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Remarks on the Laughlin theory of the fractionally quantized Hall effect

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An extension of the Laughlin theory of the anomalous quantum Hall effect to allow for integer numerators for the occupation fraction is proposed.

Laughlin has, in a fascinating paper,[1] proposed a new form of condensed state of a two-dimensional electron gas in a large magnetic field, where the separation of Landau levels is large compared with all other energies. Laughlin's theory gives a natural explanation for special stability at densities such that the occupation fraction ν is

$$\nu=\frac{1}{m}=\frac{1}{2i+1} \quad , \tag{1}$$

where m is an odd integer and i is any integer.

This theory was formulated to explain the experiments of Tsui, Störmer, and Gossard[2] which initially demonstrated plateaus in σ_{xy} and minima in σ_{xx} at $\nu=\frac{1}{3}$ and $\frac{2}{3}$. Now it is becoming clear that several values of ν of the form

$$\nu=\frac{p}{m}=\frac{p}{2i+1} \quad , \tag{2}$$

with $m=3$, 5, and 7 and $p=2$, 3, or 4, are also showing anomalies. The unmodified Laughlin theory does not explain $p\neq 1$ or possibly $(1-m)$. The purpose of the present paper is to propose a modification of this theory which may permit $p\neq 1$.

The Laughlin theory consists initially of a proposed ground-state wave function. In the gauge and the units in which

$$A=\vec{r}\times\hat{n} \quad ,$$

($\hat{n}$ a unit vector in the Z direction, $\vec{r}$ the position vector in units of the magnetic length in the x-y plane), if we define $\vec{r}_i=(x_i,y_i)$, $Z_i=x_i+iy_i$,

$$\Psi_L^{(m)}=\prod_{i>j}^{N}(Z_i-Z_j)^m \exp\left(-\sum_i^N\frac{|Z_i|^2}{2}\right) \quad . \tag{3}$$

Since all terms are of the form

$$\prod_i (Z_i)^{p_i^*}\exp\left(\frac{-|Z_i|^2}{2}\right) \quad ,$$

this Jastrow-like function is automatically made up only of states in the bottom Landau level.

Laughlin also notes that one can construct an excited state with effective charge $e^*=1/m$ by carrying out what is essentially a singular gauge transformation: We pick a point Z_0 and require that the wave function acquire an extra 2π of phase as one carries Z_i around a circuit around Z_0. One simple way to do this is as Laughlin does: to insert adiabatically a single extra quantized flux line at Z_0. Another way would be simply to multiply the initial wave function by $\prod_i(Z_i-Z_0)$: If we wish, Z_0 may be chosen at the special point $Z_0=0$. One may do this m times; after the mth, the introduction of an extra electron at the point 0 returns one locally to precisely the same wave function. In essence, this construction demonstrates that the Laughlin wave function has a *discrete* broken symmetry and a discrete order-parameter space consisting of m points. In this respect it precisely corresponds to a $1/m$ commensurate charge-density wave (CDW), which has also m possible local ground states. As is true of the CDW, strictly speaking, only by eliminating boundaries are the m possibilities truly equivalent, since at a real boundary the net charge must be adjusted in fractional amounts.

Drawing on the analogy with a charge-density wave, we point out another peculiarity of the state containing a charge $1/m$ excitation. Let us define an N-electron operator A_N^{0+} which creates the Laughlin ground state Ψ_m^L when applied to the vacuum Ψ_0. We can also define a "translation" operator $T_{1/m}$ which introduces a $1/m$ excitation when applied to A_N^{+0}:

$$\Psi_{1/m}=(T_{1/m}^{-1}A_N^{+0}T_{1/m})\Psi_0 \quad . \tag{4}$$

The essence of the argument about finite broken symmetry is that

$$(T_{1/m})^m\doteq 1 \quad , \tag{5}$$

where by $\doteq$ I mean except for the necessity to add a single electron at the origin, $(T_{1/m})^m$ leaves the state everywhere else unchanged.

