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Depinning and wetting transitions in one and two dimensions

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Possible phase transitions and the critical properties of liquid films on top of substrates are discussed. Our ideas seem to explain a disagreement with mean-field theory in a recent experiment by Kwon *et al.* Connection with models studied in the context of instantons is pointed out.

I. INTRODUCTION

There has been recent interest in the statistical physics of fluid films epitaxially adsorbed on substrates.[1,2] The question of wetting of a three-component system near the critical point of mixing of two of the components had been studied by Cahn.[1] More recently, DeOliveira and Griffiths[3] (DeOG) and Pandit, Schick, and Wortis[4] (PSW) studied the systematics of thick epitaxial films. DeOG pointed out the role played by the roughening transition on layer formation. PSW discussed possible phase diagrams for different substrate strengths. A self-consistent calculation connecting layer formation and the roughening transition was carried out by Weeks.[5] The question of surface effects on critical behavior had been discussed earlier in the context of magnetic transitions.[6] DeOG and PSW in fact based their calculations on Ising models. This analogy is further explored by Nakanishi and Fisher[7] who analyzed the global phase diagram of wetting on the basis of Landau phenomenological theory.

The wetting transition is concerned with whether or not a liquid layer forms on (wets) the substrate. The thickness of the liquid layer need not be infinite after wetting. The structure of this layer has been discussed by Widom[8] with Landau-type arguments. A similar question has also been discussed by Sheng[9] and Allender *et al.*[10] and experimentally investigated by Miyano[11] in surface phenomena in nematic liquid crystals. The way in which the film becomes infinitely thick has been called the depinning (DP) transition and has been discussed extensively in one dimension by Abrahams,[12] Chui and Weeks,[13] Burkhardt,[14] Van Leeuwen and Hilhorst,[15] Chalker,[16] and Vallade and Lajzerowicz.[17] It is also called critical wetting by PSW. All these calculations agree with each other. They found that the film thickness goes to infinity as $(T-T_{\rm DP})^{-1}$. In two dimensions there is no exact solution. Vieira and Burkhardt[18] have performed self-consistent-field calculations. They found no phase transition for the Gaussian model. For the solid-on-solid model they found a phase transition such that the film thickness goes to infinity as $\ln(T-T_{\rm DP})$, in agreement with the prediction of Widom and Cahn. Chalker[19] has performed a duality-type transformation and found a pinning transition in all cases if the pinning potential possesses a repulsive core. Kroll and Lipowsky[20] have performed a renormalization-group analysis and found that the film thickness in the two dimensions (2D) goes to infinity as $(T-T_{\rm DP})^{-1}$. The relevance of the depinning transition to magnetic systems has been discussed by Lipowsky.[21]

In the presence of an external gravitational potential, the width of the film thickness is always finite. This width was calculated in mean-field theory by de Gennes[22] using the ideas of Cahn and Widom but incorporating the long-range nature of the van der Waals forces between the substrate and the epitaxy. The quantitative value of this width disagrees with that found experimentally by Kwon *et al.*[23]

Actually, very similar models have been studied in the context of droplet formation[24] in three dimensions (3D) and "instantons" in four dimensions[25] (4D), respectively. Our picture is as follows: The interface spends most of its time away from the substrate. Every once in a while, it comes close to the substrate via the formation of a kind of 2D domain or instantons, in which the film is thinner than the surrounding area. This corresponds closely to droplet formation in 3D in the sense that a droplet is a 3D domain in which the density is higher than the surrounding volume. Using the basic idea of these approaches here, we try to provide a physical picture of the depinning transition in 2D and resolve some of the disagreement mentioned. Let us outline the contents and summarize our results here.

