PHYSICAL REVIEW B VOLUME 28, NUMBER 4 15 AUGUST 1983

Dispersion of magnetoplasmons in layered systems

S. Das Sarma
Center for Theoretical Physics, Department of Physics and Astronomy,
University of Maryland, College Park, Maryland 20742
(Received 14 March 1983)

Dispersion relations for magnetoplasmons are derived for both type-I and type-II superlattice systems within the simple model in which thickness of individual charged layers is neglected. The magnetic field is taken to be in the direction of the superlattice growth, and coupling to LO phonons is retained in the theory. General equations valid within the random-phase approximation are derived, and analytic results correct up to second order in two-dimensional wave number are obtained for the magnetoplasmon dispersion.

I. INTRODUCTION

Much attention[1-7] has recently been focused on the collective excitation spectrum of a system of a large number of equally spaced, parallel two-dimensional electron layers. Very recent interest arises from the experimental observation[1-5] of collective modes in the semiconductor superlattice system both in the presence and in the absence of an external magnetic field. In a recent publication[6] detailed theoretical description of the collective excitations in semiconductor superlattices was provided with particular emphasis on the situation without any magnetic field. Subsequent experimental observation[4] of the plasmon mode in GaAs-$Al_xGa_{1-x}As$ superlattices by a light scattering experiment confirmed the theoretical predictions rather well, and a satisfactory picture of the collective excitation spectrum in layered systems in the absence of any magnetic field has thus emerged.

However, the situation in the presence of an external magnetic field is not so clear. In particular, a magnetoplasmon mode has been observed[1-3] in light scattering spectroscopy of a GaAs-$Al_xGa_{1-x}As$ superlattice. Even though this mode obeys the simple theoretical prediction in its dependence on electron density in the layers, it is found to have rather anomalous magnetic field dependence. Defining $\omega_0^2=\omega_m^2-\omega_c^2$, where ω_m is the experimentally observed magnetoplasmon frequency and ω_c is the cyclotron frequency, one expects ω_0 to be independent of the magnetic field in a leading-order calculation. Experimentally, ω_0 is found to be weakly magnetic field dependent and its value in the long-wavelength limit seems to be in slight quantitative (but *not* qualitative) disagreement with the prediction of the simple theory.[6-8] This is surprising since the simple theory describes the situation in the absence of any magnetic field quite well.

To understand the reason for this discrepancy and in order to provide a complete description of the magnetoplasmon dispersion in these systems, we treat in this paper the problem of collective excitations in semiconductor superlattices in the presence of a strong, external magnetic field. For the superlattice we employ the simplest possible model[6] in which the thickness of individual charged layers is neglected and the system is taken to consist of a periodic array of an infinite number of charged layers (of zero thickness). The layers are considered to occupy the xy plane and the separation between adjacent layers in the z direction is taken to be a length "a." For a type-I superlattice all layers are identical, containing charge carriers of two-dimensional density n per unit area and of effective mass m. For a type-II superlattice, alternate layers contain two different types of carriers (which may be electrons and holes or electrons with different effective masses) with two-dimensional densities n_1 and n_2 and effective masses m_1 and m_2, respectively. The superlattice period (in z direction) is thus a and $2a$ for the types-I and -II systems, respectively. A constant external magnetic field B is assumed to exist along the z direction. We employ self-consistent field random-phase approximation (RPA) as discussed in detail in Ref. 6. We restrict ourselves only to the nonretarded limit ($c\rightarrow\infty$) in this paper. In Sec. II we discuss the type-I superlattice, whereas in Sec. III we discuss the type-II system. We conclude in Sec. IV with a brief discussion of the experimental situation.

II. MAGNETOPLASMONS OF TYPE-I SUPERLATTICE

Following Ref. 6 it is straightforward to write down the general dispersion relation for the magnetoplasmon in type-I superlattice system within the RPA:

$$1-\frac{2\pi e^2}{\kappa(\omega)q}\Pi(q,\omega)S(q,k_z)=0\ , \tag{1}$$

where $\kappa(\omega)=\kappa_\infty(\omega^2-\omega_L^2)/(\omega^2-\omega_T^2)$ is the background lattice dielectric constant, and ω_L and ω_T are, respectively, the LO- and the TO-phonon frequencies. The wave number q is the two-dimensional (2D) wave number in the plane of the layer, whereas k_z is the wave number in the superlattice direction which is defined in the range $0\leq k_z\leq 2\pi/a$ by virtue of the periodicity in z direction. The electronic polarizability function $\Pi(q,\omega)$ is given in the presence of the strong magnetic field by

