Mechanism of symmetry breaking in the spherical limit: Phase coherence in finite-size systems

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The appearance of symmetry breaking in the thermodynamic limit is studied from a dynamical point of view. We solved the time-dependent Landau-Ginzburg model in the spherical limit and show that the finite-size system develops a phase-coherence phenomenon analogous to that appearing in lasers. Below the critical point of the infinite system the orientational diffusion of the magnetization becomes the slowest mode and its time scale is well separated from all the other relaxational mechanisms. As expected, this phase diffusion freezes in completely in the thermodynamic limit.

I. INTRODUCTION

Symmetry breaking in statistical physical systems is usually understood as the separation of the phase space into equivalent regions. The system moves from one region to the other in a time much greater than the typical observation times, and so the ensemble averages are meaningful only if they are restricted to one of the equivalent regions. As a consequence, some averages which would vanish by symmetry requirements might assume nonzero values and this is referred to as symmetry breaking.[1,2] Within such a broad definition symmetry breaking takes place not only in systems with infinite degrees of freedom displaying phase transitions, but also in systems in which an instability phenomenon is connected with only a finite number of degrees of freedom. In the case of phase transitions, the concept of symmetry breaking becomes unambiguous once the thermodynamic limit is taken, because in that limit the transition time between the different regions of the phase space is infinite.

This general picture can be easily understood in the case of Ising-type ferromagnetic models by thinking of some effective free energy as a function of the magnetization (M). Above the transition point (T_0), the free energy has a single minimum at $M=0$. Going below T_0, two symmetrically positioned minima develop which are separated by a barrier, the height of which is proportional to some power of the volume of the system[3,4] so once the system finds itself around one of these minima it will stay there forever in the infinite-volume limit ($V\to\infty$).

The free-energy barrier explanation does not work when a continuous symmetry is broken. In such a case the effective free energy depends on the absolute value of the magnetization vector $M=|\vec{M}|$ and as for Ising ferromagnets, below T_0 a minimum appears at some macroscopic value M_0. There is no barrier, however, between states which differ only by the orientation of the magnetization. This continuous degeneracy and absence of barrier is well known and has been exploited to establish general results as the Goldstone theorem[5,6] and the absence of homogeneous ordering in low-dimensional systems.[7] In order to understand the mechanism of symmetry breaking in this case, one must explain how the diffusion of the phase of $\vec{M}$ slows down as the thermodynamic limit is approached.

There is at least one simple system where the diffusion of phase is understood. We are referring to the single-mode laser whose time evolution is effectively described by a particle undergoing an overdamped motion under the action of a stochastic force in the potential shown in Fig. 1. The two degrees of freedom describe the slowly varying part of the electric field, the distance from the origin being the square root of the intensity of the radiation, while the angle φ denotes the phase of the electric field. Above the threshold the intensity becomes macroscopic and the asymptotic motion is described by the free diffusion of the particle along the potential valley. Now, the slowing down of the phase is a simple consequence of the fact that the particle moves under the influence of a microscopic noise along a circle whose radius is macroscopic. The resulting phenomenon of very slow change of the phase is known as phase coherence.[8]

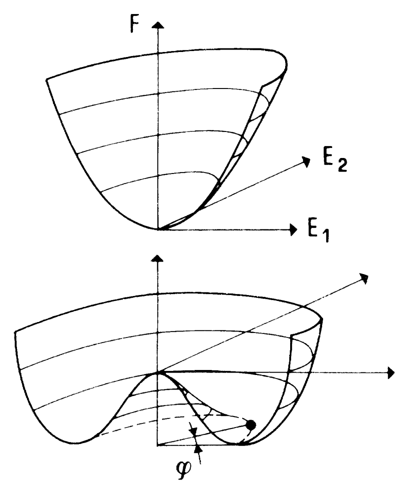

FIG. 1. Potential F as a function of the two components E_1, E_2 of the slowly varying electric field for a single-mode laser. φ is the phase of the field.

