Thermopower in high magnetic fields: Electron-phonon mass enhancement

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We calculate the adiabatic thermopower in a high magnetic field. Continuing our previous work, we derive a many-body transport equation including the magnetic field and derive the high-field conductivity tensor $\overleftrightarrow{\sigma}$. The transport equation is derived by both the Keldysh and the Kadanoff-Baym nonequilibrium formulation of many-body systems. The results published by Opsal, Thaler, and Bass are confirmed to a certain extent. Electron-phonon mass enhancement is present in the adiabatic thermopower in high magnetic fields.

I. INTRODUCTION

Thermomagnetic measurements are one kind of transport experiment. A variety of experiments may be done according to the boundary conditions on the sample and the orientations of the driving fields. In the most general case there is an electric and a magnetic field in addition to the temperature gradient. A special case of interest is when all fields are constant in time after they were switched on. Response functions, connecting the driving forces and the resulting currents, are usually calculated in the steady state, where all variations in space and time vanish. A formulation which takes into account the transient phenomena in a transport equation has been developed by the authors[1] using a method of Kadanoff and Baym[2] or the equivalent Keldysh procedure.[3]

In this paper we apply the new transport equation to the problem of low-temperature thermopower in a high magnetic field. One motivation is the experimental results for the thermoelectric power at high magnetic fields in aluminum. Opsal *et al.*[4] found a disagreement between their experimental results and an earlier theoretical prediction.[5] They resolved this discrepancy by giving an argument to include electron-phonon mass enhancement in the thermopower in a high magnetic field. This assumption is in contradiction with a statement made by Prange and Kadanoff,[5] that there is no electron-phonon mass enhancement in the thermopower. Prange and Kadanoff did not derive this result. It was a conclusion in a more general context to verify the quasiparticle approximation for an electron-phonon system. In a more elaborate analysis, which took band-structure effects into account, Opsal and Wagner[6] confirmed the results of Ref. 4.

The experimental results of Opsal *et al.* have inspired a large number of recent theoretical papers on the role of mass enhancements on the thermoelectric power.[7–9] However, none of these theoretical papers are appropriate for the actual experiments. The previous theory papers all considered the longitudinal thermopower in the absence of the magnetic field. The experiments measure the off-diagonal thermopower in the presence of a large magnetic field. The off-diagonal thermopower is zero in the absence of a magnetic field, so the previous theoretical papers never calculated the quantity which has been measured. In order to relate to the actual experiments, it is necessary to develop a transport theory for large magnetic fields ($\omega_c\tau \gg 1$). Here we report such a theory.

The usual way of calculating a Kubo formula for a response function is not convenient in a high magnetic field because it is difficult to incorporate the high-field limit into the Kubo-type expression. Using a Boltzmann equation approach, Lifshitz, Azbel, and Kaganov[10,11] did the pioneering work in thermomagnetic effects in high magnetic fields. Following Ref. 10, Wagner[12] calculated magnetothermal conductivity of tungsten, and Averbeck and Wagner[13] gave the now common formulation of the low-temperature magnetothermopower in a metal. The usual Boltzmann equation excludes many-body effects because the spectral function is considered to be a δ function.[14] In our formalism we avoid this disadvantage and keep the exact spectral function, which contains the effect of particle interactions, in the transport equation.

After a brief review on the low-temperature thermopower in a magnetic field in Sec. II we derive a many-body transport equation in Sec. III. This is a continuation of our previous paper[1] where the magnetic field was not included. This time our derivation starts from the Keldysh formulation. In Appendix A we show that both methods, Kadanoff-Baym and Keldysh, give the same many-body transport equation for the steady state. Section IV is devoted to the calculation of the conductivity tensor in

a high magnetic field. Here we also follow the procedure given in the pioneering work by Lifshitz *et al.*[10] Finally, we get an expression for the thermopower and close in Sec. V with a discussion and some concluding remarks.