$$\Pi(q,\omega)=\left[\frac{1}{2\pi l^2}\right]\sum_{n,n'}|C_{nn'}(q)|^2\frac{f_n-f_{n'}}{\hbar\omega-E_{n'}+E_n}\ , \tag{2}$$

where n,n' are Landau level indices, and $E_n=(n+\frac{1}{2})\hbar\omega_c$ is the energy of the nth Landau level with $\omega_c=eB/mc$ as the cyclotron frequency [$l=(c\hbar/eB)^{1/2}$ defines the Landau length]. In Eq. (2), f_n denotes the occupancy of the nth Landau level, whereas the coefficient $|C_{nn'}(q)|^2$ is given by

$$|C_{nn'}(q)|^2=\frac{n_2!}{n_1!}\left[\frac{q^2l^2}{2}\right]^{n_1-n_2}e^{-q^2l^2/2}\left[L_{n_2}^{n_1-n_2}\left[\frac{q^2l^2}{2}\right]\right]^2\ , \tag{3}$$

where $n_1=\max(n,n')$ and $n_2=\min(n,n')$. The function $L_m^r(x)$ is the associated Laguerre polynomial[9] defined by

$$L_m^r(x)=\frac{1}{m!}e^x x^{-r}\frac{d^m}{dx^m}(e^{-x}x^{m+r}) \quad . \tag{4}$$

In Eq. (1) the function $S(q,k_z)$, the form factor which determines the phase coherence of the collective excitation in different layers, is given by

$$S(q,k_z)=\frac{\sinh(qa)}{\cosh(qa)-\cos(k_z a)} \quad . \tag{5}$$

The first theoretical treatment of the magnetoplasmon dispersion in a type-I superlattice system was provided[8] by Kobyashi, Mizuno, and Yokota. They, however, considered only the $k_z=0$ limit in Eq. (1). In addition, they restricted themselves to the $ql \ll 1$ situation. Reference 6 generalized the calculation to nonzero k_z, but the restriction $ql \ll 1$ was retained.

In this paper we retain higher-order ql terms in the polarizability to obtain a magnetoplasmon dispersion that is second order in ql. Calculating Eq. (2) up to second order, we get

$$\Pi(q,\omega)\simeq\left[\frac{nq^2}{m}\right]\left[\frac{1-q^2b^2/2}{\omega^2-\omega_c^2}+\frac{q^2b^2/2}{\omega^2-4\omega_c^2}+O(q^4l^4)\right] \quad . \tag{6}$$

In Eq. (6), $b^2=(N+1)l^2$, where N denotes the Landau index for the highest filled level. In all the earlier work the q^2l^2 terms of Eq. (6) were neglected.

Using Eqs. (5) and (6) in Eq. (1), we get a cubic equation in ω^2:

$$(\omega^2-\omega_L^2)(\omega^2-\omega_c^2)(\omega^2-4\omega_c^2)-\omega_p^2(\omega^2-\omega_T^2)S(q,k_z)\times\left[\left[1-\frac{q^2b^2}{2}\right](\omega^2-4\omega_c^2)+\frac{q^2b^2}{2}(\omega^2-\omega_c^2)\right]=0 \quad . \tag{7}$$

In Eq. (7), $\omega_p^2=2\pi ne^2q/\kappa_\infty$ is the square of the 2D-plasmon frequency and $S\equiv S(q,k_z)$. It is straightforward to write down the three solutions to Eq. (7). These are coupled magnetoplasmon-LO phonon modes of the layered system. Instead of working with the complete solutions to Eq. (7), we will neglect the coupling between the LO phonons and the electronic collective modes in Eq. (7). This is entirely justified since in the actual experimental situation this coupling is extremely weak by virtue of the rather large difference between ω_L and the typical electronic collective excitation energies in the system (e.g., in the GaAs-$Al_xGa_{1-x}As$ system $\omega_L\sim 35$ meV, whereas the electronic modes are in the 1–20-meV range for accessible values of n and B). Neglecting this coupling we write down the electronic collective modes implied by Eq. (7):

$$\omega^2=\begin{cases} S\omega_p^2+\omega_c^2+\dfrac{3\omega_c^2\omega_p^2Sq^2b^2}{S\omega_p^2-3\omega_c^2}=\omega_m^2 \quad , \\[2ex] 4\omega_c^2-\dfrac{3\omega_c^2\omega_p^2Sq^2b^2}{S\omega_p^2-3\omega_c^2}=\omega_B^2 \quad . \end{cases} \tag{8}$$

The mode ω_m is the magnetoplasmon or the upper hybrid mode, whereas the other mode

$$\omega_B\simeq 2\omega_c-3\omega_c\omega_p^2Sq^2b^2/4(S\omega_p^2-3\omega_c^2)$$

is the Bernstein mode that carries negligible spectral weight. Bernstein modes are important only for frequencies $\omega=n\omega_c$ with $n=2$, 3, 4, etc., and ω_B is the lowest frequency mode.