The phase transition in the isotropic n-component Ginzburg-Landau model for $n \geq 2$ can also be viewed as a phase coherence phenomenon since below the critical point the orientational motion of the magnetization freezes in as the infinite-volume limit is taken. We argue that the freezing in is a direct consequence of the establishment of the macroscopic value of the length of the magnetization vector, similar to the case of the laser which can be related to the Landau-Ginzburg model with $n=2$. Our argument is based on the results we obtained in the spherical limit ($n \to \infty$), where the laser picture is shown to be still valid.

The difficulty of investigating the freezing-in lies in the treatment of the dynamics of a finite-volume system and in the following of the changes in the relaxational properties as $V \to \infty$. We choose to study the purely relaxational time-dependent Ginzburg-Landau (TDGL) model in the $n \to \infty$ limit because it has long been recognized[9] that the resulting time-dependent spherical model has relatively simple relaxational properties.[10] As it turns out, this feature of the model is maintained in the finite-volume case as well.

In order to have as close analogy as possible with the laser case, first we treat the TDGL model by decoupling the homogeneous mode of the system. This is then an n-component generalization of the laser model and describes systems where the decoupling is physically possible, such as in some autocatalytic systems[11] where n might take arbitrary values. The generalization has some pedagogical appeal too, since the volume dependence of its relaxational properties are transparent (Sec. II). It should be noted, however, that in this model all the fluctuations disappear as $V \to \infty$, so its time evolution becomes deterministic. For that reason one might question the validity of even the qualitative conclusions about the freezing in of the phase diffusion. As it turns out, however, the inclusion of all the modes of the system (Sec. III) does not alter the picture obtained from the simple model. As usual, the fluctuations only shift the instability point and make the effect completely disappear below certain spatial dimension which is two in this case.

II. SPHERICAL LIMIT OF THE LASER INSTABILITY

The transient radiation phenomenon in the single-mode laser is usually understood in terms of the following Langevin equation which describes the time-evolution of the slowly varying part of the electrical field in the laser cavity:

$$E_j = bE_j(a - E^2) + \eta_j \ , \tag{1}$$

where $E_j (j=1,2)$ is the suitably scaled electric field, and the parameters a,b and the strength of the noise term can be expressed through the parameters of the laser.[12]

In order to relate Eq. (1) to the $n=2$ case of the TDGL model, let us write down the Langevin equation defining the latter in a finte volume ($V=L^d$) with periodic boundary conditions

$$\dot{S}_j(\vec{q},t) = -[(q^2+r_0)S_j(\vec{q},t)] + u\sum_{i=1}^{n}\frac{1}{V^2}\sum_{\vec{q}\,',\vec{q}\,''} S_i(\vec{q}\,',t)S_i(\vec{q}\,'',t) \times S_j(\vec{q}-\vec{q}\,'-\vec{q}\,'',t) + \eta_j(\vec{q},t) \ . \tag{2}$$

Here $S_j(\vec{q},t)$ is the Fourier transform of the jth component of an n-component field $\tilde{S}(\vec{x},t) \equiv \{\tilde{S}_j(\vec{x},t), j=1,\dots,n\}$

$$\tilde{S}_j(\vec{x},t) = \frac{1}{V}\sum_{\vec{q}} e^{i\vec{q}\cdot\vec{x}} S_j(\vec{q},t) \ , \tag{3}$$

where $\vec{q} \equiv \{q_\alpha,\ \alpha=1,\dots,n\}$ and $q_\alpha = 2\pi k/L$ with k taking integer values.

The random force η_i is Gaussian-Markoffian,

$$\langle \eta_i(\vec{q},t)\eta_j(\vec{q}\,',t')\rangle = \epsilon V \delta_{\vec{q},\vec{q}\,'}\delta_{ij}\delta(t-t') \ , \tag{4}$$

with $\epsilon = 2k_B T$, T being the temperature. There is a variety of models which reduce to Eq. (2) in the continuum limit. The parameters r_0 and u are related to the original parameters and usually depend on the temperature. Here we shall adopt the view that r_0 and ϵ are two independent control parameters and we choose u to be $u=1/n$.