The basic point we wish to make is that if it is like a CDW displacement operator, $T_{1/m}$ displaces the entire wave function in an essential way, so that

$$(T_{1/m}^{-1}A_N^{+0}T_{1/m})A_N^{0+}\neq 0 \quad , \tag{6}$$

or in some sense, the commutator of $T_{1/m}$ with A_N^0 is of order N, not of order unity. This is quite different from a conventional excitation, which is confined to a local region, and therefore the ground state with an excitation is unchanged almost everywhere (or, in the case of a delocalized excitation, is only changed by order $1/N$ everywhere). T is like an operator which rotates the *total* spin of a ferromagnet by a *finite* angle, changing the wave function of every electron, rather than like a single spin-wave operator which reverses only one electron.

The key point, then, is that it is possible to construct a state in which we create *first* the N-electron ground state of Laughlin, and then create N *more* electrons in the state created by $(T_{1/m}A_N^{0+}T_{1/m}^{-1})$. This is like filling $1/m$ of the potential wells of a commensurate CDW with one set of electrons, and then $1/m$ displaced wells with a second set. The result is a state

$$\Psi_{2/m}=T_{1/m}A_N^{0+}T_{1/m}^{-1}A_N^{0+}\Psi_0 \quad , \tag{7}$$

with $2/m$ electrons per flux quantum and, by the same arguments as Laughlin gave, this state also has an energy gap for excitations, some of which are fractionally charged.

At present one must take the basic assumption (6) as essentially an ansatz. One way of giving a rough justification is this: A single-particle determinantal state which can be thought of as the "parent" of Laughlin's state is the wave function

$$\begin{vmatrix} 1 & 1 & \cdots & 1 \\ (Z_1)^m & (Z_2)^m & \cdots & (Z_N)^m \\ (Z_1)^{2m} & (Z_2)^{2m} & \cdots & (Z_N)^{2m} \\ (Z_1)^{3m} & (Z_2)^{3m} & \cdots & (Z_N)^{3m} \\ \vdots & \vdots & & \vdots \\ (Z_1)^{NM} & (Z_2)^{NM} & \cdots & (Z_N)^{NM} \end{vmatrix} \exp\left(\frac{-\sum |Z_i|^2}{2}\right) . \quad (8)$$

This state is not orthogonal to Laughlin's, and one might suppose that as the interactions are gradually turned on it might adiabatically transform into Laughlin's state, which has the same total angular momentum. [We note that (7) is closely related to the wave function proposed by Thouless and Tao.[3]] The state created by multiplying (8) by $\prod_i Z_i$ has one extra unit of angular momentum for each electron, in close analogy to the modified Laughlin state. Every single-electron state is explicitly orthogonal to those in the original wave function, and hence the two states can be simultaneously occupied, each with N electrons, without *any* modification. We can then imagine that interactions carry the $2N$ state adiabatically into the Laughlin-type state which we have been discussing.

Are there any experimental consequences of the present hypothesis? One peculiarity of our way of constructing the $2/m$ state is that it intrinsically contains a defect, which is required by a topological condition. Another is that it is very probable that the $2/m$ gap is smaller than the $1/m$ gap, since if the CDW analogy is any good at all, there is some overlap of the two operators making it less comfortable to fill two of the m substrates rather than one. *A fortiori,* $3/m$ (for $m > 5$, since otherwise this is just two hole states) will be weaker still. Finally, and possibly most usefully, we are proposing the existence of charge $1/m$ defects in the $2/m$ state, but these defects do *not* have symmetry-related states on either side: They are a kind of phase boundary and may have peculiar properties. Corresponding to this, there are two inequivalent $\frac{2}{5}$ states, and three inequivalent $\frac{2}{7}$ states, etc: $m-1/2$ states of $2/m$. There are four symmetry inequivalent $\frac{3}{7}$ states connected by various kinds of strange defects, and so on.

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[1]R. B. Laughlin, Phys. Rev. Lett. 50, 1395 (1983).
[2]D. C. Tsui, H. L. Störmer, and A. C. Gossard, Phys. Rev. Lett. 48, 1559 (1982); also especially Phys. Rev. Lett. (in press).
[3]R. Tao and D. J. Thouless, Bull. Am. Phys. Soc. 28, 364 (1983).