Physical interpretation of weak localization: A time-of-flight experiment with conduction electrons

Gerd Bergmann
Institut für Festkörperforschung der Kernforschungsanlage Jülich, D-5170 Jülich, West Germany
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The resistance of two-dimensional electron systems such as thin disordered films shows deviations from Boltzmann theory, which are caused by quantum corrections and are called "weak localization." Theoretically, weak localization is originated by the Langer-Neal graph in the Kubo formalism. In the present paper this graph is translated into a transparent physical picture. It represents an interference experiment with conduction electrons split into pairs of waves interfering in the backscattering direction. The intensity of the interference (integrated over the time) can be easily measured by the resistance of the film. A simple derivation for this quantum correction to the resistance is given. A magnetic field introduces a magnetic phase shift in the electronic wave function and suppresses the interference after a "flight" time proportional to $1/H$. Therefore, the application of a magnetic field allows observation of the fate of the electron as a function of time. Spin-orbit coupling rotates the spin of the electrons and yields an observable destructive interference, thereby demonstrating the change of sign of the electron-spin function by rotation. Magnetic impurities destroy the coherence of the phase. Therefore, with magnetoresistance measurements one can determine the inelastic lifetime, the spin-orbit coupling time, and the magnetic scattering time of the conduction electrons.

I. INTRODUCTION

The resistance of a disordered two-dimensional electron system like a thin film shows interesting anomalies at low temperature. This was first pointed out by Abrahams *et al.*,[1] who concluded that a two-dimensional conductor with a finite concentration of defects becomes an insulator at $T=0$ K. Anderson *et al.*[2] and Gor'kov *et al.*[3] calculated that at low but finite temperature the conductance is not constant but has a temperature-dependent correction

$$\Delta L = L_{00} p \ln T + \text{const}, \quad L_{00} = e^2/2\pi^2\hbar \; . \tag{1}$$

Here p is of the order of unity (see below) and $L_{00} \approx 80$ $k\Omega^{-1}$ is a conductance which depends only on universal constants. Altshuler *et al.*[4] showed that the weak localization is sensitive to a magnetic field and gives a negative magnetoresistance in a field range where usual orbital effects are negligible, i.e., the product $\omega_c \tau_0 \ll 1$ (ω_c is the cyclotron frequency and τ_0 indicates the elastic lifetime of the conduction electrons). Hikami *et al.*[5] found that the sign of the correction ΔL is changed when the spin-orbit coupling is sufficiently strong. A further consequence is a change in the sign of the magnetoresistance. Prior to the theoretical magnetoresistance calculations, Kawaguchi *et al.*[6] found such an anomalous behavior in Si inversion layers, and the author[7] in thin Pd films. In the latter experiment the destructive influence of paramagnetic impurities was observed. Meanwhile the theoretical predictions for the magnetoresistance of weak localization have been investigated experimentally for MOS inversion layers,[8–14] and for thin films.[15–33] In particular, experiments on quench-condensed metal films show surprising good agreement with the theory.[25,27] However, weak localiza-

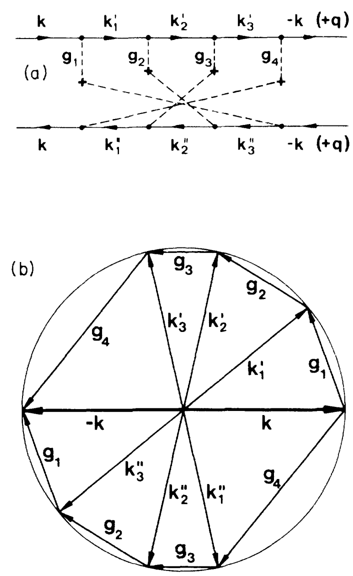

FIG. 1. (a) Fan diagram, introduced by Langer and Neal, which allows calculations of quantum corrections to the conductance within the Kubo formalism. (b) Physical interpretation of the fan diagram in (a). The electron in the eigenstate of momentum k is scattered via two parallel series of intermediate scattering states: $\vec{k}\rightarrow\vec{k}'_1\rightarrow\vec{k}'_2\rightarrow\cdots\rightarrow\vec{k}'_{n-1}\rightarrow\vec{k}'_n=-\vec{k}$ and $\vec{k}\rightarrow\vec{k}''_1\rightarrow\vec{k}''_2\rightarrow\cdots\rightarrow\vec{k}''_{n-1}\rightarrow\vec{k}''_n=-\vec{k}$ into the state $-\vec{k}$. The change of momentum is $\vec{g}_1,\vec{g}_2,\ldots,\vec{g}_{n-1},\vec{g}_n$ for the first series and $\vec{g}_n,\vec{g}_{n-1},\ldots,\vec{g}_2,\vec{g}_1$ for the second one. The amplitudes in the final state $-\vec{k}$ are identical, $A'=A''=A$, and interfere constructively, yielding an echo in backscattering direction which decays as $1/t$ in two dimensions. Only for times longer than the inelastic lifetime τ_i the coherence is lost and the echo disappears.