Assuming that the amplitude of the $q=0$ mode is macroscopic,

$$S_i(\vec{q}=0,t) = V\sigma_i \ , \tag{5}$$

and that it decouples from all the other modes, Eq. (2) yields

$$\dot{\sigma}_i = -\left[r_0 + \frac{1}{n}\sum_{i=1}^{n}\sigma_i^2 \right]\sigma_j + \bar{\eta}_j \ , \tag{6}$$

where the strength of the noise is now inversely proportional to the volume

$$\langle \bar{\eta}_i(t)\bar{\eta}_j(t')\rangle = \epsilon_0\delta_{ij}\delta(t-t') \ , \quad \epsilon_0 = \frac{\epsilon}{V} \ . \tag{7}$$

As can be seen, for $n=2$ Eq. (6) is just the laser equation [Eq. (1)].

The study of the properties of Eq. (6) is simplified in the spherical limit ($n \to \infty$) because in that limit the fluctuations of the modulus squared of $\{\sigma_i\}$ normalized by n

$$l^2 = \frac{1}{n}\sum_{i=1}^{n}\sigma_i^2 \tag{8}$$

disappear.[13] So l^2 can be replaced by its time-dependent average value and one can easily write down the equations governing the time evolution of the inverse susceptibility

$$r(t) = r_0 + \left\langle \frac{1}{n}\sum_{i=1}^{n}\sigma_i^2 \right\rangle = r_0 + l^2(t) \tag{9}$$

and of the magnetization

$$m(t) = \langle \sigma_i \rangle \ . \tag{10}$$

The averages are, as usual, over the initial conditions and

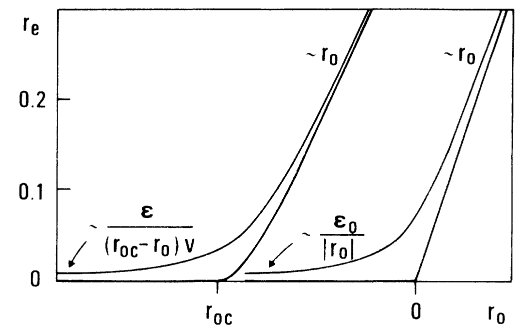

FIG. 2. Stationary value of the susceptibility r_e as a function of r_0 for $\epsilon_0=10^{-2}$ and $\epsilon_0=0$ for the homogeneous mode (right curves) and $\epsilon=10^{-2}$ and $\epsilon=0$ with all the modes included (left curves).

the noise, though one should note that the time evolution of $l^2(t)$ is deterministic,[13] so its average is taken only over the initial conditions. The equations are as follows:

$$\dot{m}=-rm \ , \tag{11}$$

$$\dot{r}=-2r(r-r_0)+\epsilon_0 \ . \tag{12}$$

The stationary properties of this "reduced" spherical model defined by Eqs. (11) and (12) are simple; the magnetization vanishes and the stationary value of susceptibility is given by

$$r_e=\tfrac{1}{2}[r_0+(r_0^2+2\epsilon_0)^{1/2}] \ . \tag{13}$$

In Fig. 2 r_e is drawn for $\epsilon_0=0$ and $\epsilon_0=10^{-2}$. As can be seen, in the $V\to\infty$ limit ($\epsilon_0=0$) the system undergoes an instability at $r_0=0$ and the ordering consists of changing the stationary value of $l_e^2=r_e-r_0$ from $l_e^2=0$ for $r_0>0$ to $l_e^2=|r_0|$ for $r_0<0$. This instability gets smeared out by the presence of the noise (finite volume) but the main feature remains: Around $r_0\simeq 0$ there is a steep change in the behavior of l_e^2 from being proportional to the noise ($l_e^2\simeq\epsilon_0/r_0$ for $r_0>0$) to being independent of it ($l_e^2\simeq|r_0|$ for $r_0<0$). The change takes place in a region $|\Delta r_0|\le(\epsilon_0)^{1/2}$.

