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Anomalous quantum Hall effect—origin of fractional Hall steps

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It is shown that properly accounting for the magnetic compressibility of the two-dimensional electron system can lead to a monotonic quantum Hall coefficient with rational step heights.

Von Klitzing, Dorda, and Pepper[1] showed that the Hall effect in a Si inversion layer is quantized at low temperatures into steps which are very accurately *integer* multiples of e^2/h. The steps occur when the Fermi level is near a gap between Landau levels, the integer giving the number of filled levels. Tsui, Stormer, and Gossard[2] find in $GaAs_xGa_{1-x}As$ superlattices that, when less than one Landau level is occupied, Hall-effect steps occur when the filling factor $\nu = nch/eB$ has simple fractional values $(\frac{1}{3},\frac{2}{3})$, and that the step height equals $\nu e^2/h$. The origin of these *fractional* Hall steps is an outstanding theoretical problem. Tsui, Stormer, and Gossard suggest that the steps are associated with new gaps, possibly related to a Wigner crystallization, but previous theoretical studies[3–6] have suggested that whenever the Fermi level falls in a gap, the Hall effect will be quantized in *integer* multiples of e^2/h. On the contrary, we show that fractional Hall steps arise naturally if the compressibility of the Wigner lattice is taken into account, as suggested by Widom.[7]

For a system of electrons in the presence of both a crystalline lattice and a magnetic field, energy gaps occur whenever[8]

$$n\Omega = M + N\phi/\phi_0 \ ; \tag{1}$$

here Ω is the area of a unit cell, $\phi = B\Omega$ is the flux, $\phi_0 = hc/e$ is the magnetic flux quantum, and M and N are integers. References 4–6 show that, within these gaps, $\sigma_H = Ne^2/h$ *if* Ω *is fixed.* Since these Hall steps are all integral in height, and vary nonmonotonically with ν, this result has been taken[6,9] as evidence that the Wigner crystal cannot explain the observations of Refs. 1 and 2. However, Ω *is not constant* in a Wigner crystal; instead, $n\Omega = \text{const}$, usually taken to be 1.

The correct result is most easily seen by following Streda's derivation.[5] Both the ordinary and the anomalous quantum Hall effects (QHE) can be derived from the Streda-Widom formula[5,10,11]:

$$\sigma_H = ec\left(\frac{\partial n}{\partial B}\right)_{\mu=E_F} \ . \tag{2}$$

If the Fermi level lies in a gap between Landau levels, the ordinary QHE follows: $\partial n/\partial B = n/B = m/\phi_0$, when m is the number of filled Landau levels. Hence $\sigma_H = me^2/h$.

In a superlattice gap *with fixed* Ω, differentiation of Eq. (1) yields $\sigma_H = Ne^2/h$, as discussed above. But if we fix $n\Omega$, differentiation of Eq. (1) yields $\partial\phi/\partial B = 0$ in a gap (if $N \neq 0$). In turn, this means $\partial n/\partial B = n/B$, or

$$\sigma_H = \nu e^2/h \ . \tag{3}$$

This is the correct result for a Wigner lattice, and is consistent with experimental observations.

There are, however, two theoretical objections to identifying the Wigner lattice with the observed phenomena:

(i) The Wigner lattice also has a field-independent gap, given by Eq. (1) with $M=1$, $N=0$, and this is the largest gap in the spectrum.[12] If the Fermi level lies within this gap (as assumed in Ref. 12), it is easy to show that Eq. (3) still holds, but the Hall effect is not quantized. It may, however, be possible to lower the energy of the system by taking $n\Omega < 1$ (a charge-density-wave-like state),[13] for ϕ/ϕ_0 close to a low-order rational number (e.g., close to $\frac{1}{3}$). In this case, the Fermi level would miss the $M=1$, $N=0$ gap, but still pass through an $N \neq 0$ gap.

(ii) Recent calculations[9,14] suggest that the ground state of the electrons in a magnetic field is not a Wigner crystal. Laughlin[14] has found a variational ground state of lower energy than the Wigner lattice of Ref. 12. This ground state is a quantum fluid and the fractional Hall steps are explained as being due to fractionally charged excitations of the system.

It should be pointed out that the only property of the Wigner lattice that we use is that, within a gap, $n/B = \text{const}$ as B changes. Any system which has this property will satisfy Eq. (3). But the Streda-Widom formula is valid for any gap in the spectrum of a system of interacting electrons, and it says that *if there is a Hall step* corresponding to the gap, then n/B is constant as long as the Fermi energy lies in the gap. Presumably, it is also true in Laughlin's treatment that, within the ground state, the flux per electron, B/n, is constant.

For a system of noninteracting electrons, the Hall coefficient must[3,4] be quantized in integer multiples of e^2/h. We have shown that this result is not valid for interacting electrons. We have also become aware of a paper by Halperin[15] which reaches similar conclusions.

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