Using Eq. (5) in Eq. (8), we obtain the explicit magnetoplasmon dispersion relation correct to $O(q^2b^2)$. It is interesting to consider the strong-coupling ($qa\ll 1$) and the weak-coupling ($qa\gg 1$) situations explicitly. For $qa\ll 1$, Eq. (5) gives

$$S\simeq qa(1-q^2a^2/6)(1+q^2a^2/2-\cos k_z a)^{-1} \quad ,$$

whereas for $qa\gg 1$, we have $S\simeq 1$. The corresponding magnetoplasmon dispersion relations are

$$\omega_0=\begin{cases} (\omega_m^2-\omega_c^2)^{1/2}=\omega_p\left[qa\left[1-\dfrac{q^2a^2}{6}\right]\left[1+\dfrac{q^2a^2}{2}-\cos(k_z a)\right]^{-1}\right]^{1/2}\left[1+\dfrac{3\omega_c^2q^2b^2/2}{qa\,\omega_p^2[1+q^2a^2/2-\cos(k_z a)]^{-1}-3\omega_c^2}\right] \quad \text{for } qa\ll 1 \quad , & (9)\\[2ex] \omega_p\left[1+\dfrac{3\omega_c^2q^2b^2}{2(\omega_p^2-3\omega_c^2)}\right] \quad \text{for } qa\gg 1 \quad . & (10)\end{cases}$$

Equation (10) gives the magnetoplasmon dispersion in the weak-coupling ($qa\gg 1$) purely 2D limit, whereas Eq. (9) contains the 3D limit as a special case when $k_z=0$. Equations (7)–(9) are the important new results of this paper for the type-I superlattice system.

III. MAGNETOPLASMONS OF TYPE-II SUPERLATTICE

In constrast to the type-I superlattice system, collective excitations in the type-II system have not been studied in any great detail either experimentally or theoretically. The theoretical work[6,10] that is known to us concentrates mostly on the situation without any magnetic field except for a very brief discussion in Ref. 6. Experimental work is almost nonexistent except for Ref. 5 reporting the observation of a helicon wave in a InAs-GaSb superlattice. We do not know of any experimental observation of plasmons or magnetoplasmons in a type-II superlattice system (e.g., the InAs-GaSb system[5] or GaAs n-i-p-i system[11]). However, experimental efforts are underway[12] trying to observe collective excitations in n-i-p-i superlattices.

Following Ref. 6 we write down the general dispersion relation for the magnetoplasmon in a type-II superlattice:

$$\left[1-\frac{2\pi e^2}{\kappa q}\Pi_1(q,\omega)S_1(q,k_z)\right]\left[1-\frac{2\pi e^2}{\kappa q}\Pi_2(q,\omega)S_1(q,k_z)\right]-\left[\frac{2\pi e^2}{\kappa q}\right]^2\Pi_1(q,\omega)\Pi_2(q,\omega)S_2^2(q,k_z)=0 \quad . \tag{11}$$

Here, Π_1 and Π_2 are the polarizability functions of the carriers in the two different kinds of two-dimensional layers forming the type-II system. Equation (6) gives Π_1 and Π_2 with the electron density n_i, effective mass m_i, and the Landau length b_i (with $i=1,2$ as appropriate), replacing n, m, and b, respectively. The form factors S_1 and S_2 are given by

$$S_1=\frac{\sinh(2qa)}{\cosh(2qa)-\cos(2k_za)}\ ,$$
$$S_2=\frac{2\sinh(qa)\cos(k_za)}{\cosh(2qa)-\cos(2k_za)}\ . \tag{12}$$

We neglect coupling with the LO phonons for the sake of simplicity (their inclusion is straightforward within the formalism as indicated in Sec. II). We also ignore the small variation in the background lattice constant κ between the two materials and take it to be a constant.