tion is more than just a new effect in the resistance of low-dimensional systems which can be well described by theory. It offers a new method to measure characteristic times of the conduction electrons such as inelastic lifetime, spin-orbit coupling time, and magnetic scattering time. The physical reason is that weak localization corresponds to a time-of-flight experiment with conduction electrons.

The correction to the "classical" (Boltzmann) resistance is calculated in the Kubo formalism and given by the so-called fan diagram [see Fig. 1(a)]. This fan diagram was first considered by Langer and Neal[34] more than 15 years ago and it is intensively studied for two-dimensional systems in connection with superconducting fluctuations and the Maki-Thomson graph.[35,36] The translation of this electron-hole propagator in terms of a physical picture is not obvious. I want to show in this paper that the fan diagram in the Kubo formalism has a very transparent meaning. It describes a process which is quite identical with a common interference experiment with particle waves. The (integrated) interference intensity is measured by an extremely simple "counter," the resistance of the system. By the application of a magnetic field the interference is strongly modified since it changes the phase of the electronic wave function. As we will see below the magnetic field allows observation of the scattering of the electrons as a function of time. We will come to these conclusions by a one-to-one translation of the fan diagram into a physical picture.

II. THE ECHO OF A SCATTERED CONDUCTION ELECTRON

At low temperature one has to distinguish between two different lifetimes of the conduction electrons, the elastic lifetime τ_0 and the inelastic lifetime τ_i. Here τ_0 is the lifetime of the electron in an eigenstate of momentum, whereas τ_i is the lifetime in an eigenstate of energy. Already at 4 K, the latter can exceed the former by several orders of magnitude. As a consequence of this, an electron in state k can be scattered by the impurities without losing its phase coherence. Owing to the statistical distribution of the impurities, the multiple scattered waves form a chaotic pattern. The usual Boltzmann theory neglects interferences between the scattered partial waves and assumes that the momentum of the electron wave disappears exponentially after the time τ_0 [or τ_{tr} (transport mean free path). In the following consideration we assume s scattering so that τ_0 and τ_{tr} are equal.] This is, however, not quite correct. There is a coherent superposition of the scattered electron wave which results in backscattering of the electron wave and lasts as long as the coherence of the scattered wave is not destroyed.

The correction to the conductance is given by the fan diagram [Fig. 1(a)]. The corresponding scattering process is described in Fig. 1(b) in $\vec{k}$ space. We consider at the time $t=0$ an electron of momentum $\vec{k}$ which has the wave function $\exp(ikr)$. The electron $\vec{k}$ is scattered after the time τ_0 into a state k'_1, after $2\tau_0$ into the state k'_2, etc. There is a finite probability that the electron will be scattered into the state $-\vec{k}$; for example, after n scattering events. This scattering sequence,

$$\vec{k}\to\vec{k}'_1\to\vec{k}'_2\to\cdots\to\vec{k}'_{n-1}\to\vec{k}'_n=-\vec{k}\ , \tag{2a}$$

is drawn in Fig. 1(b) in the $\vec{k}$ space. The momentum transfer is $\vec{g}_1,\vec{g}_2,\ldots,\vec{g}_n$. There is an equal probability for the electron k to be scattered in n steps from the state $\vec{k}$ into $-\vec{k}$ via the sequence

$$\vec{k}\to\vec{k}''_1\to\vec{k}''_2\to\cdots\to\vec{k}''_{n-1}\to\vec{k}''_n=-\vec{k}\ , \tag{2b}$$

where the momentum transfer is $\vec{g}_n,\vec{g}_{n-1},\ldots,\vec{g}_1$. This complementary scattering series has the same changes of momentum in opposite sequence. If the final state is $-\vec{k}$, then the intermediate states for both scattering processes lie symmetric to the origin. The important point is that the amplitude in the final state $-\vec{k}$ is the same for both scattering sequences. This is caused essentially by the proportionality of the final amplitude to the product of the matrix elements, i.e., $\prod V(g_i)$—where $V(g_i)$ is the Fourier component of the scattering potential—and this product is the same for both sequences. Secondly, the transition probability is identical because of the symmetry of the two complementary processes. In addition, the energy of the corresponding intermediate states is the same so that the time-dependent phase changes $(Et/\hbar)$ are identical.