II. THERMOPOWER

We consider an electrically conducting bar which is subjected to two conditions: (1) a temperature gradient $\vec{\nabla}T$, and (2) no current can flow through its ends because of insulation. Then an electric field $\vec{E}$ is developed whose magnitude is proportional to ΔT. Including the additional effects of a magnetic field $\vec{H}$ the thermopower tensor $\overleftrightarrow{S}(H)$ is defined by

$$\vec{E}=\overleftrightarrow{S}(H)\cdot\vec{\nabla}T \ . \tag{1.1}$$

If there is no magnetic field present and the temperature gradient has a x component only the thermopower S_d is given by

$$S_d(H=0)=\frac{E_x}{\nabla_x T}=S_{xx}(H=0) \ , \tag{1.2}$$

where $S_{xx}(H=0)$ is a component of the thermopower tensor with no magnetic field. By adding a magnetic field along the z direction a secondary temperature gradient in the y direction is created. The thermopower is now

$$S_d(H)=\frac{E_x}{\nabla_x T}=S_{xx}(H)-S_{xy}(H)\frac{\sigma_{yx}(H)}{\sigma_{xx}(H)} \ , \tag{1.3}$$

where $\overleftrightarrow{\sigma}$ is the conductivity tensor.

The low-temperature experimental data are consistent with the equation[13,15]

$$S_d(H)=A(H)T+B(H)T^3 \ . \tag{1.4}$$

The first term is due to electron diffusion and the second is attributed to phonon drag. T is the temperature in degrees Kelvin. We are interested in the first term only and therefore omit the phonon-drag term. How to extract the different contributions from experiment is given in Ref. 15.

If the scattering mechanisms are elastic the thermopower tensor is given by the Mott relation[16]

$$\overleftrightarrow{S}(H)=eL_0T\left[\frac{d}{d\omega}\ln\overleftrightarrow{\sigma}^H(\omega)\right]_{\omega=\epsilon_f} \ . \tag{1.5}$$

L_0 is the Lorentz number ($L_0=2.44\times10^{-8}$ V^2K^{-2}), e the charge of an electron, and ϵ_F the Fermi energy. $\overleftrightarrow{\sigma}^H(\omega)$ is an energy-dependent part of the conductivity tensor $\overleftrightarrow{\sigma}^H$,

$$\overleftrightarrow{\sigma}^H=e^2\int\frac{d\omega}{2\pi}\left[-\frac{\partial n_F}{\partial\omega}\right]\overleftrightarrow{\sigma}^H(\omega) \ , \tag{1.6}$$

with the Fermi distribution function $n_F(\omega)$. Rigorously, Eq. (1.5) applies to electron-impurity scattering only, which is the dominant part at low temperatures. However, for $kT\ll\epsilon_F$ the electron-phonon scattering can be considered as quasielastic because electrons are scattered by phonons only in a small $2kT$-wide shell around ϵ_F. The change of energy is small.

Setting the magnetic field in a direction z of twofold or higher symmetry and neglecting components of the form $\sigma_{iz}, i=x,y$, Averbeck and Wagner[13] derived in the high-field limit

$$S_d(H\to\infty)=eL_0T\left[\frac{d}{d\omega}[2\ln\sigma_{xy}^H(\omega)-\ln\sigma_{yy}^H(\omega)]\right]_{\omega=\epsilon_F} \ . \tag{1.7}$$

Although (1.7) is sufficient for a calculation, the difference ΔS_d between the low-field ($H=0$) and high-field ($H\to\infty$) limits of $S_d(H)$ provides a quantity which is almost insensitive to the impurity concentration of the sample[13] and is therefore closely related to the host properties. It was ΔS_d which gave strong evidence that electron-phonon mass enhancement is involved in the magnetothermopower. Experiments were done on both polycrystalline and crystallic based aluminum alloys. A disagreement between experimental data and theoretical predictions was explained by electron-phonon mass enhancement. In a more heuristic way Opsal *et al.*[4] derived

$$\begin{aligned} &S_d(H\to\infty)\to(1+\lambda)S_d(H\to\infty) \ , \\ &\lambda=-\left[\frac{\partial}{\partial\omega}\Sigma_{e\text{-ph}}(\vec{k}_F,\omega)\right]_{\omega=\epsilon_F} \ , \end{aligned} \tag{1.8}$$

where $\Sigma_{e\text{-ph}}$ is the electron self-energy attributed to electron-phonon interaction. Because of its slow variation on the wave vector, it is usually restricted on the Fermi surface with wave vector k_F. In Sec. IV we will give a confirmation of that result based on a microscopic calculation of the conductivity in a high magnetic field.