In Sec. II we shall set up our model and make connection with works of earlier workers. In Sec. III we shall briefly recapitulate the results for the short-range pinning potential in 1D. New results for different pinning potentials are discussed. The possibility of observing a resonance of the transfer matrix in the depinned state is mentioned. In Sec. IV our results for 2D depinning will be presented. We found that just as in 1D a phase transition will occur only if the pinning potential has a repulsive core. In the absence of any excess chemical or gravitational potential the interface width grows as $\ln(T-T_{\rm DP})$. In the presence of an external gravitational potential, the functional dependence of the thickness on this potential is the same as that predicted by the mean-field theory under some conditions. However, its magnitude is reduced by a "tunneling factor" which is quite substantial in general.

This reduction may explain the disagreement of Kwon's experiment with mean-field predictions. There are also conditions under which the functional dependence according to mean-field theory is not obeyed.

II. MODEL

In this paper we shall always assume that one is above the roughening temperature. The liquid-height profile above the substrate is described by a set of height variables (h_i) where i may be a 1D or 2D index. The energy functional E of the system is described by a surface-tension term E_1 that tends to keep the surface flat and a pinning potential E_2 that keeps the liquid close to the substrate. Specifically, we have $E=E_1+E_2$,

$$\begin{aligned} E_1 &= \frac{J}{2}\sum_{i,\delta}(h_i-h_{i+\delta})^2 , \\ E_2 &= \sum_i V_0(h_i)+\frac{w}{2}(h_i+1/2)^{-2} . \end{aligned} \tag{1}$$

Here V_0 is an exponentially short-ranged attractive potential of range b with a repulsive core. For simplicity we take the short-range part of E_2 to be a square well of magnitude A. The exponential part of V_0 we represent as $\bar{V}_0\exp(-h_ir)$. The second term in E_2 comes from the van der Waals interaction. w is a constant. According to Ref. 22, w is proportional to $-(\alpha_l-\alpha_s)(\alpha_g-\alpha_s)$ where α_l, α_g, and α_s are the polarizibilities of the liquid, gas, and substrate, respectively. We expect w to be positive and have assumed so here. This need not be true in general, however. J is proportional to the interface tension, as is discussed by Weeks.[26] The dependence of E_2 on h_i is illustrated in Fig. 1. We shall assume the intercolumn distance δ to be unity. One can regard Eq. (1) as the starting point of our model. However, it may be worthwhile to recapitulate the connection of E_2 with previous works using Landau phenomenological theory. (E_1 has been discussed by Weeks.[26]) Let us first ignore the van der Waals interaction. By denoting the concentration at a distance z from the substrate by $c(z)$ and ignoring lateral fluctuations, the energy U of the system can be written in the notation of Cahn as

$$U=\Phi(c_s)+\int_0^\infty[\Delta f+K(dc/dz)^2]dz . \tag{2}$$

Here c_s is the concentration of the fluid at the surface. $\Phi(c)$ is the short-range substrate-fluid interaction energy. Δf is the free-energy difference between the liquid and the gas. $K(dc/dz)^2$ is an elastic energy. Typically Δf has a double hump. Equation (2) has been studied intensively in the context of domain walls.[27] In general, by minimizing (2), the qualitative behavior of $c(z)$ is as illustrated in Fig. 2 where the energy dependence of that configuration on h drops off exponentially fast. Our model in Eq. (1) is valid when the domain-wall thicknesses Δ_1,Δ_2 are much less than the height variable h. From the domain-wall work, we expect Δ_1,Δ_2 to be proportional to $(m_1/K)^{1/2}$ and $(m_2/K)^{1/2}$ where m_1,m_2 are the curvature of Δf at the first and second minimum. The energy of the configuration in Fig. 1 is a compromise between the substrate-fluid energy $\Phi(c_s)$ and the elastic energy $a(m_1k)^{1/2}+b(m_2k)^{1/2}$ (a,b are constants of the order of unity). The energy of this configuration as a function of h is what we have called $V_0(h_i)$ in Eq. (1). It is obvious that before the wetting transition, it will be attractive so that h is small. As one goes toward and passes wetting, it will be less attractive. As we shall learn later, the film thickness h becomes infinite when this potential is still attractive.