The change in l_e^2 affects the relaxational properties of the system drastically. As one can see by solving Eq. (12), the relaxation of $r(t)$ [and consequently of $l(t)$] is governed by a single relaxation time

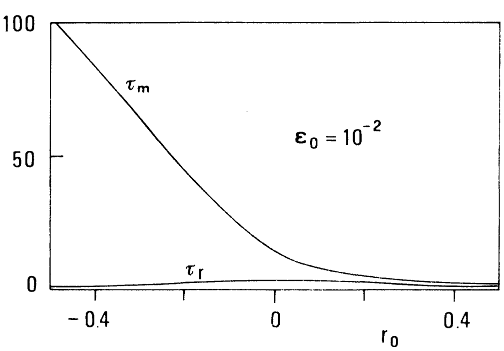

FIG. 3. Relaxation times of the susceptibility τ_r and the magnetization τ_m as a function of r_0 for the homogeneous case.

$$\tau=\frac{1}{2(r_0^2+2\epsilon_0)^{1/2}} \ , \tag{14}$$

which becomes small in both the "ordered" $[r_0\ll-(\epsilon_0)^{1/2}]$ and in the "disordered" $[r_0\gg(\epsilon_0)^{1/2}]$ region as shown in Fig. 3.

For $t>\tau_r$, the relaxation of m is determined by r and from Eq. (11)

$$\tau_m\simeq\frac{1}{r_e} \ . \tag{15}$$

In the ordered region—which will be our concern in the following—the two time scales are well separated (Fig. 3) with

$$\tau_m\simeq\frac{2|r_0|}{\epsilon_0}\gg\tau_r\simeq\frac{1}{2|r_0|} \ . \tag{16}$$

The sudden increase of τ_m is a direct consequence of the increase of l_e^2. To understand this, imagine the field to be freely diffusing on an n-dimensional sphere of radius $R=\sqrt{n}\,l_e$. The initial phase coherence is then lost (i.e., the magnetization will be relaxed to zero) when the average square displacement is of the order $R^2=nl_e^2$. This implies that by this time every component has an average displacement l_e^2. Since in the ordered region $l_e^2\simeq|r_0|$, the result for τ_m [Eq. (16)] is consistent with this picture of diffusion with the strength of the stochastic force ϵ_0 giving the diffusion coefficient.

It is also easy to discuss the initial-time behavior of the magnetization. From Eq. (11) we have

$$m(t)=m(0)\exp\left[-\int_0^t dt' r(t')\right] \ , \tag{17}$$

and writing Eq. (12) in the form

$$\int_0^t dt' r(t')=\tfrac{1}{2}\ln\frac{r(t)-r_0}{r(0)-r_0}-\frac{\epsilon_0}{2}\int_0^t\frac{dt'}{r(t')-r_0} \ , \tag{18}$$

we arrive at the following result:

$$m(t)=m(0)\left[\frac{r(t)-r_0}{r(0)-r_0}\right]^{1/2}\exp\left[-\frac{\epsilon_0}{2}\int_0^t\frac{dt'}{r(t')-r_0}\right] \ . \tag{19}$$

This expression can be made more transparent by using the fact that $r(t)$ relaxes in a time τ_r,

$$m(t)=m(0)\left[\frac{r(t)-r_0}{r(0)-r_0}\right]^{1/2}\exp\left[-\frac{\epsilon_0}{2}\int_0^{\tau_r}\frac{dt'}{r(t')-r_0}\right]\times\exp\left[-\frac{\epsilon_0(t-\tau_r)}{2(r_e-r_0)}\right] \ . \tag{20}$$