III. MANY-BODY TRANSPORT EQUATION

The solution of a classical transport equation provides a distribution function $f(\vec{R},\vec{k},T)$ in real space $\vec{R}$, momentum space $\vec{k}$, and time T. Once $f(\vec{R},\vec{k},T)$ is known, one can calculate the kinetic coefficients. In a many-body system we have to add the energy ω as an independent variable, because

momentum and energy ω are in general not related in a simple way. The spectral function $A(\vec{k},\omega)$ has a finite width. A way of finding a transport equation for this new many-body distribution function $f(\vec{R},T,\vec{k},\omega)$ was proposed by Kadanoff and Baym,[2] Keldysh,[3] and Craig.[17] We find that the main problem is to solve the set of equations [compare Eq. (2.1) of Ref. 1]

$$\begin{aligned} G^r &= G_0^r + G^r\Sigma^r G_0^r \ , \\ F &= (1+G^r\Sigma^r)F_0(1+\Sigma^a G^a)+G^r\Omega G^a \ , \\ G^< &= \tfrac{1}{2}(F-G^r+G^a) \ . \end{aligned} \tag{2.1}$$

In addition, we have[18]

$$\begin{aligned} G^r &= G-G^< = -\bar{G}+G^> \ , \\ G^a &= G-G^> = -\bar{G}+G^< \ , \\ F &= G+\bar{G} = G^<+G^> \ , \end{aligned} \tag{2.2}$$

as well as

$$\begin{aligned} \Sigma^a &= \Sigma+\Sigma^> = -(\bar{\Sigma}+\Sigma^<) \ , \\ \Sigma^r &= \Sigma+\Sigma^< = -(\bar{\Sigma}+\Sigma^>) \ , \\ \Omega &= \Sigma+\bar{\Sigma} = -(\Sigma^<+\Sigma^>) \ . \end{aligned} \tag{2.3}$$

The many-body distribution function is related to the Green's function $G^<$. The superscripts r and a denote, respectively, the retarded and advanced Green's function. Quantities with a bar are calculated with an anti-time-ordering operator. For more details we refer to the original work by Keldysh[3,19] or the paper by Blandin *et al.*[20] Products in (2.1) shall be understood as integrals.

Combining the Eqs. (2.1) and (2.2), we derive an equation for the Green's function of interest $G^<$ (there is a similar equation for $G^>$)

$$G^< = (1+G^r\Sigma^r)F_0(1+\Sigma^a G^a)-G^r\Sigma^< G^a \ . \tag{2.4}$$

In our previous paper[1] we used the equations of Kadanoff and Baym to calculate $G^<$. Here we like to follow a different approach, which is closely related to the Keldysh formulation of the problem. We will show in Appendix A, however, that the Kadanoff-Baym procedure gives the very same result for the steady-state solution. To complete our system of equations we rewrite the first of the equations (2.1),

$$G^r = G_0^r + G_0^r\Sigma^r G^r \ , \tag{2.5}$$

and add both of them:

$$G^r = G_0^r + \tfrac{1}{2}(G^r\Sigma^r G_0^r + G_0^r\Sigma^r G^r) \ . \tag{2.6}$$

A similar equation holds true for the advanced Green's function. Equations (2.4) and (2.6) are exact. To find an approximate solution we make a gradient expansion at the right-hand sides. To reach this end we go from the arguments $\vec{r}_1,t_1,\vec{r}_2,t_2$ to center-of-mass (c.m.) and relative coordinates $\vec{R},T$ and $\vec{r},t$ given by the transformation

$$\begin{aligned} \vec{R} &= \tfrac{1}{2}(\vec{r}_1+\vec{r}_2), \quad \vec{r} = \vec{r}_1-\vec{r}_2 \ , \\ T &= \tfrac{1}{2}(t_1+t_2), \quad t = t_1-t_2 \ . \end{aligned} \tag{2.7}$$

This step is justified if the Green's functions have separate contributions varying slowly on a macroscopic scale. This is certainly true if we introduce a potential varying slowly in time and space.