Since the final amplitudes A' and A'' are phase coherent and equal, $A'=A''=A$, the total intensity is

$$|A'+A''|^2=|A'|^2+|A''|^2+A'^*A''+A'A''^*=4|A|^2\ . \tag{3}$$

If the two amplitudes were not coherent then the total scattering intensity of the two complementary sequences would only be $2|A|^2$. This means that the scattering intensity into the state $-\vec{k}$ is by $2|A|^2$ larger than in the case of incoherent scattering. This additional scattering intensity exists only in the backscattering direction. For other states at the Fermi surface, sufficiently far away from $-\vec{k}$, there is only an incoherent superposition of every two sequences (with momentum transfer in the opposite sequence) and therefore as an average the scattering intensity per sequence with n scattering processes is only $|A|^2$.

The fan diagram in Fig. 1(a) gives just the product A'^*A'', i.e., the interference intensity. It consists of two parts, the upper electron propagator and the lower hole propagator. The upper one yields the amplitude of the electron k which is scattered into the state $-\vec{k}$ via the scattering sequence ('). If we invert the direction of the arrows for the lower propagator then it yields the amplitude of the electron $\vec{k}$ which is scattered into the state $-\vec{k}$ via the scattering sequence (''). The reversed direction of the arrows (i.e., it is a hole propagator) yields the complex conjugate of the amplitude.

At high temperature the scattering processes are partial-

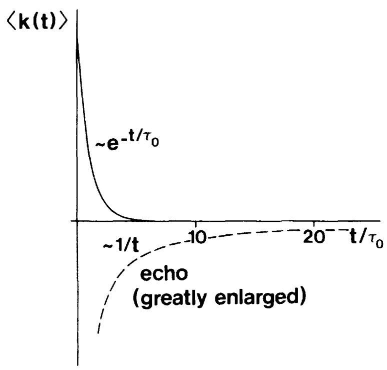

FIG. 2. Contribution of the electron state k to the momentum as a function of time. The original state and its momentum decay exponentially within the time τ_0 (s scattering assumed). But an echo with the momentum $-\vec{k}$ is formed which depends on time as $1/t$. This echo reduces the contribution of the electron to the current and yields a correction to the resistance which is proportional to $\ln(\tau_i/\tau_0)$.

ly inelastic. As a consequence the amplitudes A' and A'' lose their phase coherence (after the time τ_i) and the intensity of the backscattered value is only $2|A|^2$, i.e., the coherent backscattering disappears after the time τ_i. In Fig. 2 the contribution of the (original) electron $\vec{k}$ to the momentum is plotted as a function of the time. $\langle \vec{k}(t)\rangle$ is the expectation value of the momentum. The original momentum decays within the elastic lifetime. At later times a momentum in the opposite direction is formed; this decreases inversely proportional to the time. One obtains an echo of the original state $\vec{k}$ in the opposite direction, which vanishes only when the two processes lose their coherence. Obviously the integrated momentum of the electron $\vec{k}$ decreases with increasing τ_i. In the following we treat this scattering semiquantitively.

After the elastic lifetime τ_0 the electron k is scattered into Z intermediate states. The amplitude in the intermediate state k_1' is $(1/\sqrt{Z})e^{i\delta_1}$ where $e^{i\delta_1}$ is essentially given by $V(g_1)/|V(g_1)|$. The intensity in the next intermediate state k_2' at the time $2\tau_0$ is Z^{-2}. After n scattering processes the intensity in the final state $-\vec{k}$ is Z^{-n} and the amplitude $Z^{-n/2}\exp(i\sum\delta_\nu)$. The second scattering series yields the same amplitude. The cross product or interference term is $A'^*A''+A'A''^*=2Z^{-n}$. Now we have to sum over all intermediate states. This yields the factor $\frac{1}{2}Z^{n-1}$. ($\frac{1}{2}$ occurs because the two complementary series appear twice in the sum.) Therefore, the coherent additional backscattering intensity is Z^{-1}. It is independent of the number of intermediate scattering states n and equal to the scattering intensity from $\vec{k}$ into $\vec{k}_1'$. This intensity is, of course, completely calculated in evaluating the diagram with the appropriate rules. However, one can easily estimate this intensity in a rather direct and less formal manner.