In some experiments the substrate itself can also move. J in Eq. (1) then corresponds to the "effective mass," i.e., $J^{-1}=J_1^{-1}+J_2^{-1}$, where J_1,J_2 are the interface tensions of the substrate-liquid and the "liquid-vapor" interfaces, respectively.

In some problems there is an external chemical potential $h\Delta\mu$ that keeps the film from becoming infinitely thick. $\Delta\mu$ may come from an external gravitational field as in the experiment of Kwon *et al.*[23]

III. ONE DIMENSION REVISITED

All the 1D calculations focus on the transfer matrix, which is essentially identical to the Hamiltonian of a quantum particle in a potential. The phase transition is determined by whether or not the potential can support a bound state. When the potential can no longer support a bound state there can still be resonances. We shall explore what this means physically in this section. All previous work was carried out for short-range potential. In this section we shall examine an example that illustrates the effect of the long-range potential on the critical behavior.

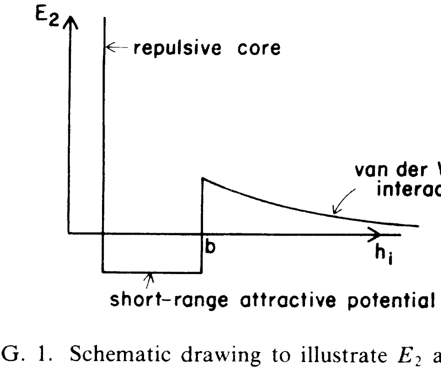

FIG. 1. Schematic drawing to illustrate E_2 as a function of h_i.

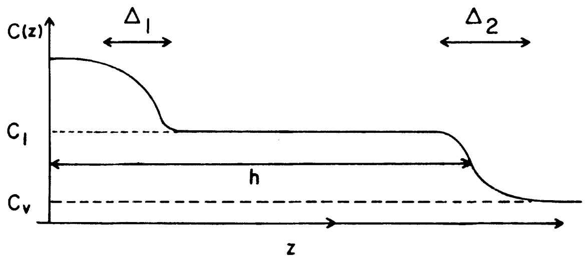

FIG. 2. Schematic illustration of the density profile of a fluid film absorbed on a substrate at $z=0$.

We are interested in the partition function

$$Z=\sum_{\{h_i\}}\exp(-\beta E)\ . \tag{3}$$

This can be written in terms of the transfer matrix T as

$$Z=\mathrm{Tr}T^n\ ,$$

$$T(h,h')=\exp[-\beta J(h-h')^2]\exp[-\beta V_0(h)]\ . \tag{4}$$

In the continuum limit the eigenvalue equation for the transfer matrix becomes

$$(4\beta J)^{-1}\partial_z^2\psi+\beta V_0\psi=\lambda\psi\ . \tag{5}$$

We approximate V_0 by ignoring its exponential tail. For a short-range attractive potential backed by an impenetrable barrier, a bound state begins to form at some critical potential strength, V_c. The wave function at large z behaves as $\exp\{-[z(\lambda)]^{-1/2}\}$. The thickness of the film is inversely proportional to $\lambda^{1/2}$ and hence to $(A-V_c)^{-1}$. It is interesting to know if the critical exponent depends on the form of the potential. The eigenvalue equation for the long-range attractive potential $B(z+1/2)^{-2}$ can still be solved exactly. The eigenfunction is now the Bessel function $z^{1/2}K_{(1/4-\gamma)^{1/2}}(-\lambda^{1/2}(z+\frac{1}{2}))$ where $\gamma=4\beta^2JB$. For a bound state to occur $(\frac{1}{4}-\gamma)^{1/2}$ must be imaginary. Let $(\frac{1}{4}-\gamma)^{1/2}=i\nu$, whence ν would be real. The eigenvalue is then determined from the boundary condition $\psi(0)=0$. From the small distance expansion of $K_{i\nu}(z)$ we get $\lambda=-16\exp[-2\pi(\gamma-\frac{1}{4})^{-1/2}]$. Note that the critical coupling γ_c is just $\frac{1}{4}$. Hence in this case the thickness of the film, which is proportional to $\lambda^{-1/2}$, is proportional to $\exp[+2\pi(\gamma-\gamma_c)^{-1/2}]$. This behavior for a long-range attractive potential is quite different from that of the short-range attractive potential.