In the new notation the Green's function $G^<$ reads, for example,

$$G^<(\vec{R},T,\vec{r},t) = G^<(\vec{R}+\tfrac{1}{2}\vec{r},T+\tfrac{1}{2}t,\vec{R}-\tfrac{1}{2}\vec{r},T-\tfrac{1}{2}t) \ . \tag{2.8}$$

The gradient expansion of (2.4) is a straightforward calculation and we will give the result only. We collect all terms to first order in derivatives with respect to c.m. coordinates and take a Fourier transform with respect to $\vec{r}$ and t:

$$G^<(\vec{R},T,\vec{k},\omega) = \int d^3r \int dt\, e^{i\omega t}e^{-i\vec{k}\cdot\vec{r}} \times G^<(\vec{R},T,\vec{r},t) \ . \tag{2.9}$$

The first term in (2.4) gives a vanishing contribution. The solution does not depend on its initial state, since the system has no memory. This feature is in accordance with the Kadanoff-Baym[2] result. Finally omitting the arguments, we have

$$\begin{aligned} G^< = &-G^a\Sigma^< G^r + i\tfrac{1}{2}[G^a,G^r]\Sigma^< - i\tfrac{1}{2}[G^r,\Sigma^<]G^a \\ &+ i\tfrac{1}{2}[G^a,\Sigma^<]G^r \ . \end{aligned} \tag{2.10}$$

The products are now simple multiplications and $[X,Y]$ is a generalized Poisson bracket:

$$\begin{aligned} [X,Y] &= [X,Y]_{\omega,T} - [X,Y]_{\vec{k},\vec{R}} \ , \\ [X,Y]_{U,V} &= \frac{\partial X}{\partial U}\frac{\partial Y}{\partial V} - \frac{\partial Y}{\partial U}\frac{\partial X}{\partial V} \ . \end{aligned} \tag{2.11}$$

The gradient expansion of (2.6) is a bit more cumbersome because the retarded self-energy Σ^r has contributions from the external fields whereas $\Sigma^<$ is determined by the particle interactions alone. In the notation we introduced in Ref. 1 this means $\Sigma^<$ has only two-point contributions while Σ^r has both one- and two-point parts. We distinguish a one- and two-point function by the number of time arguments it contains.

As is common in transport phenomena, we use the external field in a semiclassical approximation.[16] Thus we get for the one-point contribution, if we describe the external field by a vector potential[21]

$$\phi(\vec{r},t)=i\frac{e}{mc}\vec{A}(\vec{r},t)\cdot\vec{\nabla}_r+\frac{e^2}{2mc^2}A^2(\vec{r},t)\ . \tag{2.12}$$

We have already shown that our theory is gauge invariant. The special choice of the vector potential is no restriction.

Separating the one- and two-point contributions we can write (2.6) symbolically:

$$G^r=G_0^r+\tfrac{1}{2}(G^r\phi G_0^r+G_0^r\phi G^r+G^r\Sigma^r G_0^r+G_0^r\Sigma^r G^r)\ . \tag{2.13}$$

Now that Σ^r does not contain the external fields anymore, it is determined by the particle interactions of the system. Therefore, it is similar to $\Sigma^<$, a two-point function only. The one-point function ϕ gives all information on the external fields. From (2.12) we see that ϕ is an operator rather than a plain function. We must take heed of that fact. In the same order we used before, we have

$$\begin{aligned}G^r(\vec{R},T,\vec{k},\omega)=G_0^r(\vec{k},\omega)\Bigg[1-\frac{e}{mc}\vec{k}\cdot\vec{A}(\vec{R},T)G^r(\vec{R},T,\vec{k},\omega)\\ +\frac{e^2}{2mc^2}A^2(\vec{R},T)G^r(\vec{R},T,\vec{k},\omega)+\Sigma^r(\vec{R},T,\vec{k},\omega)G^r(\vec{R},T,\vec{k},\omega)\Bigg]\\ +\frac{e}{4mc}\vec{\nabla}_R\{G_0^r(\vec{k},\omega)[G^r(\vec{R},T,\vec{k},\omega),\vec{A}(\vec{R},T)]-G^r(\vec{R},T,\vec{k},\omega)[G_0^r(\vec{k},\omega),\vec{A}(\vec{R},T)]\\ -\vec{A}(\vec{R},T)[G_0^r(\vec{k},\omega),G^r(\vec{R},T,\vec{k},\omega)]\}\ .\end{aligned} \tag{2.14a}$$

