrow$) both down ($\downarrow\downarrow$), and symmetrically mixed ($\uparrow\downarrow+\downarrow\uparrow$), respectively, and where $\hat{e}$ and $\hat{f}$ are orthogonal unit vectors in orbit or spin space.

Without a magnetic field

$$\Delta_\uparrow = \Delta_\downarrow = \Delta_3 = \Delta_0 \ ,$$

i.e., the gap is isotropic. The equilibrium order parameter is then (apart from a global phase factor) a real spin-orbit rotation matrix.[2,6,7] If special frames of reference are chosen in spin and orbit space, it reads

$$\bar{n}^0_{\alpha i} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} , \tag{2.2}$$

where Greek and Roman indices refer to spin space and orbit space, respectively. Thereby, spin and orbit space are independent (neglecting dipole interaction) and the frames of the two spaces are connected by an arbitrary rotation matrix R_{ij} (which contains three arbitrary parameters). One can choose for R_{ij} the representation

$$R_{ij} = \begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix} = R_{\alpha\beta} \ , \tag{2.3}$$

with the constraints (which are not independent from one another

$$\sum_i l_i^2 = \sum_i m_i^2 = \sum_i n_i^2 = 1 \ ,$$

$$\sum_i l_i m_i = \sum_i l_i n_i = \sum_i n_i m_i = 0 \ ,$$

and

$$l_1 l_2+m_1 m_2+n_1 n_2=l_1 l_3+m_1 m_3+n_1 n_3$$
$$=l_2 l_3+m_2 m_3+n_2 n_3=0\ .$$

The equilibrium order parameter now takes the form $n^0_{\alpha i}=\bar{n}^0_{\alpha j} R_{ji}=R_{\alpha i}$ or $n^0_{\alpha i}=\bar{n}_{\beta i} R_{\alpha\beta}=R_{\alpha i}$. Both forms of $n^0_{\alpha i}$ are equivalent, since rotations of spin space against orbit space are equivalent to rotations of orbit space against spin space.

A magnetic field along the quantization axis $\hat{d}$ (hats $\hat{}$ denote unit vectors in the following) influences the three Cooper-pair species in a different manner. As is well known from the A phase the up pairs are enhanced and the down pairs suppressed (or vice versa) so that $\Delta_\uparrow \neq \Delta_\downarrow$. This effect shows up most clearly in the existence of the A_1 phase. The difference $\Delta_\uparrow - \Delta_\downarrow$ will become manifest[1] either in very high magnetic fields ($H \gg 10$ kG) at low temperatures or very close to T_{A2}. Therefore, in the B phase, effects which are based on $\Delta_\uparrow \neq \Delta_\downarrow$ are likely not measurable, since for $H > 10$ kG, the B phase does not exist.[42] However, for completeness, we will take into account $\Delta_\uparrow \neq \Delta_\downarrow$ but we will state, for experimentally accessible results, what is obtained in the limit $\Delta_\downarrow = \Delta_\uparrow$.

In addition, Δ_3, the gap of the pairs with symmetrically opposite spin, is lowered in a strong magnetic field. This effect takes place at lower-field strengths and gives rise to measurable effects for $H \gtrsim 1$ kG. The difference $\Delta_0 - \Delta_3$ is related to the Zeeman energy[2] $\gamma\hbar H$ (γ is the gyromagnetic ratio), i.e.,

$$1-(\Delta_3/\Delta_0)^2 \approx (\gamma\hbar H/\Delta_0)^2$$

roughly.

For $\Delta_\uparrow \neq \Delta_\downarrow$ and $\Delta_3 \neq \Delta_0$ the equilibrium order parameter (2.1) can be written in the form

$$n^0_{\alpha i} \sim \begin{bmatrix} \Delta_1 & +i\Delta_2 & 0 \\ -i\Delta_2 & \Delta_1 & 0 \\ 0 & 0 & \Delta_3 \end{bmatrix}, \tag{2.4}$$

with $\Delta_1 \equiv \frac{1}{2}(\Delta_\uparrow + \Delta_\downarrow)$ and $\Delta_2 \equiv \frac{1}{2}(\Delta_\uparrow - \Delta_\downarrow)$, if special frames are used. Again, the different frames of spin space and orbit space are oriented arbitrarily with respect to each other. If we choose the 3 axis in spin space to be set by the external magnetic field, the most general form of the equilibrium order parameter is

$$A^0_{\alpha i} = [2(\Delta_1^2+\Delta_2^2)+\Delta_3^2]^{-1} n^0_{\alpha i} e^{i\varphi}\ ,$$

with $n^0_{\alpha i} \equiv \bar{n}^0_{\alpha j} R_{ji}$ [cf. (2.2) and (2.3)] or explicitly

$$n^0_{\alpha i} = \begin{bmatrix} \Delta_1 l_1 - i\Delta_2 m_1 & \Delta_1 l_2 - i\Delta_2 m_2 & \Delta_1 l_3 - i\Delta_2 m_3 \\ i\Delta_2 l_1 + \Delta_1 m_1 & \Delta_1 m_2 + i\Delta_2 l_2 & \Delta_1 m_3 + i\Delta_2 l_3 \\ \Delta_3 n_1 & \Delta_3 n_2 & \Delta_3 n_3 \end{bmatrix}. \tag{2.5}$$

Note that $A^0_{\alpha i}$ is no longer a rotation matrix as is the case for small or vanishing magnetic fields. The order parameter $A^0_{\alpha i}$ minimizes the Ginzburg-Landau free-energy functional, which was already shown explicitly (for the special case $\Delta_2=0$) by Fetter.[43] Since the external field defines a fixed direction in spin space, transverse rotations of spin space are not equivalent to rotations in orbit space and an equilibrium order parameter $\tilde{A}^0_{\alpha i}$ defined by $\tilde{A}^0_{\alpha i} \sim n^0_{\beta i} R_{\alpha\beta}$ is not equivalent to $A^0_{\alpha i}$ defined above, and does not minimize the free energy.

The degeneracy still contained in (2.5) is lifted by the magnetic dipole interaction which fixes the rotation matrix R_{ij}. Minimizing the dipole interaction energy[2]

$$F_D = \tfrac{3}{10} g_D (A_{ii} A^*_{jj} + A_{ij} A^*_{ji} - \tfrac{2}{3} A_{ij} A^*_{ij})$$

one obtains $l_1 = m_2 = \cos\theta_0$, $l_2 = -m_1 = \sin\theta_0$, $n_3 = 1$, and $m_3 = l_3 = n_1 = n_2 = 0$, with

$$\cos\theta_0 = -\tfrac{1}{4} \Delta_1 \Delta_3 (\Delta_1^2 - \Delta_2^2)^{-1}\ .$$

Again, this result reduces for $\Delta_2=0$ to the expression given by Fetter.[43] For $H \to 0$, the "magic" angle $\theta_0 = \cos^{-1}(-\frac{1}{4})$ is regained, while for $\Delta_3 \to 0$ one has $\theta_0 \to 90°$.

For these special values of l_i, m_i, and n_i, the 3 axes of spin and orbit space coincide (and are given by the direction of the external field) and both spaces are twisted against each other about the 3 axis at the angle θ_0. The structure of the equilibrium order parameter then reads

$$n^0_{\alpha i} = \begin{bmatrix} \Delta_1 \cos\theta_0 + i\Delta_2 \sin\theta_0 & \Delta_1 \sin\theta_0 - i\Delta_2 \cos\theta_0 & 0 \\ -\Delta_1 \sin\theta_0 + i\Delta_1 \cos\theta_0 & \Delta_1 \cos\theta_0 + i\Delta_2 \sin\theta_0 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{2.6}$$

It should be noticed that the order parameter is now complex and nonunitary. The gap is no longer isotropic, but

$$|\Delta(k)|^2 = \Delta_3^2 k_z^2 + (\Delta_1^2 + \Delta_2^2)(k_x^2 + k_y^2)\ .$$

Of course, for $H \to 0$, i.e., $\Delta_1 \to \Delta_0$, $\Delta_2 \to 0$, and $\Delta_3 \to \Delta_0$, Eq. (2.2) is reobtained and the gap becomes isotropic. Thus the presence of a strong magnetic field changes the structure of the equilibrium order parameters and, as a consequence, the hydrodynamic equations.

In deriving the equilibrium order parameter the magnetic dipole energy was treated only perturbatively. In the same spirit we will proceed in hydrodynamics,[5,10] e.g., we disregard the magnetic dipole energy while looking for the spontaneously broken symmetries and the appropriate hydrodynamic variables. The influence of the magnetic dipole interaction on the dynamics of these variables is then obtained adding the dipole energy to the free energy.[44] This will be done in Sec. V. Up to that point we neglect the dipole interaction in the hydrodynamic equations.

In ^{3}He-B without magnetic fields there are four spontaneously broken continuous symmetries, gauge symmetry (1), and rotational invariance of spin space against orbit space (3). The hydrodynamic variables connected with these broken symmetries are the phase change $\delta\varphi$ (1) and three independent elements of $\delta n_{\alpha i}$ (cf. Refs. 6 and 7). The latter can be described as two fluctuations $\delta\hat{d}_i$ of the quantization axis $\hat{d}$ ($\hat{d}\cdot\delta\hat{d}=0$) and one fluctuating rotation angle $\delta\theta$ about the axis $\hat{d}$. In the presence of a magnetic field $\vec{H}$ ($\|\hat{d}$) the number of spontaneous broken symmetries (and appropriate hydrodynamic variables) is unchanged, since the equilibrium order parameter, again, contains an arbitrary phase and an arbitrary matrix [cf. (2.5)] with three independent elements. However, the physical interpretation of the appropriate hydrodynamic variables $\delta\varphi$, $\delta\theta$, and $\delta\hat{d}_i$ is different and more complicated as in the case of small or zero magnetic field. Let us first consider the "longitudinal" variables $\delta\varphi$ and $\delta\theta$, which reduce in the case of small magnetic field to the ordinary phase variable and to the variable describing rotation about the magnetic field, respectively. In strong magnetic fields their behavior is more complicated, as can be seen by the appropriate commutation relations with the generators of continuous symmetries. (For a proper operator definition of $\delta\varphi$ and $\delta\theta$, cf. Appendix A.)