The potential in Eq. (1) consists of a short-range attractive part and a repulsive part that subsides as $(z+\frac{1}{2})^{-2}$. The eigenvalue equation can be solved first in the region where the short-range potential is absent. The eigenfunction is just $(z+\frac{1}{2})^{1/2}K_{(\gamma+1/4)^{1/2}}(\lambda^{1/2}(z+\frac{1}{2}))$. If we assume that the short-range potential is a square well with depth A, the small distance solution can also be obtained as $\sin z(A-E)^{1/2}$. The eigenvalue can then be obtained by matching the logarithmic derivative at the point c where the two potentials joined,

$$(A-\lambda)^{1/2}\cot[(A-\lambda)^{1/2}c]$$
$$=\lambda^{1/2}K'_{(\gamma+1/4)^{1/2}}(\lambda^{1/2}c)/K_{(\gamma+1/4)^{1/2}}(\lambda^{1/2}c)\ . \tag{6}$$

If the system is barely bound, λ is small, $K_\nu(\lambda^{1/2}C)\approx(c\lambda^{1/2})^{-\nu}$, and

$$(A-\lambda)^{1/2}\cot[c(A-\lambda)^{1/2}]\approx 1-(1+4\gamma)^{1/2}\ . \tag{7}$$

The critical γ_c is determined by

$$A^{1/2}\cot(cA^{1/2})\approx 1-(1+4\gamma_c)^{1/2}\ . \tag{8}$$

Note that $cA^{1/2}$ must be larger than $\pi/2$, otherwise no bound state occurs. The critical behavior can be obtained by considering small changes of γ from γ_c. The same result as for the short-range potential is obtained.

Even when no bound state can occur it is still possible to have a resonance. We have not been able to find any previous discussion of the resonance of the transfer matrix. It is of interest to ask if there are interesting physical consequences. The answer is yes. The correlation function of an operator P is given by

$$\langle P(0)P(r)\rangle=\sum_n\exp[-\beta(E_n-E_0)]r\,|\langle 0|P|n\rangle|^2\ , \tag{9}$$

where $|n\rangle$ are the eigenstates of the transfer matrix. The density of states near a resonance is proportional to the energy derivative of the phase shift $d\eta/dE$. Since $\eta\propto\tan^{-1}[\Gamma/(E-E_r)]$, $d\eta/dE\propto\Gamma/[\Gamma^2+(E-E_r)^2]$. Hence there is an enhanced contribution to the correlation function from the resonance. This contributes the following term to the correlation function:

$$\langle P(0)P(r)\rangle\ \propto\ \exp\{[-(E_r-E_0)+i\Gamma]\beta\}r\ .$$

The Fourier transform of this term can presumably be measured and one obtains a term of the form $[(q-\Gamma\beta)+i\beta(E_r-E_0)]^{-1}$, where q is the wave vector. We thus get a Lorentzian peak located at $\Gamma\beta$ with width βE_r.

IV. DEPINNING IN TWO DIMENSIONS

Let us first present a physical argument for the critical properties of the depinning transition. We shall then make it more precise by using ideas from the study of instantons.

Owing to thermal fluctuations in the lateral direction, the interface will consist of portions that are close to the substrate and portions that are not. Suppose at some point P the interface is around the mean distance l_0 from the substrate. Owing to thermal fluctuations, the root-mean-square deviation of the film thickness away from its mean value l_0 grows as one moves away from P. When this deviation becomes of the order of l_0 itself, there is a significant probability that the interface would come close to the surface. When this happens because of the hard core of the substrate, the interface must turn around at the expense of some elastic energy. On the other hand, due to the pinning potential some energy is saved. When the net energy is negative, the interface will be pinned. Let us first determine the cost in the elastic energy.