The long-time decay of $m(t)$ is given by the second exponential and since $r\ll|r_0|$, it corresponds to the diffusive behavior discussed above. The initial relaxation of $m(t)$ can be very different according to the initial value of $r(t)$. It is remarkable, however, that if $r(0)$ is not very far from its equilibrium value, i.e., $r(0)\ll|r_0|$, then $m(t)$ approaches a value that depends only on $m(0)/l(0)$, which can be considered to be the cosine of the initial "phase" of the field

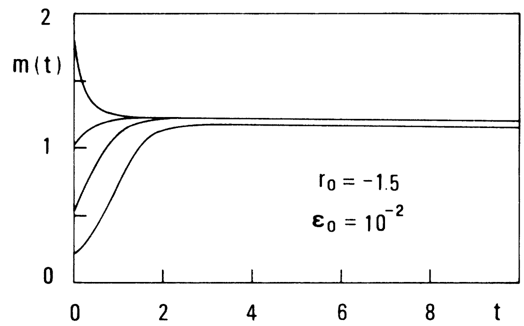

FIG. 4. Metastabilitylike behavior in the magnetization for various initial values. The lowest curve refers to an initial state close to the unstable point.

$$m_{\rm in} = m(0)\left[\frac{r_e - r_0}{r(0)-r_0}\right]^{1/2} e^{-\epsilon_0/4r_0^2}$$
$$\simeq m(0)\frac{|r_0|^{1/2}}{[r(0)-r_0]^{1/2}} = \frac{m(0)}{l(0)} l_e \; . \tag{21}$$

So we arrive at the result that the relaxation of the magnetization takes place in two stages. First its amplitude approaches its equilibrium value on a time scale τ_r and then the orientational diffusion takes place on a time scale τ_m. Since the two time scales are well separated for large volumes, the behavior of the systems appears metastable if the observation times t are $\tau_r \ll t_0 \ll \tau_m$. One can see this clearly on Fig. 4 where the system seems to be ordered on the time scale of observation. This "ordering" disappears necessarily as we approach the instability point ($r_0 = 0$) because there τ_r and τ_m are of the same order (see Figs. 3 and 5).

The metastability phenomenon disappears also far below the instability point if the system is around or at an unstable state initially. In our case the state $\sigma_i = 0$ [$l_e^2 \sim 0$, $r(0) \sim r_0$] will be the unstable one (see Fig. 1 for $n = 2$). Close to the unstable point small fluctuations will give rise to large changes in the phase and, as a consequence, the initial spread of the phase limits the intermediate values of the magnetization (see Fig. 4). This is nothing else than a manifestation of the well-known amplification of fluctuations in a system decaying from an unstable state.

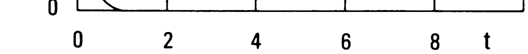

FIG. 5. Metastability and its disappearance as the instability point $r_0 = 0$ is approached.

III. DYNAMICS OF THE SPHERICAL MODEL IN A FINITE VOLUME

We extend now our treatment to the case where all the modes are coupled, Eq. (2). First we establish some results on the effect of the finite volume on the static properties of the system. The inverse susceptibility r satisfies the following self-consistency equation[14]:

$$r_e = r_0 + \frac{\epsilon}{2Vr_e} + \frac{\epsilon}{2V}\sum_{\vec{q}}{}' \frac{1}{q^2 + r_e} \; , \tag{22}$$

where the prime on the sum means that the $q = 0$ term is excluded and appears separately. In order to write this equation in a form closer to the infinite-volume case let us introduce

$$r_{0c} = -\frac{\epsilon}{2V}\sum_{\vec{q}}{}' \frac{1}{q^2} \; , \tag{23}$$

which becomes the critical value of r_0 in the $V \to \infty$ case. Then Eq. (22) takes the form

$$r_e = r_0 - r_{0c} + \frac{\epsilon}{2Vr_e} - \frac{\epsilon r_e}{2V}\sum_{\vec{q}}{}' \frac{1}{q^2(q^2 + r_e)} \; . \tag{24}$$