In spin space, $\delta\theta$ and $\delta\varphi$ break rotational invariance, since

$$\langle [\delta\theta, m_\alpha H_\alpha] \rangle = -2i\gamma \;,$$
$$\langle [\delta\varphi, m_\alpha H_\alpha] \rangle = 2i\gamma\tilde{\beta}_1 \;, \tag{2.7}$$

with

$$\tilde{\beta}_1 = \frac{3\Delta_1 |\Delta_2|}{\Delta_3^2 + 2(\Delta_1^2+\Delta_2^2)} = \frac{\frac{3}{4}|\Delta_\uparrow^2 - \Delta_\downarrow^2|}{\Delta_3^2+\Delta_\uparrow^2+\Delta_\downarrow^2} \;.$$

In addition, $\delta\theta$ and $\delta\varphi$ break gauge invariance

$$\langle [\delta\theta, N] \rangle = 2i\beta_1 \;,$$
$$\langle [\delta\varphi, N] \rangle = -2i \;, \tag{2.8}$$

with

$$\beta_1 = \frac{\Delta_1 |\Delta_2|}{\Delta_1^2+\Delta_2^2} \;.$$

It is possible to use linear combinations of $\delta\theta$ and $\delta\varphi$,

$$\delta\tilde{\varphi} = \frac{1}{1-\beta_1\tilde{\beta}_1}\delta\varphi + \frac{\tilde{\beta}_1}{1-\beta_1\tilde{\beta}_1}\delta\theta \;,$$
$$\delta\tilde{\theta} = \frac{1}{1-\beta_1\tilde{\beta}_1}\delta\theta + \frac{\beta_1}{1-\beta_1\tilde{\beta}_1}\delta\varphi \;, \tag{2.9}$$

which have the properties of being gauge invariant ($\delta\tilde{\theta}$), or a scalar quantity in spin space ($\delta\tilde{\varphi}$). For microscopic calculations this may be an advantage. For the derivation of the hydrodynamic equations, however, we will use $\delta\theta$ and $\delta\varphi$ as variables, since these variables are defined as in the B phase without magnetic fields. By the smallness of β_1 and $\tilde{\beta}_1$, the discrimination of $\delta\varphi, \delta\theta$ and $\delta\tilde{\varphi}, \delta\tilde{\theta}$ is, in any case, rather academic. Of course, experimentally accessible results do not depend on which linear combination is chosen.

By the presence of an external magnetic field $\vec{H}$, ^{3}He-B is no longer "isotropic," but there is a preferred direction in the spin space defined by the unit vector $\hat{H}_\alpha$. Because of the intricate connection of spin space and orbit space in ^{3}He-B there is also a preferred direction in orbit space defined by the unit vector $\hat{d}_i = \hat{H}_\alpha n^0_{\alpha i} \Delta_3^{-1}$. Thus ^{3}He-$B$ becomes anisotropic in spin space and even in orbit space due to an external magnetic field. This is in contrast to ^{3}He-A, where an external magnetic field leads to a biaxiality in spin space, but does not change the uniaxiality of orbit space, which is already set by the $\hat{l}$ vector. As already stated above, $\hat{d}$ is parallel to $\hat{H}$ due to the dipole interaction.

The variables $\delta\theta$ and $\delta\varphi$ also break rotational invariance in orbit space. We find

$$\langle [\delta\theta, \vec{L}\cdot\hat{d}^0] \rangle = 2in_3 \;,$$
$$\langle [\delta\varphi, \vec{L}\cdot\hat{d}^0] \rangle = -2i\tilde{\beta}_1 n_3 \;, \tag{2.10}$$

where n_3 is the cosine of the angle between $\vec{H}$ and $\vec{L}$. This angle ψ is just the angle between the 3 axis in spin space and the 3 axis in orbit space, if the representation (2.5) is used. Of course, in the true equilibrium state there is $n_3 = 1$ due to the dipole interaction and

$$\langle [\delta\theta, \vec{L}\cdot\hat{d}^0] \rangle = 2i \;,$$
$$\langle [\delta\varphi, \vec{L}\cdot\hat{d}^0] \rangle = -2i\tilde{\beta}_1 \;. \tag{2.11}$$

Since $\delta\theta$ (and $\delta\varphi$) breaks more than one symmetry, we can construct linear combinations of symmetries, which are spontaneously broken, too. This is not the case for the symmetry with the generator $\vec{S}\cdot\hat{H} + \vec{L}\cdot\hat{d}^0$, which is broken only externally.

Instead of

$$\delta\theta \sim \tfrac{1}{2}\hat{H}_\alpha \epsilon_{\alpha\beta\gamma}(\delta A^*_{\beta i} A^0_{\gamma i} + \text{c.c.})$$

[Appendix A, (A3)] we could have defined a variable

$$\delta\theta' \sim \tfrac{1}{2}\hat{d}_i \epsilon_{ijk}(\delta A^*_{\alpha j} A^0_{\alpha k} + \text{c.c.}) \;,$$

which is connected to the same broken symmetries as $\delta\theta$. Again all linear combinations of $\delta\theta$ and $\delta\theta'$ are equally well suitable as hydrodynamic variables (except for $\delta\theta + \delta\theta'$). The difference between $\delta\theta$ and $\delta\theta'$ is that the former describes longitudinal rotations, i.e., rotations about $\hat{H}$, of spin space against orbit space, and the latter describes longitudinal rotations, i.e., rotations about $\hat{d}$, of orbit space against spin space.

Of the transverse rotations (i.e., rotations of $\hat{H}$ or of $\hat{d}$) only those which are in orbit space (i.e., described by δd_i with $\delta\vec{d}\cdot\hat{d} = 0$) are hydrodynamic, since rotations in spin space (rotations of $\hat{H}$) are connected with a finite energy even in the homogeneous limit. The hydrodynamic variables δd_i (for a proper operator definition cf. Appendix A) break rotational symmetry spontaneously in orbit space, since

$$\langle [\delta d_i, L_j \epsilon_{ijk} \hat{d}^0_k] \rangle = i \left[1 + \frac{\Delta_3^2}{\Delta_1^2 + \Delta_2^2} \right] \;, \tag{2.12}$$

but not gauge symmetry, since

$$\langle [\delta d_i, N] \rangle = 0 \;.$$

Of course, $\delta\hat{d}_i$ break rotational symmetry in spin space too. However, that symmetry is already broken externally. Having specified the spontaneously broken symmetries and the associated hydrodynamic variables we proceed in deriving the linear hydrodynamic equations (Secs. III and IV). For the nonlinear theory we refer to Ref. 40, which is supplemented in Appendix C. For simplicity, we also refrain from giving higher-order gradients (cf. Refs. 45–47). The dipole interaction will be neglected in Secs. III and IV but will be introduced perturbatively in Sec. V.

III. STATIC EQUATIONS

In addition to the hydrodynamic variables $\delta\theta$, δd_i, and $\delta\varphi$, which are connected with broken symmetries and which we have discussed in the preceding section, the conserved quantities are hydrodynamic variables too. The latter are mass density ρ, energy density ϵ, momentum density $\vec{g}$, and the (longitudinal) magnetization density $\vec{m}\cdot\hat{H}\equiv M$.

The angular momentum conservation is treated by symmetrization of the stress tensor.[48,29,10] The transverse magnetization is not conserved because of the external magnetic field and leads to modes $\omega(k)$ with a gap for $k\to 0$ (Larmor frequency).

Assuming local thermodynamic equilibrium the change of entropy density σ is given by the Gibbs relation

$$T d\sigma = d\epsilon - \mu d\rho - \vec{v}^{(n)}\cdot d\vec{g} - (h-H)dM - \vec{\lambda}^{(s)}\cdot d\vec{\nabla}\varphi - \psi_i d\nabla_i\theta - \phi_{ij} d\nabla_j d_i \ . \tag{3.1}$$

The thermodynamic conjugates (generalized forces) T temperature, μ chemical potential, $\vec{v}^{(n)}$ normal velocity, and h longitudinal molecular magnetic field, and $\vec{\lambda}^{(s)}$, ϕ_{ij}, and ψ_i are thereby defined as partial derivatives of ϵ (or σ). They are connected with the variables by equations of state, which govern the statics of the system.