To simplify the discussion, we would initially ignore the long-range tail of E_2 and assume it to be zero for $h>b$. The elastic energy expended in turning the interface around is proportional to the mean-square slope $s^2=|\vec{\nabla}h|^2$ of the interface. Let us estimate s^2 by considering two points, one of which, such as P mentioned before, has the interface at around the mean distance l_0 from the substrate (see Fig. 3). If the distance between P and Q is x, then the mean slope of the interface between P and Q is l_0/x. The mean-square slope s^2, to which the elastic energy is proportional, is then given by $s^2=(l_0/x)^2+c^2$ where c is the standard deviation of the slope in the absence of the substrate. It remains to evaluate x. To do this we ask how the mean-square fluctuation of the film

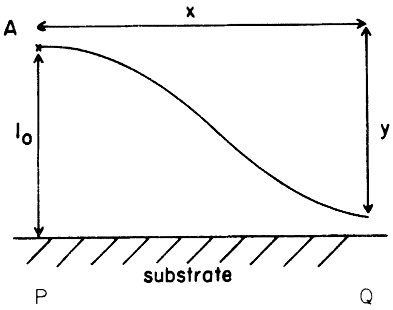

FIG. 3. Illustration of the interface at some distance 0 from the substrate coming close to the substrate at a lateral distance x due to thermal fluctuation.

thickness $\langle(\delta h)^2\rangle$ grows as we move away from the point P where we know that the film thickness h has its mean value l_0. In one dimension we have $\langle(\delta h)^2\rangle \sim \ln x$, whereas, in two, we would have $\langle(\delta h)^2\rangle \sim \ln x$. If we apply our previous criterion that x is typically the distance at which $\langle \delta h^2 \rangle$ is of the order of l_0^2 itself, we would get $x \sim l_0^2$ in 1D and $x \sim \exp(l_0^2)$ in 2D. Hence the elastic energy is of the order of $J(c^2 + l_0^{-1})$ in 1D and $J[c^2 + l_0^2 \exp(-2l_0^2)]$ in 2D. Owing to the pinning potential, the interface gains an energy $-Ab$. The interface becomes depinned when the sum of the above two energies becomes positive. From this we get

$$l_0 \propto \begin{cases} (A - v_c)^{-1} \text{ in 1D, where } v_c = Jc^2/b \\ \ln^{1/2}(A - v_c) \text{ in 2D} \end{cases} \tag{10}$$

if we ignore the pre-exponential l_0 dependence in one of the terms in the elastic energy in this crude estimate.

In the above calculation we have assumed that E_2 is zero for distances longer than b. In most physical problems $V_0(h)$ has an exponential dependence on h for large h. Even if one ignores the long-range van der Waal energy and sets $w=0$, the pinning energy gain, mentioned above, will become $-Ab + \bar{V}_0 \exp(-\kappa l_0)$, were $\bar{V}_0$ and κ are constants. In 2D, this added term would eventually become larger than the term $J l_0^2 \exp(-2l_0^2)$ in the elastic energy for a sufficiently large l_0 and takes over. When this happens, we have l_0 proportional to $\ln(A - v_c)$ instead.[1,8] Hence our results in 2D for the critical behavior agree with previous mean-field calculations when $w=0$. They disgree with the renormalization-group result of Kroll and Lipowsky. In 1D this argument gives the same result as previous calculations. By a similar argument, we also expect that any attractive pinning potential of the form Dh^{-p} will change the pinning energy gained to $-Ab + Dl_0^{-p}$. In 2D the added term would dominate the exponential elastic energy term for sufficiently large l_0 near the transition. Hence, the critical behavior of the thickness becomes $l_0 \propto (A - V_c)^{-1/p}$. In 1D it would have no effect, unless $p<1$, which is unlikely. Let us now reformulate the above calculation in a more precise way.

