Analytic vortices and magnetic resonances in rotating superfluid ^{3}He-A. II.

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The earlier analysis of the vortex lattices in rotating ^{3}He-A is extended to lower temperatures. It is shown that, contrary to the earlier assertion, the observed satellite frequencies lie in between those associated with the circular and the hyperbolic vortices when the temperature-dependent corrections are properly included. Therefore, we conclude that the vortex lattice of the circular-hyperbolic type is indeed realized in the Helsinki experiment.

I. INTRODUCTION

In an earlier paper[1] (which will be referred to as I hereafter), we have studied the two-dimensional array of analytic vortices[2] in rotating superfluid ^{3}He-A in the presence of a strong axial magnetic field and we find that (a) the circular-hyperbolic vortex lattice has the lower free energy in the vicinity of the Ginzburg-Landau regime (this is an extension of the result due to Fujita *et al.*[2] in the presence of a magnetic field) and that (b) the observed satellite resonance frequencies in a recent experiment by Hakonen *et al.*[3,4] are consistent with the satellite resonance associated with the radial vortex. Therefore, it is quite puzzling why the radial vortex, which has the highest free energy, was observed.

In order to resolve this puzzle, we shall extend our analysis of the nuclear magnetic resonances (NMR) in the presence of analytic vortices to lower temperatures. For this purpose, we make use of the equilibrium vortex solutions found in I where the lowest-order corrections in ϵ ($\equiv 1-T/T_c$) are included. These vortex configurations are determined by making use of the generalized Ginzburg-Landau free energy due to Cross.[5] As already stated in I, the conclusion (a) is still valid when the lowest-order corrections in ϵ are included. On the other hand, we discover that the observed satellite frequencies[4] lie in between those corresponding to the circular votex and those corresponding to the hyperbolic vortex, if the lowest-order corrections in ϵ are properly accounted. Therefore, we believe now that the observed satellite peaks[3,4] consist of two satellites; the one due to the circular vortices and the other due to the hyperbolic vortices. This interpretation also explains the unusually large width of the observed satellites. Finally the observed intensity[4] of the satellite signal is also consistent with the present interpretation. We expect that, if the interpretation is correct, the satellite resonance associated with the analytic vortices will split into two separate satellite resonances at sufficiently low temperatures. We have also calculated the longitudinal resonance satellite frequencies, of which detection will further confirm the existence of the underlying vortex lattice.

Since the analysis of the vortex configurations have been already done in the Appendix of I, we shall limit our consideration to the satellite resonance frequencies. Further, we limit ourselves to the lowest-order correction in ϵ, which appears to be adequate in the temperature regime of the present interest (for example, $0<\epsilon<0.4$).

II. MAGNETIC RESONANCES

We shall study the temperature dependence of the satellite resonance frequencies due to analytic vortices, when the lowest-order corrections in $\epsilon=1-T/T_c$ are included. We shall consider the three types of vortices separately.

A. Circular vortex

The circular vortex is an analytic vortex with the lowest free energy among three analytic vortices to be considered.[1,2] The transverse and the longitudinal satellite frequencies are determined by the eigenvalues of the following equations[1,6]:

$$\lambda_g g = -\frac{1}{r}\frac{\partial}{\partial r}(r g_r) + (1-2\cos^2\beta) g \ , \tag{1}$$

$$\lambda_f f = -\frac{1}{r}\frac{\partial}{\partial r}(r f_r) + (1-\cos^2\beta) f \ , \tag{2}$$

as

$$\omega_t^{\text{sat}} = (\omega_0^2 + R_t^2 \Omega_A^2)^{1/2} \ , \tag{3}$$

$$\omega_l^{\text{sat}} = R_l \Omega_A \ , \tag{4}$$

where

$$R_t^2 = \lambda_g \text{ and } R_l^2 = \lambda_f \tag{5}$$

and ω_0 is the Larmor frequency and Ω_A is the Leggett frequency in ^{3}He-A.

Here the length r is measured in the unit of $\xi_\perp$ (10 μm), the dipole coherence length, and $\cos\beta$ is given by[1]

$$\cos\beta = e^{-(\eta r)^2} \tag{6}$$

with

$$\eta^{-2} = \tfrac{3}{2} + 2(\tfrac{1}{3}A_1 - B_1)\epsilon \tag{7}$$

and

$$A_1 = \frac{F_1}{1+F_1/3}, \quad B_1 = \frac{F_1^a}{1+F_1^a/3} \ , \tag{8}$$

and F_1 and F_1^a are the Fermi-liquid coefficients.