For $r_0 < r_{0c}$ the only positive solution of this equation is of the order of $1/V$ and for large volume we have

$$r_e \approx \frac{\epsilon}{2V(r_{0c} - r_0)} \; . \tag{25}$$

For $r_0 \gg r_{0c}$, on the other hand, r_e becomes

$$r_e \approx r_0 \; , \tag{26}$$

apart from corrections of the order of $1/V$. Therefore, similar to the uniform mode case one can see a sudden change in the behavior of r_e (r_0). If we remember that the normalized modulus squared of the field is

$$l^2(t) = \frac{1}{n}\sum_{i=1}^{n} \frac{1}{V}\sum_{\vec{q}} \langle S_i(\vec{q},t) S_i(-\vec{q},t)\rangle \; , \tag{27}$$

and in equilibrium it is related to r_e (r_0) by

$$l_e^2 = r_e - r_0 \; , \tag{28}$$

we can apply again the geometrical picture worked out in Sec. II. We can distinguish a disordered region ($r_0 \gg r_{0c}$), where $l_e^2 \sim 1/V$ and an ordered region ($r_0 \ll r_{0c}$), with $l_e^2 \approx |r_0|$. The transition between these regions can be seen on Fig. 2, which shows also that the effect of the finite volume is very similar to the uniform mode case.

Turning to dynamical properties, let us consider Eq. (2) in the spherical limit. To derive an equation of motion, considerations introduced previously can be taken over also in the case of finite volume. Assume the initial state to be a homogeneous one, the average of one of the components ($i = 1$) of the field to be macroscopic and of the order $n^{1/2}$. Then we can write

$$S_1(\vec{x},t) = (V\sqrt{n})m(t) + L(\vec{x},t) \; , \tag{29}$$

where $m(t) = \langle S_1(\vec{x},t)\rangle / V\sqrt{n}$ with the angular brackets denoting again the average over the noise and the initial conditions. Then averaging Eq. (2) with $j=1$ and $q=0$ and collecting the terms of the order $n^{1/2}$, we obtain

$$\dot{m}(t)=-\Gamma(t)m(t)\ , \tag{30}$$

where

$$\Gamma(t)=r_0+m^2(t)+\frac{1}{V}\sum_{\vec{q}}C(\vec{q},t)\ , \tag{31}$$

and $C(\vec{q},t)$ is the Fourier transform of the transverse correlation function $C(\vec{x},t)=\langle S_i(\vec{x},t)S_i(0,t)\rangle$ $(i=2,\ldots,n)$.

The equation for $C(q,t)$ is obtained by solving Eq. (2) for $i\neq 1$, under the assumption that the transverse components of the field $S_{i\neq 1}(\vec{x},t)$ are of the order of n^0. The result is

$$C(\vec{q},t)=C(\vec{q},0)\exp\left[-2\int_0^t ds[q^2+\Gamma(s)]\right] +\epsilon\int_0^t dt'\exp\left[-2\int_0^t ds[q^2+\Gamma(s)]\right]. \tag{32}$$

Now from Eqs. (30)–(32) one obtains a single nonlinear integro-differential equation for m,

$$\dot{m}(t)=-\left[r_0+m^2(t)+\frac{1}{V}\sum_{\vec{q}}C(\vec{q},0)e^{-2q^2t}\frac{m^2(t)}{m^2(0)} +\frac{\epsilon}{V}\sum_{\vec{q}}\int_0^t dt' e^{-2q^2(t-t')}\frac{m^2(t)}{m^2(t')}\right]m(t)\ . \tag{33}$$