Taking into account the symmetry of the variables we find two separate sets of variables and conjugates, which are decoupled from each other. For the quantities, even under time reversal, we have

$$\begin{aligned}
\delta T &= T_0 C_v^{-1}\delta\sigma + \zeta_1\delta M + \zeta_3\delta_\rho \ ,\\
\delta\mu &= \zeta_4\delta\rho + \zeta_2\delta M + \zeta_3\delta\sigma \ ,\\
\delta h &= \chi_{\|}^{-1}\delta M + \zeta_2\delta\rho + \zeta_1\delta\sigma \ ,
\end{aligned} \tag{3.2}$$

where $C_v^{-1}, \zeta_{1,2,3,4}$ and $\chi_{\|}^{-1}$ are susceptibilities which have to satisfy the relations $C_v>0$, $\chi_{\|}>0$, $\zeta_4>0$, $\zeta_4 T_0 C_v^{-1}>\zeta_3^2$, $\zeta_4\chi_{\|}^{-1}>\zeta_2^2$, and $T_0 C_v^{-1}\chi_{\|}^{-1}>\zeta_1^2$. For the remaining quantities which are odd under time reversal the most general ansatz reads [$\delta\theta$ and δd_i are odd because of the $\hat{H}$ or $\hat{d}^0$ factor in the definitions (A2) and (A3)]

$$\begin{aligned}
g_i &= \rho_{ij}^n V_j^n + \rho_{ij}^s\nabla_j\varphi + \rho_{ij}^d\nabla_j\theta + C_{ij}^{(1)}(\mathrm{curl}\,\vec{d})_j \ ,\\
\lambda_i^{(s)} &= f_{ij}^s\nabla_j\varphi - \rho_{ij}^s V_j^n + f_{ij}^d\nabla_j\theta + C_{ij}^{(2)}(\mathrm{curl}\,\vec{d})_j \ ,\\
\psi_i &= q_{ij}\nabla_j\theta + f_{ij}^d\nabla_j\varphi - \rho_{ij}^d V_j^n + C_{ij}^{(3)}(\mathrm{curl}\,\vec{d})_j \ ,\\
\phi_{ij} &= K_{ijlm}\nabla_m d_l + \epsilon_{ijk}(C_{km}^{(1)}V_m^n + C_{km}^{(2)}\nabla_m\varphi + C_{km}^{(3)}\nabla_m\theta) \ .
\end{aligned} \tag{3.3}$$

All material tensors are of the axial form

$$\rho_{ij} = \rho_{\|}\hat{H}_i\hat{H}_j + \rho_{\perp}(\delta_{ij} - \hat{H}_i\hat{H}_j)$$

and clearly show the anisotropy of the B phase in high magnetic fields. Here we are able to reduce the number of unknown parameters from 21 to 13 by the observation that $\vec{g}$ is not only a variable but also the current of the mass density ρ. Under a Galilean transformation with velocity $\vec{U}$ the current $\vec{g}$ has to change into $\vec{g}+\rho_0\vec{U}$.

Therefore, $\vec{g}$ must have the form

$$\vec{g} = \rho_0\vec{V}^n + \vec{Z} \ , \tag{3.4}$$

where $\vec{Z}$ is Galilean invariant and does not contain $\vec{V}^n$. The most general possibility left for $\vec{Z}$ is

$$Z_i = \alpha_{ij}^{(1)}\lambda_j^{(s)} + \alpha_{ij}^{(2)}\psi_j \ . \tag{3.5}$$

Comparing (3.5) with the exact commutator relations (2.8) we can conclude $\alpha_{ij}^{(1)}=\delta_{ij}$ and $\alpha_{ij}^{(2)}=\beta_1\delta_{ij}$, and finally $\vec{g}$ reads

$$\vec{g} = \rho_0\vec{V}^{(n)} + \vec{\lambda}^{(s)} + \beta_1\vec{\psi} \ . \tag{3.6}$$

Equation (3.6) is compatible with Eqs. (3.3) if and only if

$$\begin{aligned}
\delta_{ij}\rho_0 &= \rho_{ij}^n + \rho_{ij}^s + \beta_1\rho_{ij}^d \ ,\\
f_{ij}^d + \beta_1 q_{ij} &= \rho_{ij}^d \ ,\\
f_{ij}^s + \beta_1 f_{ij}^d &= \rho_{ij}^s \ ,\\
C_{ij}^{(3)} &= \beta_1^{-1}(C_{ij}^{(2)} - C_{ij}^{(1)}) \ .
\end{aligned} \tag{3.7}$$

The equations of state for the variables that are odd under time reversal then take the final form

$$\begin{aligned}
g_i &= \rho_{ij}^n V_j^n + \rho_{ij}^s\nabla_j\varphi + \rho_{ij}^d\nabla_j\theta + C_{ij}^{(1)}(\mathrm{curl}\,\vec{d})_j \ ,\\
\lambda_i^{(s)} &= \rho_{ij}^s(\nabla_j\varphi - V_j^n) + (\rho_{ij}^d - \beta_1 q_{ij})(\nabla_j\theta - \beta_1\nabla_j\varphi) + C_{ij}^{(2)}(\mathrm{curl}\,\vec{d})_j \ ,\\
\psi_i &= \rho_{ij}^d(\nabla_j\varphi - V_j^n) + q_{ij}(\nabla_j\theta - \beta_1\nabla_j\varphi) + \beta_1^{-1}(C_{ij}^{(2)} - C_{ij}^{(1)})(\mathrm{curl}\,\vec{d})_j \ ,\\
\phi_{ij} &= K_{ijlm}\nabla_m d_l + \epsilon_{ijl}[C_{lm}^{(1)}V_m^n + C_{lm}^{(2)}\nabla_m\varphi + \beta_1^{-1}(C_{lm}^{(2)} - C_{lm}^{(1)})\nabla_m\theta] \ ,
\end{aligned} \tag{3.8}$$

where the susceptibilities have to satisfy (in order to guarantee thermodynamic stability)

$$\begin{aligned}
&\rho_{\|,\perp}^n > 0 \ ,\\
&\rho_{\|,\perp}^s > \beta_1(\rho_{\|,\perp}^d - \beta_1 q_{\|,\perp}) \ ,\\
&q_{\|,\perp} > 0 \ ,
\end{aligned}$$

and

$$\rho_{\|,\perp}^s q_{\|,\perp} > \rho_{\|,\perp}^d(\rho_{\|,\perp}^d - \beta_1 q_{\|,\perp}) \ ,$$

etc. They are connected with the total density ρ_0 by (3.7),

$$\rho_0 = \rho_{\|}^n + \rho_{\|}^s + \beta_1\rho_{\|}^d = \rho_{\perp}^n + \rho_{\perp}^s + \beta_1\rho_{\perp}^d \ ,$$

leaving 13 susceptibilities.

The Galilean invariance of $\vec{Z}$ (3.5) can now be used to derive the behavior of $\nabla_i\theta$ under Galilean transformation. Under the assumptions (which are proven in Appendix B) that $\nabla_i\varphi$ is a true Galilean velocity ($\nabla_i\varphi\to\nabla_i\varphi - U_i$) and that $\mathrm{curl}\,\vec{d}$ is Galilean invariant we find from (3.5) and (3.8)

$$\nabla_i\theta \to \nabla_i\theta + \beta_1 U_i \ , \tag{3.9}$$

under a Galilean transformation with velocity $\vec{U}$. This behavior is proven directly in Appendix B. For $H\to 0$, $\nabla_i\theta$, is of course, Galilean invariant. [In the A phase in high magnetic fields (but not in the A_1 phase) there exists a variable with similar properties.[36]]

For $H\to 0$, Eqs. (3.8) have to reduce to the well-known formulas for the B phase in zero or vanishing magnetic field (cf., e.g., Ref. 6). Thus we have to conclude that in the limit $H\to 0$

$$\rho_\perp^n \to \rho_n \ , \quad \rho_\parallel^n \to \rho_n \ , \quad \rho_\parallel^s \to \rho_s \ , \quad \rho_\perp^s \to \rho_s \ ,$$

which suggests

$$\rho_{\perp,\parallel}^{n,s}(H) = \rho_{n,s} + H^2 \tilde{\rho}_{\perp,\parallel}^{n,s} \ .$$

In addition, ρ_{ij}^d, $C_{ij}^{(1)}$, and $C_{ij}^{(2)}$ have to vanish, which is achieved by

$$C_{\perp,\parallel}^{(1,2)}(H) = H^2 \tilde{C}_{\perp,\parallel}^{(1,2)} \text{ and } \rho_{\perp,\parallel}^d(H) = H^2 \tilde{\rho}_{\perp,\parallel}^d \ .$$

Since $\beta_1 \sim H^1$ for $H \to 0$ we can conclude that $\beta_1^{-1}(C_{ij}^{(1)} - C_{ij}^{(2)})$ vanishes also in that limit. In small but nonzero field, the H^2 contributions are highly nonlinear and are usually omitted; their existence is based on $H \neq 0$. The quantities ψ_i and ϕ_{ij} (or $\delta\theta$ and δd_i) cannot be defined separately in zero field; therefore,

$$M_{ijklmq}(\nabla_k R_{ij})(\nabla_q R_{lm})$$

enters the free energy,[6] where δR_{ij} are the fluctuations of the real rotation matrix R_{ij}^0 (or n_{ij}^0) and the tensor M contains two independent susceptibilities M_1 and M_2. Since $\delta\theta$ and δd_i are related to δR_{ij} by $\delta d_i \sim H_j \delta R_{ij}$ and $\delta\theta \sim H_i \epsilon_{ijk} \hat{n}_{lj}^0 \delta R_{lk}$ the independent elements contained in q_{ij} and K_{ijlm} ($q_\perp$, $q_\parallel$, K_1, K_2, K_3) reduce in the limit $H \to 0$ to linear combinations of M_1 and M_2 (containing also the Leggett angle θ_0).