For the calculation of Z we consider the scattering from the state $\vec{k}$ into the state $\vec{k}_1'$. This state is an intermediate state for the scattering sequence which does not have to conserve the energy (sometimes called the virtual scattering process). Since the lifetime in an eigenstate of momentum is τ_0 the intermediate state can lie within $\pi\hbar/(\tau_0)$ of the Fermi energy (because of the uncertainty principle). This corresponds to a smearing of the Fermi sphere by π/l (l indicates the mean free path of the conduction electrons). Therefore, the available area in $\vec{k}$ space is $2\pi k_F\pi/l=2\pi^2k_F/l$ and Z is obtained by multiplication with the density of states in $\vec{k}$ space, i.e., $(2\pi)^{-2}$.

The coherent backscattering is not restricted to the exact state $-\vec{k}$; one has a small spot around the state $-\vec{k}$ which contributes. The determination of the area of this spot can be obtained by a heuristic consideration. In Appendix A another derivation of I_{coh} is given which considers the loss of phase coherence and corresponds directly to the evaluation of the fan diagram. During the time $t=n\tau_0$ the electron has propagated by diffusion in space only over a distance of $X=(Dn\tau_0)^{1/2}$. Here D is the diffusion constant which has (in two dimensions) the form $D=v_F^2\tau_0/2$. Coherent interference is only possible in this area. This weakens the requirement that the final state is $-\vec{k}$. Neighboring states with $-\vec{k}+\vec{q}$ remain coherent in this restricted area as long as $Xq<1$, i.e., $q^2<1/Dn\tau_0$. This corresponds to about $\pi(Dn\tau_0)^{-1}/(2\pi)^2$ states. The spot of coherent final states has a finite area but shrinks with increasing time. Therefore, the portion of coherent backscattering is given by

$$I_{\text{coh}}=[\pi/(Dt)](2\pi^2k_F/l)^{-1}$$
$$=\tau_0/\pi k_Flt=\hbar/2\pi E_Ft\ . \tag{4}$$

In the presence of an external electrical field the conduction electrons contribute to the current. However, the echo, i.e., the coherent backscattering reduces the current and therefore the conductance. A pulse of the electric field generates a short current (for the time τ_0 in the direction of the electric field and then a reversed current which decays as $1/t$. The dc conductance is obtained by integrating the momentum over time. For the normal contribution this yields $k\tau_0$ and for the echo $[\tau_0/(\pi k_Fl)]\ln(\tau_i/\tau_0)$. Therefore, the electron in the state k contributes to momentum

$$k\tau_0[1-1/(\pi k_Fl)\ln(\tau_i/\tau_0)]\ . \tag{5}$$

The contribution of the electron k to the current is reduced by the factor in the brackets and the conductance is decreased by the same factor as follows:

$$L=(ne^2\tau_0/m)[1-(1/\pi k_Fl)\ln(\tau_i/\tau_0)]$$
$$=ne^2\tau_0/m-(e^2/2\pi^2\hbar)\ln(\tau_i/\tau_0)\ , \tag{6}$$

with $n=2\pi k_F^2/(2\pi)^2$. This correction to the conductance was introduced by Anderson *et al.*[2] and Gor'kov *et al.*[3]

The important consequence of the above consideration is that the conduction electrons perform a typical interference experiment. The (incoming) wave $\vec{k}$ is split into two complementary waves $\vec{k}'_1$ and $\vec{k}''_1$. The two waves propagate individually, experience changes in phase, spin-orientation, etc., and are finally unified in the state $-\vec{k}$ where they interfere. The intensity of the interference is simply measured by the resistance. In the situation which has been discussed above the interference is constructive in the time interval from τ_0 to τ_i. It is only slightly more complicated than a usual interference experiment because one has a large number of pairs of complementary waves.