It is important to recognize that a simple change of variables

$$y(t)=\frac{m^2(0)}{m^2(t)} \tag{34}$$

linearizes Eq. (33),

$$\tfrac{1}{2}\dot{y}=r_0 y+m^2(0)+\frac{1}{V}\sum_{\vec{q}}C(\vec{q},0)e^{-2q^2t} +\frac{\epsilon}{V}\sum_{\vec{q}}\int_0^t dt' e^{-2q^2(t-t')}y(t')\ , \tag{35}$$

and then it can be solved by Laplace transformation. Introducing the definition

$$Y(z)=\int_0^\infty dt\, e^{-zt}y(t)\ , \tag{36}$$

we obtain from Eq. (35)

$$Y(z)=\frac{\dfrac{1}{2}+\dfrac{m^2(0)}{z}+\dfrac{C(0,0)}{Vz}+\dfrac{1}{V}\sum_{\vec{q}}{}'\dfrac{C(\vec{q},0)}{z+2q^2}}{\dfrac{z}{2}-r_0-\dfrac{\epsilon}{Vz}-\dfrac{\epsilon}{V}\sum_{\vec{q}}{}'\dfrac{1}{z+2q^2}}\ , \tag{37}$$

where the $q=0$ terms are separated explicitly. Of course, since $y(t)=m^2(0)/m^2(t)$ and $m(t)\to 0$ as $t\to\infty$, we expect $y(t)$ to be an increasing function. The asymptotic behavior will be determined by the only singularity (a simple pole) on the $\mathrm{Re}(z)>0$ half-plane located at

$$z_0=2r_e\ . \tag{38}$$

One can obtain this result by noting that the condition of vanishing of the denominator of $Y(z)$ coincides with the static self-consistency equation provided $z/2$ is replaced by r. Thus we arrive at the result that the long-time behavior of $m(t)$ is given by

$$\frac{m(t)}{m(0)}=\frac{1}{\sqrt{y(t)}}\sim e^{-r_e t}\ , \tag{39}$$

and the asymptotic behavior is completely analogous to the homogeneous mode case. Since for $r_0\ll r_{0c}$, $l_e^2\approx|r_0|$, the relaxation time can be expressed as

$$\tau_m=\frac{1}{r_e}\approx\frac{2(r_{0c}-r_0)V}{\epsilon}\approx\frac{2l_e^2}{\epsilon/V} \tag{40}$$

and can be interpreted again as the orientational diffusion of the magnetization.

It is important to understand whether the orientational diffusional mode is separated from all the other relaxational mechanisms in the system. This can be examined by looking at the other singularities of $Y(z)$. For finite volume they are all simple poles on the $\mathrm{Im}(z)=0$, $\mathrm{Re}(z)<0$ axis and the distance between the poles is of the order $q^2\sim L^{-2}$. The short-time behavior of $y(t)$ is related to the large-z form of the Laplace transform. For $t\ll L^2$ the region of interest is $z\gg L^{-2}$ where the sums in Eq. (37) can be replaced by integrals, and as a consequence the initial-time evolution of the system is the same as in the infinite volume. The volume starts to be important for $t\approx L^2$. Since the pole on the negative half-plane closest to the origin is also at a distance of the order of L^{-2}, we see that for times $t\gg L^2$ all the relaxational processes died out, except the orientational diffusion which has a time scale $\tau_m\sim L^d$ [Eq. (40)] well separated from the other ones for $d>2$. So for observation times $\Delta t\ll L^d$ we expect metastablelike behavior, analogous to the homogeneous-mode case (see Fig. 4). However, this metastable-like behavior appears only above dimension $d=2$, i.e., the same condition under which the infinite system has a phase transition.

Finally we mention that the above results imply that the finite-size anlaysis of the Monte Carlo simulation of systems with broken continuous symmetry should be carried out especially carefully. In order to extract those properties of the system which remain intact in the thermodynamic limit, one must choose an observation time which is much larger than the relaxation time of the observed quantities but much smaller than the characteristic time of the phase diffusion. In principle, the phase-diffusion problem can be eliminated by increasing the size of the system. In reality, however, the relaxation times of the system might be very large, as in spin-glasses for example, and then the size needed to separate the phase diffusion from other processes might be prohibitively large. In such cases one must devise other methods to separate the phase diffusion part of the relaxation.[15]

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