IV. DYNAMICAL EQUATIONS

According to the Gibbs relation (3.1) the equations of motion are

$$
\begin{aligned}
&\dot{\rho} + \nabla_i g_i = 0 \ , \\
&\dot{g}_i + \nabla_j \sigma_{ij} = 0 \quad \text{with } \nabla_j \sigma_{ij} = \nabla_j \sigma_{ji} \\
&\dot{\sigma} + \nabla_i j_i^\sigma = \frac{R}{T} \ , \\
&\dot{M} + \nabla_i j_i^M = 0 \ , \\
&\dot{\varphi} + I_\varphi = 0 \ , \\
&\dot{\theta} + Y = 0 \ , \\
&\dot{d}_i + X_i = 0 \ .
\end{aligned}
\tag{4.1}
$$

Hereby the currents are defined. For the reversible parts of the currents we obtain

$$
\begin{aligned}
g_i^R &= \rho_0 V_i^n + \lambda_i^{(s)} + \beta_1 \psi_i \ , \\
I_\varphi^R &= \mu - \tilde{\beta}_1 \gamma h + \lambda \hat{d}_i^0 \epsilon_{ijk} \nabla_j V_k^n \ , \\
Y^R &= \beta_1 \mu + \gamma h + \beta_2 T - \delta_1 \epsilon_{ijk} \hat{d}_i^0 \nabla_j V_k^n \ , \\
j_i^{\sigma R} &= \sigma_0 V_i^n + \beta_2 \psi_i \ , \\
j_i^{MR} &= m_0 V_i^n + \gamma \psi_i - \gamma \tilde{\beta}_1 \lambda_i^{(s)} \ , \\
\sigma_{ij}^R &= p \delta_{ij} - \delta_1 \epsilon_{ijk} \hat{d}_k^0 \nabla_m \psi_m \\
&\quad - \lambda \hat{d}_p^0 (\epsilon_{pik} \delta_{jl} + \epsilon_{pjk} \delta_{il}) \nabla_k \lambda_l^{(s)} \ , \\
&\quad + \varphi_{ikjl} \nabla_j V_l^n + \tfrac{1}{2} \alpha_{lji} \nabla_m \phi_{lm} \ , \\
X_i^R &= \delta_2 \epsilon_{ijk} \hat{d}_j^0 \nabla_l \phi_{kl} - \alpha_{ijk} \nabla_j V_k^n \ ,
\end{aligned}
\tag{4.2}
$$

with

$$
\begin{aligned}
\varphi_{ijkl} = {} & \varphi_1 (\hat{d}_p^0 \epsilon_{pjl} \delta_{ik}^\kappa + \hat{d}_p^0 \epsilon_{pil} \delta_{jk}^\kappa + \hat{d}_p^0 \epsilon_{pjk} \delta_{il}^\kappa + \hat{d}_p^0 \epsilon_{pik} \delta_{jl}^\kappa) \\
& + \varphi_2 (\hat{d}_i^0 \hat{d}_k^0 \hat{d}_p^0 \epsilon_{pjl} + \hat{d}_j^0 \hat{d}_k^0 \hat{d}_p^0 \epsilon_{pil} + \hat{d}_i^0 \hat{d}_l^0 \hat{d}_p^0 \epsilon_{pjk} + \hat{d}_j^0 \hat{d}_l^0 \hat{d}_p^0 \epsilon_{pik}) \ ,
\end{aligned}
$$

$$\alpha_{ijk} = \alpha_1 \delta_{ij}^\kappa \hat{d}_k^0 + \alpha_2 \delta_{ik}^\kappa \hat{d}_j^0 \quad \text{with } \delta_{ij}^\kappa \equiv \delta_{ij} - \hat{d}_i^0 \hat{d}_j^0$$

and γ the gyromagnetic ratio, p the isotropic pressure, m_0 the longitudinal equilibrium magnetization $m_0 = \vec{m} \cdot \vec{H}$, and $\beta_1, \tilde{\beta}_1$ defined by (2.7) and (2.8).

The expression for g_i^R was already derived in the preceding section. The first terms of j_i^{MR}, $j_i^{\sigma R}$, and σ_{ij}^R are the usual convective ones.

The phenomenological terms proportional to β_2, δ_2, and φ_1, φ_2 contain four reversible transport parameters (cf. Appendix A). All other terms follow from Eqs. (2.7)–(2.12). Especially we find $\lambda = \frac{1}{2}\tilde{\beta}_1 + \tilde{\lambda}$, $\delta_1 = \frac{1}{2} + \tilde{\delta}_1$, and

$$\alpha_1 - \alpha_2 = \frac{1}{4} \left[1 + \frac{\Delta_3^2}{\Delta_1^2} \right] \ ,$$

where $\tilde{\lambda}$ and $\tilde{\delta}_1$ are contributions due to collisions [cf. (A7)]. For $H \to 0$ Eq. (4.2) reduces to the well-known results for the B phase without magnetic fields.[6,7] [The additional term contained in Ref. 17, $\dot{\eta}_i \sim (\text{curl}\vec{v}^n)_i$, is represented in Eq. (4.2) by the terms proportional to δ_1 and to $\alpha_2 - \alpha_1$.]

For the irreversible parts of the currents we obtain

$$
\begin{aligned}
j_i^{\sigma D} &= -\kappa_{ij} \nabla_j T - \kappa'_{ij} \nabla_j h \ , \\
j_i^{MD} &= -\mu_{ij} \nabla_j h - \kappa'_{ij} \nabla_j T \ , \\
Y^D &= -\nu \nabla_j \psi_j - \eta_7 \nabla_j \lambda_j^{(s)} - \eta_{ij}^8 \nabla_j V_i^n \ , \\
I_\varphi^D &= -\eta_7 \nabla_k \psi_k - \zeta \nabla_i \lambda_i^{(s)} - \xi_{ij} \nabla_j V_i^n \ , \\
\sigma_{ij}^D &= -\nu_{ijkl} (\nabla_k V_l^n + \nabla_l V_k^n) - \xi_{ij} \nabla_l \lambda_l^{(s)} \\
&\quad - \eta_{ij}^8 \nabla_k \psi_k - \xi_{ijk} \nabla_l \phi_{kl} \ , \\
X_i^D &= -\eta \nabla_k \phi_{ik} - \xi_{kji} \nabla_k V_j^n \ ,
\end{aligned}
\tag{4.3}
$$

where the material parameters are of the axial form with the preferred axis given by $\hat{d}^0$, e.g.,

$$\kappa_{ij} = \kappa_\parallel \hat{d}_i^0 \hat{d}_j^0 + \kappa_\perp (\delta_{ij} - \hat{d}_i^0 \hat{d}_j^0) \ .$$

Positivity of entropy production requires the constraints

$$\kappa_{\|,\perp}>0\ ,\quad \mu_{\|,\perp}>0\ ,\quad \nu>0\ ,$$
$$\zeta>0\ ,\quad \nu_2+\nu_4>0\ ,$$
$$2\nu_1+2\nu_5+\nu_2-\nu_4>0\ ,$$
$$\kappa'^{\,2}_{\|,\perp}<\kappa_{\|,\perp}\mu_{\|,\perp}\ ,\quad \eta_7^2<\nu\zeta\ ,$$
$$(\eta_\perp^8)^2<2\nu(\nu_2+\nu_4)\ ,$$
$$(\eta_\|^8)^2<2\nu(2\nu_1+2\nu_5+\nu_2-\nu_4)\ .$$

The terms $\sim\kappa'_{ij}$, $\sim\eta_7$, and $\sim\eta_{ij}^8$ owe their existence to the magnetic field and vanish with $H\to 0$. In addition, the isotropy of ^{3}He-B is regained in the limit $H\to 0$, e.g., $\kappa_\perp\to\kappa_\|$, etc.

V. MAGNETIC DIPOLE INTERACTION

In Secs. III and IV we have neglected the interaction of the magnetic moments of the ^{3}He nuclei. This magnetic dipole interaction leads to a spin-orbit coupling, which fixes the position of orbit space against spin space in equilibrium (cf. Sec. II). Thus fluctuations of orbit against spin space now relax in a finite time even in the homogeneous limit $k\to 0$, since there is an elastic energy connected with these fluctuations. Varying the magnetic dipole energy[2]

$$F_D=\tfrac{3}{10}g_D(A_{ii}A_{jj}^*+A_{ij}A_{ji}^*-\tfrac{2}{3}A_{ij}A_{ij})$$

we find ($\Delta_2=0$ in this section)

$$\delta^2F_D=\tfrac{3}{10}g_D\left[\frac{\Delta_1^2-\Delta_3^2}{2\Delta_1^2+\Delta_3^2}\psi^2+\frac{8\Delta_1^2-\frac{1}{2}\Delta_3^2}{2\Delta_1^2+\Delta_3^2}(\delta\theta)^2\right]\ , \tag{5.1}$$

where θ is the rotation angle about $\hat{H}$ (which is θ_0 in equilibrium) and where ψ is the angle between $\hat{H}$ and $\hat{d}$ (which is zero in equilibrium).[49] Thereby, ψ and $\delta\theta$ are related to the hydrodynamic variables by

$$\psi^2=[2\Delta_1^2/(\Delta_1^2+\Delta_3^2)]^2(\delta\hat{d}\,)^2$$

and $\theta-\theta_0=\delta\theta$, respectively. The elastic energy (5.1) gives rise to new contributions in the Gibbs relation (3.1),

$$Td\sigma=d\epsilon-\psi_i d\nabla_i\theta-\phi_{ij}d\nabla_j d_i-B_1\delta\theta d\theta-B_2\delta d_i d(\delta d_i)\ , \tag{5.2}$$

with

$$B_1=\tfrac{3}{5}g_D\frac{8\Delta_1^2-\frac{1}{2}\Delta_3^2}{2\Delta_1^2+\Delta_3^2}\ ,$$
$$B_2=\tfrac{3}{5}g_D\frac{4\Delta_1^4(\Delta_1^2-\Delta_3^2)}{(2\Delta_1^2+\Delta_3^2)(\Delta_1^2+\Delta_3^2)^2}\ .$$

Therefore, the magnetic dipole interaction is taken into account in all the formulas of the previous sections, if ψ_i and ϕ_{ij} are replaced by $\tilde{\psi}_i$ and $\tilde{\phi}_{ij}$, respectively,

$$\nabla_i\tilde{\psi}_i\equiv\nabla_i\psi_i-B_1\delta\theta\ ,$$
$$\nabla_j\tilde{\phi}_{ij}\equiv\nabla_j\phi_{ij}-B_2\delta d_i\ . \tag{5.3}$$

This is always possible, since in the equations of motion (but not in the currents) $\nabla_i\psi_i$ and $\nabla_j\phi_{ij}$ (and not ψ_i and ϕ_{ij} alone) occurred in Secs. III and IV.