Now we consider the motion of the conduction electron in real space. Assume that a conduction electron is injected into the system at the origin at the time $t=0$. Since the conduction electron has a very short mean free path its wave experiences a multiple scattering by the defects. This corresponds to a diffusion from impurity to impurity. The classical diffusion equation in two dimensions yields, for the probability of finding the electron at the time t at the position r,

$$p(r,t)=(1/4\pi Dt)\exp[-r^2/(4Dt)] \ . \tag{7}$$

The chance to return to the origin is given by $1/(4\pi Dt)$. In Fig. 3 a possible path is drawn for the diffusion of an electron which returns to the origin (in the sequence $0\to 1\to 2\to \cdots \to 0$). For classical diffusion one has an identical probability for the electron to propagate on the same path in the opposite direction ($0\to 1'\to 2' \to \cdots \to 0$). The two probabilities add up and contribute to the total probability of $1/4\pi Dt$. Since the electron has wavelike character one has in reality to consider two partial waves of the electron which propagate in opposite directions on the indicated path. Returned to the origin, however, their amplitudes add (instead of their intensities). It is the same physical mechanism which has been discussed in the preceding section. This picture has been used by Altshuler *et al.*[37] in studying the electric field effect on weak localization. The amplitudes A' and A'' are equal because their partial waves propagated on the same

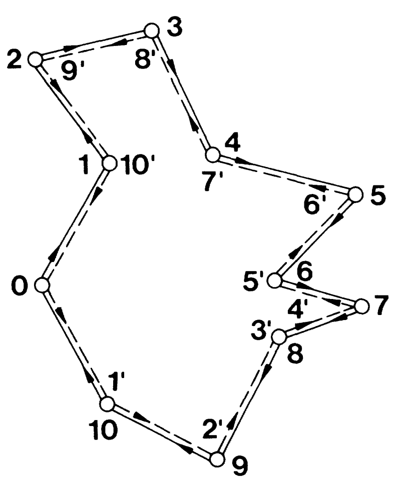

FIG. 3. Diffusion path of the conduction electron in the disordered system. The electron propagates in both directions (solid and dashed lines). In the case of quantum diffusion the probability to return to the origin is twice as large as in classical diffusion since the amplitudes add coherently.

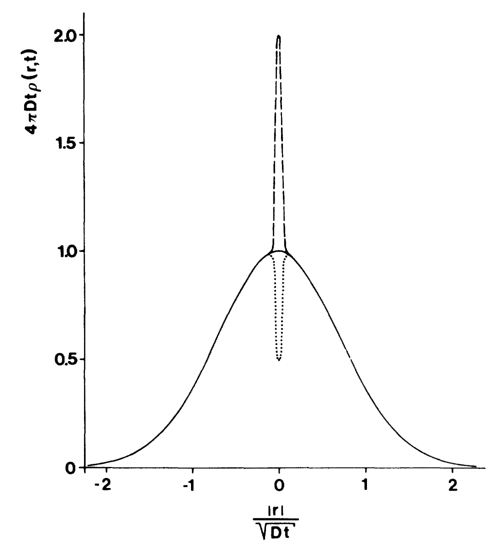

FIG. 4. Probability distribution of a diffusing electron which starts at $r=0$ at the time $t=0$. In quantum diffusion (dashed peak) the probability to return to the origin is twice as large as in classical diffusion (solid curve). Large spin-orbit coupling reduces the probability by a factor of 2 (dotted peak) and yields a weak antilocalization.

path in opposite directions, and as long as the system is time invariant, the two partial waves arrive at the origin in phase and with the same amplitude. Therefore, the intensity or probability is twice as large as in the classical diffusion problem, i.e., $1/2\pi Dt$. For the diffusion to any other point except the origin the different partial waves are generally incoherent and only their intensities add. (There is only a small reduction to compensate the increased intensity at the origin.) In Fig. 4 the classical and the quantum-diffusion probabilities are qualitatively plotted. The (dashed) peak in quantum diffusion at the origin describes a tendency to remain at or return to the origin. Since it was thought of as a precursor of localization this quantum diffusion has been called weak localization. (A localized electron would remain close to the origin.) This name is, however, questionable because in the presence of large spin-orbit coupling—as we discuss below—the quantum diffusion yields a reduced probability (dotted peak) to return to the origin, an effect one might call weak antilocalization.[38]

III. TIME-OF-FLIGHT EXPERIMENT BY A MAGNETIC FIELD

One of the interesting possibilities for an interference experiment is to shift the relative phase of the two interfering waves. For charged particles this can be easily done by an external magnetic field. In a magnetic field, however, the phase coherence of the two partial waves is weakened or destroyed. When the two partial waves surround an area F containing the magnetic flux ϕ, then the relative change of the two phases is $(2e/\hbar)\phi$. The factor of 2 arises because the two partial waves surround the area twice. (This is sometimes interpreted as if a particle with twice the electron charge surrounds the area in analogy to the double charge $2e$ in superconductivity.)