Models similar to ours have been discussed in the context of droplet formation in 3D (Ref. 24) and false vacuum of the universe in 4D.[25] The Lagrangian L studied in those cases [Ref. 25, Eq. (1.1)] is given by $L = \int [(\vec{\nabla}\psi)^2 - U(\psi)]$. $U(\psi)$ has a type of double-well structure. In the present case, h plays the role of ψ, U is replaced by E_2. The 3D or 4D domains of integration are replaced by one that is 2D. Note that E_2 has a kind of double-well structure. When $\Delta\mu$ is zero, the outer minimum is at $r=\infty$, but is very flat; for $\Delta\mu \neq 0$ this minimum is moved in. While the detail of these ideas is not directly applicable here, the essence of their approximation is. In general, one has to sum over all possible configurations $\{h_i\}$ to calculate Z. In particular, there are configurations that correspond to $h = l_0$ at some point P and $h = b$ at point Q, a distance R away from P. These correspond to the instantons. Z is approximated by summing over all configurations of this instanton gas. Near the depinning transition, the interface does not come close to the substrate often. One then is in the dilute limit of this gas. The interaction between instantons will hence be neglected. The only relevant parameter is then the free energy (chemical potential) of an isolated instanton. One can evaluate the probability of an instanton configuration by the method of steepest descent. The presence of the substrate can be neglected until the probability of the interface coming close to it becomes substantial. This determines an optimal distance R_0. For $R > R_0$, the interface must turn around and we do not have a single instanton configuration anymore. Let us write out the mathematical details. First let us assume $\Delta\mu = 0$. The energy of the domain wall or instanton consists of an elastic part E and a part due to the substrate potential. For $h > b$, the substrate part is small and slowly varying. Hence the interface energy is dominated by elastic energy E. The energy functional E is in circular coordinates, with ρ as the distance from Q,

$$E = 2\pi J \int \rho^{d-1} \left[\frac{dh}{d\rho} \right]^2 d\rho, \quad h > b \ . \tag{11}$$

One seeks a path $g_0(\rho)$ that extremizes E subjected to the boundary condition $g_0(0) = b$ and $g_0(R) = l_0$. [In the droplet language, this corresponds to picking configurations $\rho(r)$ such that for large r, ρ represents gas density while at small r, ρ represents liquid density.] This can be obtained from the Euler Lagrange equation

$$\frac{d}{d\rho} \left[\rho^{d-1} \frac{dg_0}{d\rho} \right] = 0 \ . \tag{12}$$

We get

$$g_0 = \begin{cases} \alpha\rho + \beta, & d=1 \\ \alpha \ln\rho + \beta, & d=2 \end{cases} \tag{13}$$

where α, β are constants. From the boundary condition we have $\beta = b$,

$$\alpha = \begin{cases} (-b + l_0)/R, & d=1 \\ (-b + l_0)/\ln(R/b_0), & d=2 \end{cases} \tag{14}$$

where b_0 is a lower limit cutoff of ρ.

To complete the steepest descent calculation, one considers a small deviation from the optimum path g_0, i.e., $h = g_0 + g$. The energy functional becomes

$$E = \int \rho^{d-1} \left[\frac{dq}{d} \right]^2 d\rho + \alpha^2 \int_0^R \rho^{1-d} d\rho \ . \tag{15}$$

The mean slope is

$$\left\langle \frac{dh}{d\rho} \right\rangle = \alpha R^{1-d} + c \ , \tag{16}$$

where $c = \langle dq/d\rho \rangle$ is a number of the order of unity. The free energy F_0 for these configurations is obtained by integrating over g. Hence

$$F_0 = \bar{F}_0 + \begin{cases} 2\pi J l_0^2 / R, & d = 1 \\ 2\pi J l_0^2 / \ln(R/b_0), & d = 2 \end{cases} \tag{17}$$

where F_0 is the contribution due to g. So far the value of R has not been specified. As we mentioned at the beginning of this calculation, this is determined by the requirement that the probability of the interface coming close to the substrate becomes significant. This happens when $\beta(F - F_0)$ is of the order of unity. This determines an optimum R_0 given by

$$R_0 \propto \begin{cases} 2\pi J \beta l_0^2, & d = 1 \\ 2\pi J b_0 \exp l_0^2, & d = 2 \ . \end{cases} \tag{18}$$

From this the slope s in the preceding section is obtained. The rest of the calculation is then essentially as described in the first paragraph. We add the energy gained due to the pinning potential. The net energy determines if the configurations are favorable or not.