The physical content of (5.1) or (5.3) is easily seen by solving the hydrodynamic equations (4.1)—(4.3), (3.2), and (3.8) in the homogeneous limit $k\to 0$. For the longitudinal variables $\delta\rho$, δM, $\delta\sigma$, or $\delta\theta$ one obtains equations describing (weakly) damped oscillations, e.g.,

$$\delta\ddot{\theta}+\Omega^2\delta\theta+\nu B_1\delta\dot{\theta}=0\ , \tag{5.4}$$

with

$$\Omega^2=B_1(\gamma^2\chi_\|^{-1}+2\zeta_1B_2\gamma+2\beta_1\beta_2\zeta_3+2\beta_1\gamma\zeta_2$$
$$+\beta_1^2\zeta_4+\beta_2^2T_0C_v^{-1})$$
$$\approx B_1\gamma^2\chi_\|^{-1}$$

because of the smallness of β_1 and β_2. Probing these oscillations by longitudinal NMR experiments one obtains a resonance peak at

$$\omega_f=\Omega\approx\pm(B_1\gamma^2\chi_\|^{-1})^{1/2}$$

with halfwidth $\Delta\omega_{1/2}=\nu B_1$. Thus the longitudinal NMR resonance frequency depends on the magnetic field strength. For $H\to 0$ ($\Delta_3\to\Delta_1$),

$$\omega_f^2(H=0)=\tfrac{3}{2}g_D\gamma^2\chi_\|^{-1}$$

is regained.[41,8] For $H\neq 0$

$$\frac{\omega_f^2(H)}{\omega_f^2(0)}=\frac{1}{5}\frac{16\Delta_1^2-\Delta_3^2}{2\Delta_1^2+\Delta_3^2}\frac{\chi_\|(0)}{\chi_\|(H)}\ ,$$

where the first ratio increases with increasing magnetic field to its maximum value,[41] $\frac{8}{5}$ in the case $\Delta_3=0$, while the second ratio probably decreases with increasing magnetic field.[2] In contrast to the B phase in vanishing magnetic field, there is also a transverse NMR shift in the B phase in high magnetic fields. Fluctuations of the direction of $\vec{H}$ cost magnetic dipole energy and the resonance frequency in transverse NMR experiments is shifted away from the Larmor frequency. This (positive) frequency shift $\Delta\omega_L$ is given by [cf. (5.1)]

$$(\Delta\omega_L)^2=\frac{1}{2}\gamma^2\chi_\perp^{-1}\frac{\partial^2F_D}{\partial\psi^2}=\gamma^2\chi_\perp^{-1}g_D\frac{3}{5}\frac{\Delta_1^2-\Delta_3^2}{2\Delta_1^2+\Delta_3^2}\ . \tag{5.5}$$

Of course, for $H\to 0$, $(\Delta\omega_L)^2=0$, and for $\Delta_3=0$ (planar case),

$$(\Delta\omega_L)^2=\gamma^2\chi_\perp^{-1}g_D\tfrac{3}{10}$$

is regained.[41] The ratio of the transverse NMR shift to longitudinal NMR resonance

$$\frac{(\Delta\omega_L)^2}{\omega_f^2}=\frac{\chi_\|}{\chi_\perp}\frac{\Delta_1^2-\Delta_3^2}{8\Delta_1^2-\frac{1}{2}\Delta_3^2} \tag{5.6}$$

increases from zero (for $H=0$, i.e., $\Delta_1=\Delta_3$) to $\frac{1}{8}(\chi_\|/\chi_\perp)$ for the planar case ($\Delta_3=0$).

After submitting this paper for publication, we became aware of a paper by Schopohl[50] dealing with magnetic field dependence of θ_0 and ω_f in ^{3}He-B. His findings are very similar to ours (he puts $\Delta_2=0$, our Δ_3 is called Δ_2 by him); since he gives a microscopic calculation of

$\Delta_1(H)/\Delta_3(H)$ (for which we have only a rough estimate), his paper is complementary to ours with respect to these topics.

VI. NORMAL MODES

We now turn to a discussion of the hydrodynamic excitations which occur in ^{3}He-B in high magnetic fields. For $\vec{v}^{\,n}=0$ we find an equation of the seventh order for $\omega(k)$ reflecting the seven degrees of freedom described by ρ, σ, M, $\delta\varphi$, δd_i, and $\delta\theta$. In linear order $[\omega(k)\sim k]$ three of the normal mode frequencies are zero. The remaining biquadratic equation for $\omega(k)$ reflects a complicated mixture of fourth sound and (longitudinal) spin waves. This mixing of an excitation in spin space with one in orbit-space in the lowest order of k is due to the presence of the strong magnetic field and vanishes with $H\to 0$. For the velocities $c_{s,f}$ defined by

$$\omega_{s,f}(k)=\pm c_{s,f}(\vec{k}/|\vec{k}|)k\ ,$$

we obtain ($\Delta_2=0$ in order to simplify the expressions)

$$\begin{aligned}
&c_f^2+c_s^2=k^{-2}[\zeta_4(\rho_0k^2-\hat{\rho}_n^2)+\chi_{\|}^{-1}\gamma^2\hat{q}^2]\ ,\\
&c_f^2c_s^2=k^{-4}(\zeta_4\chi_{\|}^{-1}-\zeta_2^2)[\hat{q}^2(\rho_0k^2-\hat{\rho}_n^2)-\hat{\rho}_d^2\hat{\rho}_d^2]\gamma^2\ ,
\end{aligned}$$

with

$$\begin{aligned}
&\hat{\rho}_{n,d}^2=\rho_{\|}^{n,d}k_{\|}^2+\rho_{\perp}^{n,d}k_{\perp}^2\ ,\\
&\hat{q}^2=q_{\|}k_{\|}^2+q_{\perp}k_{\perp}^2+B_1\ ,\\
&k_{\|}=\hat{d}_1^0\cdot\vec{k}\ ,\quad \vec{k}_{\perp}=\hat{d}_x^0\times\vec{k}\ .
\end{aligned}\tag{6.1}$$

The damping of these propagating modes is manifest only in the next order of k $[\mathrm{Im}\omega(k)\sim k^2]$. For $H\to 0$ spin waves and fourth sound decouple and the velocities c_f^2 and c_s^2 take the values already given in Refs. 6 and 7. For $k\to 0$ the spin-wave frequency has a gap $\omega_f^2=\chi_{\|}^{-1}\gamma^2B_1$ (longitudinal spin resonance) while the second-sound frequency tends to zero.

For the remaining modes with $\omega(k)\sim k^2$ we obtain one diffuse mode

$$\omega(k)=ik^2E(\vec{k}/k)\tag{6.2}$$

and one pair of orbit "waves"

$$\begin{aligned}
\omega_{0w}(k)={}&\frac{i}{2}\eta(\hat{K}_1^2+\hat{K}_2^2-\hat{B}^2)\\
&\pm\tfrac{1}{2}[4\delta_2^2\hat{K}_1^2(\hat{K}_2^2-\hat{B}^2)\\
&\qquad-\eta^2(\hat{K}_1^2-\hat{K}_2^2+\hat{B}^2)^2]^{1/2}\ ,
\end{aligned}\tag{6.3}$$

where

$$E(\vec{k}/k)=k^{-2}(\zeta_4\chi_{\|}^{-1}-\zeta_2^2)^{-1}(\det A)$$

with

$$A=\begin{vmatrix}\zeta_2\hat{\kappa}'^2+\zeta_3\hat{\kappa}^2 & \zeta_4 & \zeta_2\\ \zeta_1\hat{\kappa}'^2+T_0C_v^{-1}\hat{\kappa}^2 & \zeta_3 & \zeta_1\\ \chi_{\|}^{-1}\hat{\kappa}'^2+\zeta_1\hat{\kappa}^2 & \zeta_2 & \chi_{\|}^{-1}\end{vmatrix}\ ,$$

and where

$$\hat{B}^2=\frac{\hat{C}_1^2\hat{C}_2^2}{\hat{\rho}_s^2}+\hat{C}_2^2\left[\frac{\hat{\rho}_d^2}{\hat{\rho}_s^2}+\frac{\hat{C}_2^2-\hat{C}_1^2}{\beta_1\hat{C}_2^2}\right]\frac{\beta_1^{-1}\hat{\rho}_s^2(\hat{C}_1^2-\hat{C}_2^2)-\hat{C}_1^2(\hat{\rho}_d^2-\beta_1\hat{q}^2)}{\hat{\rho}_d^2(\hat{\rho}_d^2-\beta_1\hat{q}^2)-\hat{\rho}_s^2\hat{q}^2}\ .$$

Thereby,

$$\begin{aligned}
&\hat{C}_1^2=k_{\|}k_{\perp}(C_{\|}^{(1)}-C_{\perp}^{(1)})\ ,\\
&\hat{C}_2^2=k_{\|}k_{\perp}(C_{\|}^{(2)}-C_{\perp}^{(2)})\ ,\\
&\hat{K}_1^2=K_1k_{\perp}^2+K_3k_{\|}^2+B_2\ ,\\
&\hat{K}_2^2=K_2k_{\perp}^2+K_3k_{\|}^2+B_2\ ,\\
&\hat{\rho}_{s,d}^2=\rho_{\|}^{s,d}k_{\|}^2+\rho_{\perp}^{s,d}k_{\perp}^2\ ,\\
&\hat{q}^2=q_{\|}k_{\|}^2+q_{\perp}k_{\perp}^2+B_1\ ,\\
&\hat{\kappa}^{2(1)}=\kappa_{\|}^{(1)}k_{\|}^2+\kappa_{\perp}^{(1)}k_{\perp}^2\ .
\end{aligned}$$

The diffusive mode involves mainly $\delta\sigma$ and δM, while orbit "waves" are built up by δd_i, $\delta\varphi$, and $\delta\theta$. For $k\to 0$ the orbit waves have a gap and are damped due to the magnetic dipole energy

$$\omega_{0w}=B_2(i\eta\pm\delta_2)\ .$$

Since we have not taken into account the transverse magnetization $\hat{H}\times\vec{m}$ as variables,[51] the (shifted) Larmor precession discussed in Sec. V does not occur in our list of normal modes. For the same reason, the direction $\hat{d}^0=\hat{H}^0$ is held fixed, and only the dynamics of δd_i is considered. This is plausible since the Zeeman energy is much greater than the magnetic dipole energy. Note that in the A phase a similar adiabatic elimination procedure leads to the conclusion that the orbit waves acquire no gap in contrast to what is found here.