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Equations (1) and (2) are solved variationally; to the lowest order in ϵ we find

$$\lambda_g = 0.874 - 0.192(\tfrac{1}{3}A_1 - B_1)\epsilon \ , \qquad (9)$$

$$\lambda_f = 0.995 - 0.022(\tfrac{1}{3}A_1 - B_1)\epsilon \ . \qquad (10)$$

From Eqs. (9) and (10), R_t and R_l are easily obtained.

B. Hyperbolic vortex

The corresponding eigenequations are now given by [1]

$$\lambda_g g = -\frac{1}{4r}\frac{\partial}{\partial r}[r(3+\cos^2\beta)g_r] + (1-2\cos^2\beta)g \ , \qquad (11)$$

$$\lambda_f f = -\frac{1}{4r}\frac{\partial}{\partial r}[r(3+\cos^2\beta)f_r] + (1-\cos^2\beta)f \ , \qquad (12)$$

where $\cos\beta$ is still given by Eq. (6) but with

$$\eta^{-2} = 2 + (A_1 - \tfrac{11}{6}B_1)\epsilon \ . \qquad (13)$$

Then up to the lowest order in ϵ, a variational calculation yields

$$\lambda_g = 0.720 - 0.218(A_1 - \tfrac{11}{6}B_1)\epsilon \ , \qquad (14)$$

$$\lambda_f = 0.958 - 0.052(A_1 - \tfrac{11}{6}B_1)\epsilon \ . \qquad (15)$$

C. Radial vortex

The corresponding eigenequations are given by

$$\lambda_g g = -\frac{1}{2r}\frac{d}{dr}[r(1+\cos^2\beta)g_r] + (1-2\cos^2\beta)g \ , \qquad (16)$$

$$\lambda_f f = -\frac{1}{2r}\frac{d}{dr}[r(1+\cos^2\beta)f_r] + (1-\cos^2\beta)f \ , \qquad (17)$$

where $\cos\beta$ is given by Eq. (6) but with

$$\eta^{-2} = \tfrac{5}{2} + (\tfrac{4}{3}A_1 - \tfrac{5}{3}B_1)\epsilon \ . \qquad (18)$$

Variational solutions of Eqs. (16) and (17) yield

$$\lambda_g = 0.4928 - 0.223(A_1 - \tfrac{5}{4}B_1)\epsilon \ , \qquad (19)$$

$$\lambda_f = 0.889 - 0.0852(A_1 - \tfrac{5}{4}B_1)\epsilon \ . \qquad (20)$$

In the limit ϵ approaches zero (i.e., at $T = T_c$), these results agree with the results obtained in I.

III. DISCUSSION

The transverse satellite resonance frequencies observed by Hakonen *et al.*[4] are conveniently fit by

$$R_t^{\mathrm{ex}} = 0.895 - 1.24\epsilon \qquad (21)$$

for $0 \le \epsilon < 0.4$, although there is about 5% uncertainty in the coefficient of ϵ. Comparing Eq. (21) with λ_g's calculated in the preceeding section, we discover Eq. (21) lies in between the R_t's for the circular and the hyperbolic vortices at least in the limit $\epsilon = 0$. Therefore this appears to imply that the observed satellite is the overlapping of two satellites; one is due to the circular and the other due to the hyperbolic vortices. This identification becomes even more convincing due to the fact that the circular-hyperbolic pair of vortices is the basic unit to build up the two-dimensional vortex lattice with the lowest free energy. It is then quite tempting to study the temperature dependence of the observed R_t. Suppose that the observed R_t is given by the weighted average of λ_g's. Noting that the transverse resonance frequencies are given approximately by

$$\omega_t^{\mathrm{sat}} \cong \omega_0 + \lambda_g \Omega_A^2 (2\omega_0)^{-1} \qquad (22)$$

in the present circumstances ($H = 287\,0e$), and the intensity of the circular vortex signal is about 2 times larger than that of the hyperbolic vortex,[1] we obtain

$$\overline{\lambda_g} = \tfrac{1}{3}(2\lambda_g^c + \lambda_g^h)$$
$$= 0.8233 - (0.153A_1 - 0.373B_1)\epsilon \qquad (23)$$

where the superfices C and H on λ_g indicate the circular and the hyperbolic, respectively. Then the averaged R_t is given by

$$\overline{R}_t = (\overline{\lambda}_g)^{1/2}$$
$$= 0.907 - (0.0843A_1 - 0.2055B_1)\epsilon \ . \qquad (24)$$