We first use the simplest formula for the polarizability neglecting the higher-order corrections ($qb >> 1$) in Eq. (6) whence

$$\Pi_{1,2}\simeq(n_{1,2}q^2/m_{1,2})(\omega^2-\omega_{c1,c2}^2)^{-1}\ .$$

Using this and Eq. (12) in Eq. (11), we get a quadratic equation in ω^2 which can be easily solved to give the following collective modes:

$$\omega_\pm^2=\tfrac{1}{2}[\omega_{c1}^2+\omega_{c2}^2+S_1(\omega_{p1}^2+\omega_{p2}^2)]$$
$$\pm[(\omega_{c1}^2-\omega_{c2}^2+S_1\omega_{p1}^2-S_1\omega_{p2}^2)^2+4\omega_{p1}^2\omega_{p2}^2S_2^2]^{1/2}\ . \tag{13}$$

In Eq. (13), $\omega_{p1,p2}$ are 2D-plasma frequencies defined by $\omega_{p1,p2}^2=2\pi n_{1,2}e^2q/\kappa m_{1,2}$. Equation (13) defines the general magnetoplasmon modes for the type-II superlattice in the $ql << 1$ limit for arbitrary values of qa and k_za. Experimentally, one is most interested in the strong-coupling ($qa << 1$) situation. This can be easily obtained from Eq. (13) by appropriate expansion of $S_{1,2}$ as defined by Eq. (12). Let us consider the interesting case of $k_za=n\pi$ (where $n=0$, 1,2, etc.) in the strong-coupling situation ($qa << 1$). It is easy to show from Eq. (13) that one gets the following collective modes:

$$\omega_\pm^2=\tfrac{1}{2}\{(\omega_{c1}^2+\omega_{c2}^2+W_{p1}^2+W_{p2}^2)$$
$$\pm[(\omega_{c1}^2-\omega_{c2}^2+W_{p1}^2-W_{p2}^2)^2+4W_{p1}^2W_{p2}^2]^{1/2}\}\ , \tag{14}$$

where $W_{p1,p2}=(2\pi n_{1,2}e^2/\kappa m_{1,2}a)^{1/2}$ are the appropriate 3D-plasma frequencies of the system. Equation (14) is the magnetoplasmon dispersion relation for a two-component plasma in the 3D system.[13]

If we consider the $k_za=(n+\frac{1}{2})\pi$ in the strong-coupling ($qa << 1$) situation, we get

$$\omega_\pm^2=\tfrac{1}{2}\{\omega_{c1}^2+\omega_{c2}^2+qa(\omega_{p1}^2+\omega_{p2}^2)$$
$$\pm[\omega_{c1}^2-\omega_{c2}^2+qa(\omega_{p1}^2-\omega_{p2}^2)]\}$$
$$=\begin{cases}\omega_{c1}^2+qa\,\omega_{p1}^2,\\ \omega_{c2}^2+qa\,\omega_{p2}^2\ .\end{cases} \tag{15}$$

Equation (15) gives the coupled magnetoacoustic plasmon modes of the type-II superlattice in the strong-coupling limit.

Finally, taking the $qa >> 1$ limit of Eq. (13) will give us the weak-coupling limit, where we recover[14] the magnetoplasmon modes of the pure 2D layers themselves. The weak-coupling limit is not an experimentally relevant limit for these systems, particularly for the light scattering spectroscopy.

It is straightforward, but rather tedious, to obtain explicitly the higher-order ql corrections to the magnetoplasmon dispersion relations for the type-II system. One should now use the complete Eq. (6) for Π in the general dispersion relation given by Eq. (11). One immediately gets a quartic equation in ω^2 given by

$$(\omega^2-\omega_{c1}^2)(\omega^2-4\omega_{c1}^2)(\omega^2-\omega_{c2}^2)(\omega^2-4\omega_{c2}^2)-S_1\{\omega_{p1}^2(\omega^2-\omega_{c2}^2)(\omega^2-4\omega_{c2}^2)[(1-q^2b_1^2/2)(\omega^2-4\omega_{c1}^2)+(\omega^2-\omega_{c1}^2)q^2b_1^2/2]$$
$$+\omega_{p2}^2(\omega^2-\omega_{c1}^2)(\omega^2-4\omega_{c1}^2)[(1-q^2b_2^2/2)(\omega^2-4\omega_{c2}^2)+(q^2b_2^2/2)(\omega^2-\omega_{c2}^2)]\}$$
$$-(S_2^2-S_1^2)\omega_{p1}^2\omega_{p2}^2[(\omega^2-4\omega_{c1}^2)(1-q^2b_1^2/2)+(\omega^2-\omega_{c1}^2)q^2b_1^2/2][(\omega^2-4\omega_{c2}^2)(1-q^2b_2^2/2)+(\omega^2-\omega_{c2}^2)q^2b_2^2/2]=0\ . \tag{16}$$

Solutions to Eq. (16) give the coupled magnetoplasmon-Bernstein modes ($n=2$) of the type-II superlattice. We do not pursue these solutions any more because the Bernstein modes carry negligible spectral weight and the magnetoplasmon modes have almost the same dispersion as that given by Eq. (13) in the experimentally interesting $ql << 1$ situtation.