In the most general case $\vec{v}^{\,n}\neq 0$ there are at first three pairs of propagating modes built up by div $\vec{g}$, ρ, σ, T, $\delta\varphi$, and $\delta\theta$ with soundlike frequency spectra

$$\omega=\pm c(\vec{k}/k)k+iD(\vec{k}/k)k^2\ .$$

These modes are a mixing of first-sound, second-sound, and longitudinal spin waves. In the linear order of k neglecting the static coupling between δT and $\delta\rho$ (i.e., $\zeta_3=0$, $C_p=C_v$) and between δh and $\delta\rho$ (i.e., $\zeta_2=0$ or no magnetostriction) we find for the velocities

$$c_1^2=\left[\frac{\partial p}{\partial\rho}\right]_{\sigma,M}\tag{6.4}$$

and

$$[k^2(\hat{c}_2^2+\hat{c}_s^2)=\hat{\rho}_1^2\left[\chi^{-1}\left[\frac{m_0}{\rho_0}\right]^2+2s_0\zeta_1\frac{m_0}{\rho_0}+s_0^2T_0c_v^{-1}\right]$$
$$+\hat{\rho}_2^2 2\gamma\left[-\chi^{-1}\frac{m_0}{\rho_0}-s_0\zeta_1\right]+\hat{\rho}_3^2\gamma\ ,$$

$$k^4\hat{c}_s^2\hat{c}_2^2=(\hat{\rho}_2^2\hat{\rho}_2^2-\hat{\rho}_1^2\hat{\rho}_3^2)(\zeta_1^2-T_0c_v^{-1}\chi^{-1})\gamma^2s_0^2\ ,$$

where

$$\hat{\rho}_1^2=\left[\rho_{\|}^s+\frac{(\rho_{\|}^s)^2}{\rho_{\|}^d}\right]k_{\|}^2+\left[\rho_{\perp}^s+\frac{(\rho_{\perp}^s)^2}{\rho_{\perp}^d}\right]k_{\perp}^2\ ,$$

$$\hat{\rho}_2^2=\left[\rho_{\|}^d+\frac{\rho_{\|}^s\rho_{\|}^d}{\rho_{\|}^n}\right]k_{\|}^2+\left[\rho_{\perp}^d+\frac{\rho_{\perp}^s\rho_{\perp}^d}{\rho_{\perp}^n}\right]k_{\perp}^2\ ,$$

$$\hat{\rho}_3^2=\left[q_{\|}-\frac{(\rho_{\|}^d)^2}{\rho_{\|}^n}\right]k_{\|}^2+\left[q_{\perp}-\frac{(\rho_{\perp}^d)^2}{\rho_{\perp}^n}\right]k_{\perp}^2+B_1\ .$$

The remaining four modes, involving $\vec{k}\times\vec{g}$, but also $\delta\varphi$, δd_i, and $\delta\theta$, have a spectrum of the form

$$\omega(k)=\pm(A^2-B^2)^{1/2}k^2+iCk^2\ , \tag{6.5}$$

where $A(\vec{k}/k)$ contains the reversible transport parameters $\delta_{1,2}$ and $B(\vec{k}/k), C(\vec{k}/k)$, the irreversible ones (e.g., ν_α, $\xi_\perp$, η_1^8). These modes are either propagating ($A^2>B^2$) and damped in the same order of k or they are overdamped ($B^2>A^2$), i.e., diffusive. The structure of this spectrum reminds us of the transverse momentum order-parameter spectrum in nematic liquid crystals. It should be stressed that for $H\to 0$ these modes are in any case diffusive, since $A\sim\rho_\perp^d\to 0$ in that limit. In addition, the parameters $\delta_{1,2}$ do not enter the normal mode spectra for $H\to 0$.

VII. EXPERIMENTS

As has been discussed in the previous sections, there are numerous new couplings between spin-space variables and orbit-space variables in ^{3}He-B if a strong magnetic field is applied. In this section we will discuss how these new effects are accessible to experiments. In principle, the normal mode spectra given in Sec. VI contain all the necessary information. However, by the very complicated structure of those spectra it is very unlikely that it will be possible to verify all their details soon, and to measure, thereby, all phenomenological parameters. On the other hand, it should be experimentally possible to verify the existence of the density ρ^d (and, thus, of the velocity $\nabla_i\theta$) because of the mixing of fourth (second) sound with spin waves induced by ρ^d. It should especially be possible to excite fourth (second) sound by changes of the magnetization or spin waves by changes of the temperature. This

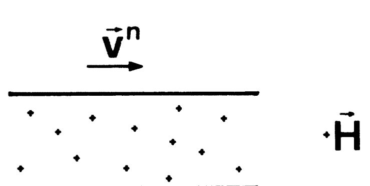

FIG. 1. Perturbations to measure $\rho_\perp^n$, $\vec{v}^{\,n}\perp\hat{H}$, cf. Eq. (A10).

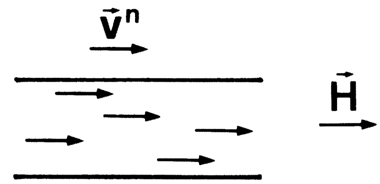

FIG. 2. Perturbations to measure $\rho_{\|}^n$, $\vec{v}^{\,n}\|\hat{H}$, cf. Eq. (A13).

would prove the existence of $\rho^d\neq 0$.

The coupling between spin space and real space in the reversible currents of ^{3}He-B in high magnetic fields [Eq. (4.2)] can—at least in principle—be measured by quasistatic experiments (magnetic fountain effect, magnetothermal effect, thermal fountain effect), which are quite similar to those described for the A phase in high magnetic fields.[36] The most easily accessible results we have given are related to NMR experiments. In longitudinal NMR experiments the resonance frequency increases with increasing magnetic field, while for transverse NMR experiments a shift from the Larmor frequency occurs in high magnetic fields. Both effects are based on the anisotropy of the gap due to $\Delta_1\neq\Delta_3$. For $\Delta_3=0.9\Delta_1(0.5\Delta_1)$ (Ref. 52) the longitudinal NMR resonance will be enhanced at 2% (11%) and the transverse NMR shift will be 1% (3%) of the longitudinal resonance frequency.

As is discussed in Appendix A the static susceptibilities, involving the linear momentum density and the hydrodynamic variables characterizing the broken symmetries, become quite complicated. However, it is possible to propose experiments which can measure the normal density parallel and orthogonal to the applied magnetic field as well as the total density. Thereby, the uniaxiality of orbit space in ^{3}He-B in high magnetic fields can be established experimentally. We have sketched the different configurations in Figs. (1)—(3).

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APPENDIX A: MICROSCOPIC DESCRIPTION

In this Appendix we will make contact with microscopic theory and thereby express the phenomenological pa-

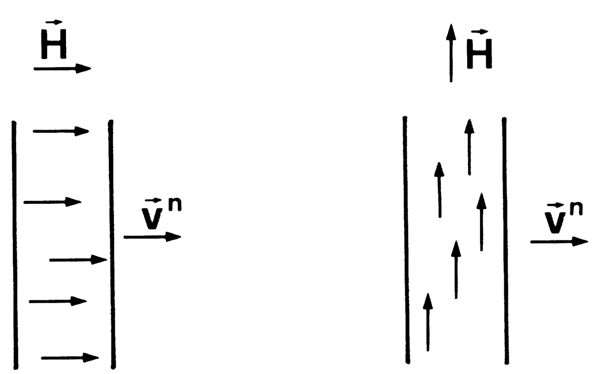

FIG. 3. Perturbations to measure ρ, cf. Eqs. (A14) and (A15).

rameters by correlation functions and response functions, which can be calculated (at least approximately) by various many-body techniques. For a detailed exposition of the method see Refs. 11 and 28.

For the operator of the order parameter we have[11]

$$A_{ij}^{\rm op}(\vec{x}) = \int_0^\infty dr\, r^2 g(r)\frac{3}{4\pi}\int d\Omega_r \psi_\alpha\left[\vec{x}-\frac{\vec{r}}{2}\right](\sigma_i\sigma_2)_{\alpha\beta} \times \frac{r_j}{|\vec{r}|}\psi_\beta\left[\vec{x}+\frac{\vec{r}}{2}\right], \tag{A1}$$

where ψ_α are bare fermion annihilation operators and $g(r)$ assures normalization of $A_{ij}^{\rm op}(\vec{x})$. The equilibrium expectation value[53] of $A_{ij}^{\rm op}(\vec{x})$ is related to the order parameter n_{ij}^0 [defined by (2.6)] by

$$A_{ij}^0 = n_{ij}^0 e^{i\varphi}[\Delta_3^2+2(\Delta_1^2+\Delta_2^2)]^{-1/2}.$$

The variables characterizing broken symmetries have the operator representation

$$\delta\theta^{\rm op}(\vec{x}) = \frac{N}{4}\hat{H}_i\epsilon_{ijk}[A_{jl}^{*0}A_{kl}^{\rm op}(\vec{x})+A_{jl}^0 A_{kl}^{\dagger \rm op}(\vec{x})], \tag{A2}$$

$$\delta\varphi^{\rm op}(\vec{x}) = \frac{1}{2i}[A_{jl}^{*0}A_{jl}^{\rm op}(\vec{x})-A_{jl}^0 A_{jl}^{\dagger \rm op}(\vec{x})], \tag{A3}$$

$$\delta d_i^{\rm op}(\vec{x}) = \frac{N}{4}\hat{d}_k^0[A_{\alpha i}^{\dagger \rm op}(\vec{x})A_{\alpha k}^0 - A_{\alpha k}^{\dagger \rm op}(\vec{x})A_{\alpha i}^0 + A_{\alpha i}^{\rm op}(\vec{x})A_{\alpha k}^{*0} - A_{\alpha k}^{\rm op}(\vec{x})A_{\alpha i}^{*0}],$$

with

$$N = [\Delta_3^2+2(\Delta_1^2+\Delta_2^2)](\Delta_1^2+\Delta_2^2)^{-1}.$$

For the conserved variables the operator representation can be found, e.g., in Ref. 11.