Since the diffusion is statistical one has for a given diffusion time t a whole range of enclosed areas for the different diffusion paths. Altshuler *et al.*[39] suggested performing such an "interference experiment" with a cylindrical film in a magnetic field parallel to the cylinder axis. Then the magnetic phase shift between the complementary waves is always a multiple of $2e\phi/\hbar$ (ϕ indicates the flux in the area of the cylinder). Sharvin and Sharvin[40] showed in an elegant experiment that then the resistance oscillates with a flux period of $\phi = h/(2e)$. However, for a thin film in a perpendicular magnetic field the pairs of partial waves enclose areas between $-2Dt$ and $2Dt$. When the largest phase shift exceeds 1, the interference is constructive and destructive as well and the average cancels. This happens roughly after the time $t_H = \hbar/4eDH$. This means essentially that the conductance correction in the field H, i.e., $\Delta L(H)$, yields the coherent backscattering intensity integrated from τ_0 to t_H,

$$\Delta L(H) \sim \int_{\tau_0}^{t_H} I_{\mathrm{coh}}\, dt \sim -L_{00} \ln(t_H/\tau_0) \ . \tag{8}$$

It is important to mention that only the amplitudes of the "scattered" waves interfere. There is no interference between the original wave function and its scattered component considered in this theory, and at these finite temperatures the coherence length, i.e., the length over which a wave packet can be defined at finite temperature and which is of the order of $\hbar v_F/k_B T$, is much smaller than the inelastic mean free path $v_F \tau_i$. (Otherwise, one is no longer in the region of weak localization.)

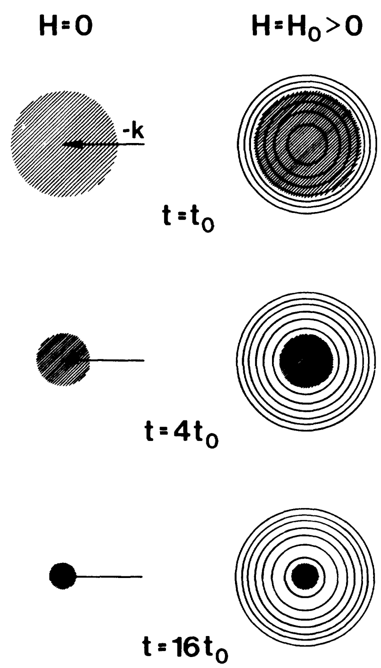

FIG. 5. Backscattering spot (close to the state $-k$) without a magnetic field (left-hand side) and in a finite magnetic field H. The spot has a finite area π/Dt which shrinks with time. In a magnetic field the coherence condition is modified and only k states which lie on "Landau"-type circles allow coherent backscattering. For large times the two interference conditions exclude each other because the spot is inside of the Landau circle and the coherent backscattering dies at a time $t_H = \hbar/4eDH$. The resistance integrates the coherent backscattering intensity in the time interval from τ_0 to t_H.

The quantitative calculation yields a quite simple result. The application of a magnetic field causes a destructive interference in the final state $-k$. But in the vicinity of $-k$ for the states $-k+q$ the interference is constructive if q lies on Landau-type circles with $(\hbar q)^2/4m = \hbar\omega_c(n+\frac{1}{2})$, where ω_c is the cyclotron frequency. The allowed states as a function of q are shown in Fig. 5. (The electron states on the "Landau circles" are not free-electron states in a magnetic field because they are centered around $-k$. Only formally they correspond to hypothetical particles with twice the electron mass.) Since, on the other hand, the width of the coherently backscattered spot shrinks with time as $1/\sqrt{Dt}$ (due to the diffusion in real space) the coherent backscattering dies out when the spot lies completely inside of the first Landau circle with the radius $\sqrt{2eH/\hbar}$. This occurs for fields of the order of $H = \hbar/4eDt$.

This means that the magnetic field allows a time-of-flight experiment. If a magnetic field H is applied, the contribution of coherent backscattering is integrated in the time interval between τ_0 and $t_H = \hbar/4eDH$. If one reduces the field from the value H' to the value H'' and measures the change of resistance this yields the contribution of the coherent backscattering in the time interval $t_{H'}$ and $t_{H''}$. In a very strong field the coherent interference is suppressed. A reduction of the field integrates the coherent backscattering and increases the resistance. If t_H exceeds the inelastic lifetime of the conducting electrons, i.e., $H < H_i = \hbar/4eD\tau_i$, then the coherence is lost anyway and magnetoresistance disappears. Since the magnetic field introduces a time t_H into the electron system all characteristic times τ_n of the electrons can be expressed in terms of magnetic fields H_n:

$$\tau_n \Longleftrightarrow H_n \ ,$$

where $\tau_n H_n = \hbar/4eD$. In a thin film this is given by $\hbar e \rho N/4$ which is of the order of 10^{-12}–$10^{-13}Ts$ (ρ is the resistivity of the film and N is the density of electron states for both spin directions). The exact formula for the magnetoconductance is (in the absence of spin-orbit coupling and magnetic scattering)

$$L(H) - L(0) = -\frac{e^2}{2\pi^2\hbar}\left[\ln\left[\frac{H_i}{H}\right] - \Psi\left[\frac{1}{2} + \frac{H_i}{H}\right]\right] . \tag{9}$$

The motion of the conduction electron in real space gives a simple criterion for the conditions under which a thin film is two dimensional. This has been discussed in detail by Altshuler and Aronov,[41] Berggren,[42] and Ovadyahu *et al.*[33] The important requirement for the quantum interference is that the electron wave function is coherent. Therefore, a system is two dimensional with respect to weak localization when this coherence volume has a two-dimensional shape. Without a magnetic field the electron diffuses during its inelastic lifetime over a distance of $(D\tau_i)^{1/2}$. If the thickness of the film is much less than this "Thouless length" then the region of coherence is two dimensional. For films thinner than 100 A thickness and at temperatures under 20 K this requirement is in general very well fulfilled. However, in strong magnetic fields the distance of coherent diffusion $(Dt_H)^{1/2}$ is much less and therefore one easily moves into the three-dimensional range. Therefore, one expects in high magnetic fields deviations from the two-dimensional formula [one has to include the sheets in $\vec{k}$ space for $k_z = v\pi/d$ (d indicates the film thickness)].

IV. SPIN-ORBIT COUPLING

One of the most interesting questions in weak localization is the influence of spin-orbit coupling. Hikami *et al.*[5] and Maekawa and Fukuyama[43] predicted that in the presence of strong spin-orbit coupling a logarithmic decrease of the resistance occurs with decreasing temperature. As a consequence the magnetoresistance should change sign as well. This prediction is contrary to the picture of localization and was one of the most exciting questions raised at the Sixteenth International Conference on Low Temperature Physics.[23] The author[25] confirmed the prediction by Hikami *et al.* For this purpose a thin Mg film has been covered by a $\frac{1}{100}$ monolayer of Au. Figure 6 demonstrates the unique influence of such a small coverage of Au on the weak localization. The magnetoresistance of the pure Mg film is plotted in the upper part of the figure while the lower part of the figure shows the strong change due to a coverage of $\frac{1}{100}$ monolayer of Au. In Ref. 25 the influence of the spin-orbit coupling has been quantitatively evaluated. As a consequence weak localization provides a new and very sensitive method to measure the spin-orbit coupling directly, i.e., with a substructure and not only by a broadening of a resonance. The natural question is, why does weak localization change to weak antilocalization in the presence of spin-orbit coupling?

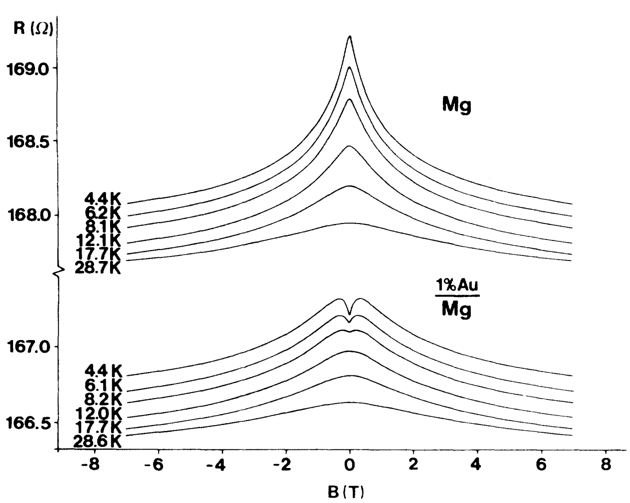

FIG. 6. Magnetoresistance of a pure Mg film at different temperatures (upper part). A superposition of $\frac{1}{100}$ atomic layer of Au (statistically) changes the behavior completely. The Au introduces a rather pronounced spin-orbit coupling which rotates the spins of the complementary scattered waves. This changes the interference from constructive to destructive.