Comparing the coefficient of ϵ with that of Eq. (21), we obtain

$$0.0843A_1 - 0.2055B_1 = 1.24 \ . \qquad (25)$$

Then making use of $F_1 = 15.66$ as tabulated by Wheatley,[7] we find

$$F_1^a = -1.82 \ . \qquad (26)$$

This F_1^a value is somewhat larger in the absolute magnitude than that deduced from the spin-wave spectrum[8] of ^{3}He-*B* and from the satellite frequency of the composite soliton[9] in ^{3}He-*B* but is not inconsistent.

Making use of these A_1 and B_1 thus determined, we have calculated R_t's and R_l's for three types of vortices, which are shown in Table I. Furthermore, the ϵ-

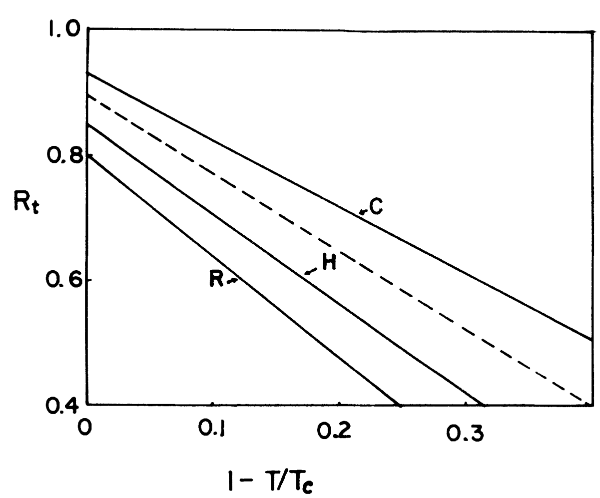

FIG. 1. R_t's associated with three types of analytic vortices are shown as function of $\epsilon = 1 - T/T_c$. The letters C, H, and R indicate the circular, hyperbolic, and radial vortices, respectively. The experimental data are shown by a broken line.

TABLE I. The predicted R_t's and R_l's, which appear in the NMR satellite frequencies, are given for three types of analytic vortices.

	R_t	R_l
Circular	$0.935-1.06\epsilon$	$0.997-0.12\epsilon$
Hyperbolic	$0.849-1.42\epsilon$	$0.979-0.305\epsilon$
Radial	$0.702-1.34\epsilon$	$0.943-0.378\epsilon$

dependent R_t's are shown in Fig. 1 together with the experimental result (21). The experimental curve lies right in the middle of the theoretical values for the circular and the hyperbolic vortices, while the one for the radial vortex is much smaller.

As to the intensity of the satellite resonances due to the analytic vortices, we can generalize the result obtained in I as

$$I_{\rm cir}=14.8(1+3.89\epsilon)(2\pi\xi^2)\;,$$
$$I_{\rm hyp}=6.41(1+2.92\epsilon)(2\pi\xi^2)\;, \tag{27}$$
$$I_{\rm rad}=2.9(1+2.34\epsilon)(2\pi\xi^2)\;,$$

where $I_{\rm cir}$, $I_{\rm hyp}$, and $I_{\rm rad}$ are the intensity of the corresponding single vortex. If the circular-hyperbolic vortex lattice is formed the total satellite intensity is given by

$$\begin{aligned} I_{\rm sat} &= \tfrac{1}{2}(I_{\rm cir}+I_{\rm hyp})\Omega/K \\ &= 10.6(1+3.6\epsilon)(2\pi\xi^2)\Omega/K \end{aligned} \tag{28}$$

with $K=h/2m_3$. Equation (28) is also quite consistent with the experimental observation.[4] Therefore we conclude that the two-dimensional vortex lattice proposed by Fujita *et al.*[2] describes the observed transverse resonance satellite satisfactorily. We predict also the longitudinal satellites, the detection of which will provide a crucial test of the theoretical model proposed here.

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