The instantaneous collisionless response of the hydrodynamic variables to external perturbations is contained in the frequency matrix $\omega_{ij}(k)$, which can be evaluated exactly because it contains equal-time commutators. We obtain for the elements involving $\delta\theta$ and $\delta\varphi$

$$\begin{aligned}
&\omega_{M\delta\varphi} = -i\gamma\beta_1 2,\\
&\omega_{g^2\delta\varphi} = -2i\beta_1 k_\perp,\\
&\omega_{\rho\delta\varphi} = 2i,\\
&\omega_{\rho\delta\theta} = -i\,\tilde{\beta}_1 2,\\
&\omega_{g^2\delta\theta} = -2ik_\perp,\\
&\omega_{M\delta\theta} = 2i\gamma.
\end{aligned} \tag{A4}$$

For those not involving $\delta\varphi$ and $\delta\theta$ we find for the elements of the frequency matrix contributing to hydrodynamics to order k^2

$$\begin{aligned}
&\omega_{Mg^1} = m_0 k_\perp,\\
&\omega_{Mg^3} = m_0 k_{||},\\
&\omega_{\rho g^1} = \rho k_\perp,\\
&\omega_{\rho g^3} = \rho k_{||},
\end{aligned} \tag{A5}$$

where $k_{||}$ and $k_\perp$ are given by $k_{||}=\vec{k}\cdot\hat{d}^0$ and

$$k_\perp = \frac{[\vec{k}-(\vec{k}\cdot\hat{d}^0)\hat{d}^0]}{|\vec{k}-(\vec{k}\cdot\hat{d}^0)\hat{d}^0|}\cdot\vec{k}, \tag{A6}$$

and g^1, g^2, and g^3 are defined as in Eq. (5.7) of Ref. 11 where $\hat{l}^0$ has to be replaced by $\hat{d}^0$, reflecting again the fact that the preferred direction in real space ($\hat{d}^0$) for ^{3}He-B in high magnetic fields is different from the preferred direction ($\hat{l}^0$) in real space in the A phase without external magnetic field[3,11] or in the presence of an external magnetic field.

The noninstantaneous, collision-dominated response is contained in the memory matrix σ_{ij}.[28,11] Its reversible part yields the following contributions to hydrodynamics (up to now only hydrodynamic systems with broken rotational symmetries in real space have obtained reversible contributions from σ_{ij}):

$$\begin{aligned}
&\sigma^{||}_{g^1g^2} = 2\varphi_1 k_\perp^2+\varphi_2 k_{||}^2,\\
&\sigma^{||}_{g^3g^2} = \varphi_2 k_\perp k_{||},\\
&\sigma^{||}_{g^2\delta\varphi} = \tilde{\lambda}k_\perp,\\
&\sigma^{||}_{g^2\delta\theta} = \tilde{\delta}_1 k_\perp.
\end{aligned} \tag{A7}$$

For the irreversible contributions of the memory matrix we have to order k^2:

$$\begin{aligned}
&\sigma^1_{MM} = \mu_{||}k_{||}^2+\mu_\perp k_\perp^2,\\
&\sigma^1_{M\sigma} = \kappa^1_{||}k_{||}^2+\kappa^1_\perp k_\perp^2,\\
&\sigma^1_{\sigma\sigma} = \kappa_{||}k_{||}^2+\kappa_\perp k_\perp^2,\\
&\sigma^1_{g^1g^1} = (\nu_2+\nu_4)k_\perp^2+\nu_3 k_{||}^2,\\
&\sigma^1_{g^2g^2} = \nu_2 k_\perp^2+\nu_3 k_{||}^2,\\
&\sigma^1_{g^3g^3} = \nu_3 k_\perp^2+(2\nu_1+2\nu_5+\nu_2-\nu_4)k_{||}^2,\\
&\sigma^1_{g^1g^3} = (\nu_3+\nu_5)k_{||}k_\perp,\\
&\sigma^1_{\delta\varphi g^1} = \xi_\perp k_\perp,\\
&\sigma^1_{\delta\varphi g^3} = \xi_{||}k_{||},\\
&\sigma^1_{\delta\varphi\delta\varphi} = \zeta,\\
&\sigma^1_{\delta\theta\delta\theta} = \nu,\\
&\sigma^1_{\delta\theta\delta\varphi} = \eta_7,\\
&\sigma^1_{\delta\theta g^1} = \eta_\perp^8 k_\perp,\\
&\sigma^1_{\delta\theta g^3} = \eta_{||}^8 k_{||}.
\end{aligned} \tag{A8}$$

As is well known the nonvanishing irreversible elements of the memory matrix σ^1 can be related to the corresponding absorptive response functions by Kubo relations. These have the same structure as in the A phase, reflecting once more the fact that the B phase in high magnetic fields is uniaxial, contrary to the B phase without external field which is isotropic in real space. For the reversible transport parameters analogous relations can be derived in the same way as in the A phase.

Finally, we consider briefly the matrix of static susceptibilities. For the longitudinal magnetization density, the density, and the entropy density, the same discussion as in the A_1 phase and the A phase applies and we have

$$\chi_{\rho\rho}^{-1}=\zeta_4\ ,\ \ \chi_{\rho\sigma}^{-1}=\zeta_3\rho\ ,\ \ \chi_{\sigma\sigma}^{-1}=TC_v^{-1}\rho^2\ ,$$
$$\chi_{\sigma M}^{-1}=\zeta_1\rho\ ,\ \ \chi_{MM}^{-1}=\chi_{\parallel}^{-1}\ ,\ \ \chi_{M\rho}^{-1}=\zeta_2\ . \tag{A9}$$

The other variables come in two different groups. Because of its behavior under parity and time reversal g^2 is completely decoupled from g^1, g^3, $\delta\varphi$, and $\delta\theta$ and we have

$$\chi_{g^2g^2}=\rho_\perp^n\ , \tag{A10}$$

which leads to $\rho_\perp^n=\lim_{k\to 0}\chi_{g^2g^2}(\vec{k}\,)$.

The second group leads to static correlation functions which become complicated for $k\to 0$ because two hydrodynamic variables characterizing broken symmetries are in this group. First we consider $\vec{v}^{\,n}=0$. The static susceptibilities of δn and $\delta\theta$ should diverge as k^{-2} for $k\to 0$. That they diverge at least as k^{-2} can be shown by an application of the Bogoliubov inequality. Therefore, we discuss the inverse susceptibilities first.

Axial symmetry requires the structure

$$\chi_{\delta\varphi\delta\varphi}^{-1}=[\rho_\parallel^s-\beta_1(\rho_\parallel^d-\beta_1 q_\parallel)]k_\parallel^2$$
$$+[\rho_\perp^s-\beta_1(\rho_\perp^d-\beta_1 q_\perp)]k_\perp^2\ ,$$
$$\chi_{\delta\theta\delta\theta}^{-1}=q_\parallel k_\parallel^2+q_\perp k_\perp^2\ , \tag{A11}$$
$$\chi_{\delta\theta\delta\varphi}^{-1}=(\rho_\parallel^d-\beta_1 q_\parallel)k_\parallel^2+(\rho_\perp^d-\beta_1 q_\perp)k_\perp^2\ .$$

We have chosen the notation to comply with Eqs. (3.8). For the corresponding susceptibilities, which have to be identical for $\vec{v}^{\,n}=0$ and $\vec{v}^{\,n}\neq 0$, we have $[\Delta=\det(\chi_{ij}^{-1})$, $i,j\equiv\delta\varphi,\delta\theta]$

$$\chi_{\delta\varphi\delta\varphi}=\frac{1}{\Delta}(q_\parallel k_\parallel^2+q_\perp k_\perp^2)\ ,$$
$$\chi_{\delta\varphi\delta\theta}=-\frac{1}{\Delta}[(\rho_\parallel^d-\beta_1 q_\parallel)k_\parallel^2+(\rho_\perp^d-\beta_1 q_\perp)k_\perp^2]\ , \tag{A12}$$
$$\chi_{\delta\theta\delta\theta}=\frac{1}{\Delta}\{[\rho_\parallel^s-\beta_1(\rho_\parallel^d-\beta_1 q_\parallel)]k_\parallel^2$$
$$+[\rho_\perp^s-\beta_1(\rho_\perp^d-\beta_1 q_\perp)]k_\perp^2\}\ .$$

If we allow for $\vec{v}^{\,n}\neq 0$ the corresponding static correlation functions involving g^1 and g^3 become rather complicated. Nevertheless, we can deduce some simple relations for $\chi_{g^3g^3}$:

$$\rho_\parallel^n=\lim_{k_\perp\to 0}\lim_{k_\parallel\to 0}\hat{d}_i\hat{d}_j\chi_{g^ig^j}(\vec{k}\,)\ , \tag{A13}$$

$$\rho=\rho_\parallel^n+\rho_\parallel^s+\beta_1\rho_\parallel^d=\lim_{k_\parallel\to 0}\lim_{k_\perp\to 0}\hat{d}_i\hat{d}_j\chi_{g^ig^j}(\vec{k}\,)\ , \tag{A14}$$

$$\rho=\rho_\perp^n+\rho_\perp^s+\beta_1\rho_\perp^d=\lim_{k_\perp\to 0}\lim_{k_\parallel\to 0}\frac{k_ik_j}{k^2}\chi_{g^ig^j}(\vec{k}\,)\ . \tag{A15}$$