V. INTERFERENCE OF ROTATED SPINS

It is a consequence of quantum theory and proved by a rather sophisticated neutron experiment that spin-$\frac{1}{2}$ particles have to be rotated by 4π to be transferred into the identical state. A rotation by 2π reverses the sign of the spin state. Weak antilocalization gives another experimental proof of this fact. In the presence of spin-orbit coupling the spin of a scattered electron is slightly rotated during each scattering event. During the whole scattering series (′) the spin orientation diffuses into a final state s' which can be obtained by a rotation T of the original spin state s ($s' = Ts$). It is straightforward to show that the finite spin state of the complementary scattering series (″) is $s'' = T^{-1}s$. Without the spin rotation the interference of the two partial waves is constructive (in the absence of an external field). In the presence of spin-orbit coupling the interference becomes destructive if the relative rotation of s' and s'' is 2π. It can be shown that for strong spin-orbit coupling the destructive part exceeds the constructive one.[38] This means that the backscattering is reduced below the statistical one. This corresponds to an echo in the forward direction and an increase of the conductance. The magnetoresistance curve in Fig. 6 for 1% of a monolayer of Au on top of Mg at 4.4 K can be interpreted as follows. In a high magnetic field where $t_H < \tau_{so}$ the spin states of the complementary states are almost unchanged and one obtains the usual negative magnetoresistance. For $t_H > \tau_{so}$ (and $t_H < \tau_i$) the interference is destructive and shows the opposite sign. For $t_H \approx \tau_{so}$ it changes sign. The resistance maximum in a finite field corresponds to a relative rotation of s' and s'' by the angle π (in an average).

VI. MAGNETIC SCATTERING

Another interesting application of weak localization is the determination of magnetic scattering by magnetic ions. The magnetic ion introduces an interaction with a conduction electron JSs, where S and s are the ion and electron spins. The magnetic ions scatter the two complementary waves differently and destroy their coherence after the magnetic scattering time τ_s. Therefore, weak localization is blocked at low temperature (where $\tau_i > \tau_s$) and the magnetoresistance curves remain broad (because only for $t_H < \tau_s$ or $H > \hbar/4eD\tau_s$ the magnetic field can overcome the destructive influence of the magnetic scattering). Such measurements yield the temperature dependence of τ_s. For $\frac{1}{1000}$ atomic layer of Fe on Mg the magnetic scattering time was determined and found to be temperature independent.[26]

I believe that one of the attractive future applications of weak localization will be the investigation of interacting magnetic systems like Kondo impurities, spin fluctuations, valence mixing, etc., where a temperature dependence of the magnetic scattering time yields an insight into the physics of the phenomena. Generally weak localization allows the measurement of characteristic electronic times which are otherwise difficult, expensive (such as neutron scattering), or impossible.

APPENDIX A: THE AREA OF THE BACKSCATTERING SPOT

We calculate the coherent backscattering intensity into the state $-\vec{k}+\vec{q}$ which is reached after n scattering processes with the transfer of momentum $\vec{g}_i$ where $\sum \vec{g}_i = -2\vec{k}_F + \vec{q}$. The sum of the momenta of the initial and final state is $+q$. The same applies for each pair of scattering states in Fig. 1(b) which lies opposite to the center, i.e.,

$$q = k_1' + k_{n-1}'' = k_2' + k_{n-2}'' = \cdots .$$

The corresponding intermediate states differ not only in momentum but also in energy (which must not be conserved). The energy difference is $\hbar q v_F$ and since the phase rotates with $Et/\hbar$ one obtains during the time τ_0 a phase difference between the two complementary waves which is $q v_F \tau_0$. The important fact is that the different intermediate states have independent directions of momentum. Therefore, the phase differences are independent in sign and value. This means that only the square of the phase shifts adds. Therefore, after the n scattering processes one obtains phase differences between the complementary waves whose width is

$$(\Delta\varphi)^2 = n(qv_F)^2 = n(\tfrac{1}{2})(v_F q)^2 = nD\tau_0 . \qquad \text{(A1)}$$

In two dimensions the average over $(v_F q)^2$ is $(v_F q)^2/2$ [and in three dimensions $(v_F q)^2/3$ but the diffusion constant absorbs the factor of the dimension]. The neighboring states of $-k$ contribute less to the coherent backscattering because they lose the phase coherence with increasing n and q. Their contribution is proportional to $\exp(-Dq^2 t)$ since $t = n\tau_0$. The area of the spot for the coherent backscattering is obtained by integration over q. In two dimensions this yields π/Dt.

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