IV. CONCLUSION

We have obtained the dispersion relations of the magnetoplasmons in type-I and -II superlattices in this paper. Our results agree with the known 2D and 3D results in the appropriate limits. We have gone beyond the leading-order result by keeping the higher-order qb correction in our formula where $b=l(N+1)^{1/2}$ in the cyclotron radius of the highest filled Landau level and q is the 2D wave number in the plane of the layer.

Experimental work[1-3] of Worlock and co-workers on a type-I superlattice shows a small variation of the quantity $\omega_0=(\omega_m^2-\omega_c^2)^{1/2}$ with the magnetic field. This is qualitatively consistent with our Eq. (9) by virtue of the higher-order ql correction explicitly retained in our calculation. However, quantitatively the experimental effect seems to be larger in magnitude than what Eq. (9) would suggest.

Specifically, the observed magnetoplasmon frequency[3] at $B=5$ T is about 9.5 meV, giving rise to an $\omega_0=(\omega_m^2-\omega_c^2)^{1/2}$ of about 5 meV. If we compare this with the $\omega_0=\omega_pS^{1/2}$ that we obtain by neglecting the correction term in Eq. (8), we find a theoretical value of 7.5 meV to be compared with the experimental value of 5 meV. By the inclusion of the dispersion correction [the second term in Eq. (8)] and use of the parameters ($qa=0.8$, $k_za=5.5$, $a=780$ Å) corresponding to the experimental situation we get $\omega_0=7.3$ meV, which is closer to the experimental value of 5 meV but still far too large. Thus at least part of the explanation of the experimental observation must lie outside the higher-order qb corrections in the dispersion relations considered in this work. Other possible mechanisms could be electron-electron interaction neglected in this paper and the finite-size effects.

ACKNOWLEDGMENTS

The work is supported by the National Science Foundation through Grant No. NSF-DMR-82-13768.

[1]J. M. Worlock, A. Pinczuk, Z. J. Tien, C. H. Perry, H. L. Stormer, R. Dingle, A. C. Gossard, W. Weigmann, and R. L. Aggarwal, Solid State Commun. 40, 867 (1981).

[2]Z. J. Tien, J. M. Worlock, C. H. Perry, A. Pinczuk, R. L. Aggarwal, H. L. Stormer, A. C. Gossard, and W. Weigmann, Surf. Sci. 113, 89 (1982).

[3]J. M. Worlock, A. C. Maciel, C. H. Perry, Z. J. Tien, R. L. Aggarwal, A. C. Gossard, and W. Weigmann, in *Proceedings of the International Conference on Applications of High Magnetic Fields in Semiconductor Physics, Grenoble, 1982,* edited by G. Landwehr (Springer-Verlag, Berlin, 1983).

[4]D. Olego, A. Pinczuk, A. C. Gossard, and W. Weigmann, Phys. Rev. B 26, 7867 (1982).

[5]J. C. Maan, M. Altarelli, H. Sigg, P. Wyder, L. L. Chang, and L. Esaki, Surf. Sci. 113, 347 (1982).

[6]S. Das Sarma and J. J. Quinn, Phys. Rev. B 25, 7603 (1982); 27, 7603(E) (1983), and references therein.

[7]W. L. Bloss, Solid State Commun. 43, 523 (1982); 44, 363 (1982).

[8]M. Kobyashi, J. Mizuno, and I. Yokota, J. Phys. Soc. Jpn. 39, 18 (1975).

[9]I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series and Products* (Academic, New York, 1965).

[10]B. Bergerson and E. Tosatti, Synth. Met. 3, 61 (1981).

[11]G. H. Dohler, H. Kunzel, D. Olego, K. Ploog, P. Ruden, H. J. Stolz, and G. Abstreiter, Phys. Rev. Lett. 47, 864 (1981).

[12]G. Abstreiter (private communications).

[13]H. Noto, J. Phys. Soc. Jpn. 36, 1137 (1974).

[14]K. W. Chiu and J. J. Quinn, Phys. Rev. B 9, 4724 (1979).