APPENDIX B: GALILEAN PROPERTIES OF $\vec{\nabla}\varphi$, $\vec{\nabla}\theta$, AND (curl $\hat{d}$)

Infinitesimal Galilean transformations with velocity $\delta\vec{U}$ are generated by the operator ($\hbar/2m=1$),

$$g=\frac{i}{2}\delta\vec{U}\cdot\int d^3x\,\psi_\alpha^\dagger(\vec{x}\,)\psi_\alpha(\vec{x}\,)\vec{x}\ . \tag{B1}$$

The order-parameter operator $A_{ij}^{\rm op}$ (A1) transforms under g like[11]

$$[g,A_{ij}^{\rm op}(\vec{x}\,)]=A_{ij}^{\rm op}(\vec{x}\,)e^{-i\vec{x}\cdot\delta\vec{U}}\ . \tag{B2}$$

This immediately yields the Galilean properties of $\delta\theta$, $\delta\varphi$, and δd_i, (A3),

$$\langle[g,\vec{\nabla}\varphi]\rangle=-\delta\vec{U}\ ,$$
$$\langle[g,\vec{\nabla}\theta]\rangle=\beta_1\delta\vec{U}\ , \tag{B3}$$
$$\langle[g,\nabla_i d_j]\rangle=0\ .$$

Thus $\vec{\nabla}\varphi$ is a true Galilean velocity as in all other phases of ^{3}He. On the contrary, $\vec{\nabla}\theta$ is only partially a Galilean velocity since $0<\beta_1<1$, while (curl $\hat{d}$)$_i$ is Galilean invariant. For $H\to 0$, $\vec{\nabla}\theta$ becomes Galilean invariant too. For the linear combinations $\delta\tilde{\varphi}$ and $\delta\tilde{\theta}$ introduced by Eq. (2.9) the Galilean properties are simply

$$\vec{\nabla}\tilde{\varphi}\to\vec{\nabla}\tilde{\varphi}-\delta\vec{U}$$

and

$$\vec{\nabla}\tilde{\theta}\to\vec{\nabla}\tilde{\theta}\ .$$

APPENDIX C: NONLINEAR THEORY

Recently, the present authors have given[40] a nonlinear hydrodynamic theory for the superfluid phases of ^{3}He including the B phase in high magnetic field. Variables which describe rotations in spin or orbit space do not commute in a nonlinear theory giving rise to the so-called "Mermin-Ho" relation. In Ref. 40 these relations are given somewhat implicitly and we will state them here explicitly:

$$\partial_1(\partial_2\varphi)-\partial_2(\partial_1\varphi)=\tilde{\beta}_1\hat{H}\cdot[(\partial_1\vec{d}\,)\times(\partial_2\vec{d}\,)]\ ,$$
$$\partial_1(\partial_2\theta)-\partial_2(\partial_1\theta)=\hat{d}\cdot[(\partial_1\vec{d}\,)\times(\partial_2\vec{d}\,)]\ , \tag{C1}$$
$$\partial_1(\partial_2\vec{d}\,)-\partial_2(\partial_1\vec{d}\,)=\hat{d}\times[(\partial_1\vec{d}\,)(\partial_2\theta)-(\partial_2\vec{d}\,)(\partial_1\theta)]\ ,$$

where $\tilde{\beta}_1$ is given in Eq. (2.7).

For the nonlinear terms in the equations of state and in the reversible and irreversible currents there is a great structural similarity between ^{3}He-A and -B (both) in high magnetic fields. By the simple replacement $\delta n\to\delta\theta$, $\delta l_i\to\delta d_i$, $l_i^0\to d_i^0$ in the nonlinear terms of Eqs. (3.18)–(3.24) of Ref. 40, one obtains the corresponding nonlinear terms for ^{3}He-B in high magnetic fields, which we, therefore, must not write down explicitly here. In Sec. IV C of Ref. 40 the variables δd_i were omitted.

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[1]J. C. Wheatley, Rev. Mod. Phys. 47, 415 (1975).
[2]A. J. Leggett, Rev. Mod. Phys. 47, 331 (1975).
[3]R. Graham, Phys. Rev. Lett. 33, 1431 (1974).
[4]M. Liu, Phys. Rev. B 13, 4174 (1976).
[5]R. Graham and H. Pleiner, Phys. Rev. Lett. 34, 792 (1975).
[6]R. Graham and H. Pleiner, J. Phys. C 9, 279 (1976).
[7]R. Graham and H. Pleiner, in *Low Temperature Physics LT-14, Proceedings LT-14, Helsinki, 1975,* edited by M. Krusius and M. Vuorio (North-Holland, Amsterdam, 1975), Vol. 1, p. 1.
[8]H. Pleiner and R. Graham, J. Phys. C 9, 4109 (1976).
[9]H. Pleiner, J. Phys. C 10, 2337 (1977).
[10]H. Pleiner, J. Phys. C 10, 4241 (1977).
[11]H. Brand, M. Dörfle, and R. Graham, Ann. Phys. (NY) 119, 434 (1979).
[12]J. M. Delrieu, J. Phys. (Paris) 38, L127 (1977).
[13]D. Lhuillier, J. Phys. (Paris) 38, L121 (1977).
[14]C. R. Hu and W. M. Saslow, Phys. Rev. Lett. 38, 605 (1977).
[15]H. E. Hall and J. R. Hook, J. Phys. C 10, L91 (1977).
[16]N. D. Mermin and T. L. Ho, Phys. Rev. Lett. 36, 594 (1976).
[17]M. Liu and M. C. Cross, Phys. Rev. Lett. 41, 250 (1978).
[18]M. Liu, Phys. Rev. Lett. 43, 1740 (1979).
[19]L. P. Kadanoff and P. C. Martin, Ann. Phys. (NY) 24, 419 (1963).
[20]P. C. Hohenberg and P. C. Martin, Ann. Phys. (NY) 34, 291 (1965).
[21]I. M. Khalatnikov, *Introduction to the Theory of Superfluidity* (Benjamin, New York, 1965).
[22]B. I. Halperin and P. Hohenberg, Phys. Rev. 188, 898 (1969).
[23]B. I. Halperin and W. M. Saslow, Phys. Rev. B 16, 2154 (1977).
[24]P. D. Fleming III and C. Cohen, Phys. Rev. B 13, 500 (1976).
[25]H. Brand and H. Pleiner, J. Phys. (Paris) 41, 553 (1980).
[26]D. Forster, T. C. Lubensky, P. C. Martin, J. Swift, and P. S. Pershan, Phys. Rev. Lett. 26, 1016 (1971).
[27]D. Forster, Ann. Phys. (NY) 84, 505 (1974).
[28]D. Forster, *Hydrodynamic Fluctuations, Broken Symmetry and Correlation Functions* (Benjamin, Reading, Mass., 1975).
[29]P. C. Martin, O. Parodi, and P. S. Pershan, Phys. Rev. A 6, 2401 (1972).
[30]F. Jähnig and H. Schmidt, Ann. Phys. (NY) 71, 129 (1972).
[31]T. C. Lubensky, Phys. Rev. A 6, 452 (1972).
[32]T. C. Lubensky, Mol. Cryst. Liq. Cryst. 23, 99 (1973).
[33]F. Prost and N. A. Clark, in *Liquid Crystals of One- and Two-Dimensional Order and Their Applications,* edited by W. Helfrich and G. Heppke (Springer, New York, 1980).
[34]F. Prost and N. A. Clark, in *Liquid Crystals,* edited by S. Chandrasekhar (Heyden, London, 1980).
[35]H. Brand and H. Pleiner, Phys. Rev. A 24, 2777 (1981).
[36]H. Brand and H. Pleiner, J. Phys. C 14, 97 (1981).
[37]H. Brand and H. Pleiner, J. Phys. (Paris) 43, 369 (1982).
[38]M. Liu, Z. Phys. B 40, 175 (1980).
[39]W. M. Saslow and C.-R. Hu, Phys. Rev. B 23, 4523 (1981).
[40]H. Pleiner and H. Brand, Phys. Rev. B 24, 6430 (1981).
[41]A. J. Leggett, J. Phys. C 6, 3187 (1973).
[42]R. C. Richardson and D. M. Lee, in *Collective Properties of Physical Systems,* edited by B. Lundqvist and S. Lundqvist (Academic, New York, 1974).
[43]A. Fetter, in *Quantum Statistics and the Many-Body Problem,* edited by S. B. Trickey, W. P. Kirk, and J. W. Dufty (Plenum, New York, 1975).
[44]A similar treatment with respect to the magnetic Zeeman energy was performed in Ref. 6, thus referring to the *B* phase in small or vanishing magnetic fields.
[45]R. Combescot, J. Phys. C 14, 4765 (1981).
[46]H. Pleiner and H. Brand, J. Phys. (Paris) Lett. 41, L491 (1981).
[47]H. Brand and H. Pleiner, Phys. Rev. B 23, 155 (1981).
[48]F.-J. Belinfante, Physica (Utrecht) 6, 887 (1939).
[49]Since we are dealing with infinitesimal angles (corresponding to linear theory), there are no problems with noncommutivity of finite angles. For the nonlinear theory, cf. Ref. 40 and Appendix C.
[50]N. Schopohl, J. Low Temp. Phys. 49, 347 (1982).
[51]Since we did not consider the transverse magnetization in Secs. III and IV, the result (5.5) cannot be obtained by inserting (5.3) into (4.1)–(4.3).
[52]Adopting the estimate $1-(\Delta_3/\Delta_0)^2 \approx (\gamma \hbar H/\Delta_0)^2$ of Ref. 2, the ratio $\Delta_3/\Delta_1 = 0.9$ (0.5) is reached for $H \approx 1$ kG (4 kG). For a detailed discussion, cf. Ref. 50.
[53]Expectation values (as indicated by angular brackets) are performed with a restricted ensemble (cf. Ref. 11).