Let us next turn our attention to the case such that $\Delta\mu$ is finite. In this case the interface is always bound. One would like to determine the height of the interface as a function of $\Delta\mu$. Again we evaluate this by the method of steepest descent. The pinning potential now consists of a double well with an inner minimum at around $h = b$ of magnitude A and an outer minimum at $h_0 \simeq (\Delta\mu/c)^{-1/3}$ of magnitude $V(h_0) = 3(\Delta\mu^2 c)^{1/3}/2$. For small deviations from h_0, the potential is given by

$$\begin{aligned} V(h) &= V(h_0) + \tfrac{3}{2}(\Delta h)^2 (\Delta\mu 4/c)^{1/3} \\ &\equiv V(h_0) + J G^2 (\Delta h)^2 \ . \end{aligned}$$

At the depinning transition A is of the order of $-Jb^2$. The favorable configurations will consist of the following. Part of the interface will be at the inner minimum while part of it will be at the outer minimum. In between, they are joined by a kind of domain wall, the instantons. The free energy per unit area of an interface at the outer minimum is approximately given by

$$F(h_0) = -\frac{T}{2} \ln(\beta J/\pi) - \frac{T}{2(2)^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} dg_x dg_y \ln[\phi(0) - \phi(g) + G^2] + V(h_0), \quad \phi(g) = 2(\cos g_x + \cos g_y) \ . \tag{19}$$

We shall assume that when the interface is located at the inner minimum, the entropy associated with it is sufficiently small to be negligible. The probabilities of a unit area of the interface at the outer and inner minimum are proportional to $\exp[-\beta F(h_0)] \equiv P_0$ and $\exp(-\beta V_0) \equiv P_i$, respectively. However, since the total interface is continuous, we now have to estimate the energy of the domain wall (W_d) joining these regions. If $P_i > P_0$, the net probability of finding the interface at the outer minimum will be reduced by the factor $\exp(-\beta W_d)$. This corresponds to the "tunneling probability" in the instanton language. Because of this factor, the average height of the interface is reduced. We think it is this factor that causes the disagreement between the experimental result of Kwon *et al.* and the mean-field estimate of de Gennes.

It is difficult to give an analytic expression for W_d valid for the entire range of the parameters. There are essentially two limits depending on the relative magnitude of Jh_0^2 and wh_0^{-2}. If $Jh_0^2 \ll wh_0^{-2}$ then the domain wall should be very thin. In the other limit it should be thick. If one is close to the critical point, then one of the interface tensions (J_i) is small. Hence the domain wall should be thin. In that case the domain-wall energy W_d per unit length is approximately $\frac{1}{2}J(h_0 - b)^2$.

We now turn our attention to the evaluation of the average height $\langle h \rangle$. Let us assume that $P_i > P_0$. First, we shall assume that $h_0 \gg b$. Since we are interested in the average height $\langle h \rangle$, even though the possibility for an interface to be at the outer minimum is small, they are the ones that provide the dominant contribution to $\langle h \rangle$. The optimum area A_0 of the interface at the outer minimum can be determined by minimizing the quantity

$$-\beta[F(h_0) - V_0]A_0 - \beta W_0 (A_0)^{1/2} + \ln A_0$$

with respect to A_0. Let us call the optimum value of this K_0. The average height $\langle h \rangle$ is then approximately equal to $h_0 \exp(-K_0)$. Presumably, the parameters in this calculation can be determined from experiments. It would be interesting to measure the interface tension J and see if the tunneling factor does account for the discrepancy between experiment and the mean-field calculation. For h_0 comparable to b, the above simple functional dependence is not obeyed, and the dependence of $\langle h \rangle$ on $\Delta\mu$ is then much more gradual.

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