Details of the calculation are delegated to Appendix B. Neglecting second-order terms in derivatives with respect to the c.m. coordinates, which is at least a good approximation due to the slow variation in these coordinates, (2.14a) can be cast into a well-known form,

$$\begin{aligned}G^r(\vec{R},T,\vec{k},\omega)=[\omega-\epsilon(\vec{k}-(e/c)\vec{A}(\vec{R},t))\\ -\Sigma^r(\vec{R},T,\vec{k},\omega)]^{-1}\end{aligned} \tag{2.14b}$$

with the free-electron energy $\epsilon(k)=k^2/2m$. This level of approximation does not contain the quantum effects due to the discrete structure of the Landau levels. Quantum oscillations are for most metals superimposed in the form of a fine structure onto the main dependence on the magnetic field and are more important in a region where the spacing between Landau levels $\hbar\omega_c$ is larger than the broadening of the Fermi surface.[22] For a generalization of the result to a more general dispersion, see the end of this section.

In (2.14b) we rederived the solution obtained by Kadanoff and Baym [compare Eq. (9.31) in Ref. 2]. Although we used a completely different method, we end up with the same leading term in a gradient expansion for the retarded Green's function. It is obvious that a similar result is valid for $G^a(\vec{R},T,\vec{k},\omega)$. We keep in mind that (2.14b) is exact in first-order derivatives with respect to the c.m. coordinates, as is (2.10), which is the equation providing $G^<(\vec{R},T,\vec{k},\omega)$. To calculate $G^<$ we can therefore insert (2.14b) into (2.10). Before we do so, we would like to discuss some other conclusions drawn from (2.10) and an analog equation for $G^>$, replacing $<$ by $>$ in (2.10).

The four Green's functions $G^>$, $G^<$, G^r, and G^a are not independent. They are related by

$$G^<-G^>=G^a-G^r=-2i\ \mathrm{Im}G^r \tag{2.15}$$

and so are the self-energies

$$\Sigma^<-\Sigma^>=\Sigma^r-\Sigma^a=2i\ \mathrm{Im}\Sigma^r\ . \tag{2.16}$$

Equations (2.15) and (2.16) can be checked by Eqs. (2.2) and (2.3). Subtracting the equations which provide $G^<$ and $G^>$, we get

$$G^<-G^>=-G^rG^a(\Sigma^<-\Sigma^>)=-2\,\mathrm{Im}\Sigma^rG^rG^a \tag{2.17}$$

using (2.16). The right-hand side defines a spectral function $A(\vec{R},T,\vec{k},\omega)$ similar to the equilibrium one. Equation (2.17) is a check whether the gradient expansion is consistent with the exact equations (2.15) and (2.16). The spectral function in our order of approximation is represented by

$$A(\vec{R},T,\vec{k},\omega)=-\frac{2\,\mathrm{Im}\Sigma^r(\vec{R},T,\vec{k},\omega)}{[\omega-\epsilon(\vec{k}-(e/c)\vec{A}(\vec{R},T))-\mathrm{Re}\Sigma^r(\vec{R},T,\vec{k},\omega)]^2+[\mathrm{Im}\Sigma^r(\vec{R},T,\vec{k},\omega)]^2}\ . \tag{2.18}$$

Now we multiply (2.10) by $2i\,\mathrm{Im}\Sigma^r$. On the left-hand side we use (2.16) and on the right-hand side, (2.17) in the first term. We get

$$G^>\Sigma^<-\Sigma^>G^<=-\mathrm{Im}\Sigma^r[G^a,G^r]\Sigma^<+\mathrm{Im}\Sigma^r[G^r,\Sigma^<]G^a-\mathrm{Im}\Sigma^r[G^a,\Sigma^<]G^r\ . \tag{2.19}$$

Equation (2.19) is close to our final result. To calculate the Poisson brackets in (2.19) we use (2.14b) for G^r and the corresponding expression for G^a

$$\begin{aligned}
&[G^a,G^r]=(G^aG^r)^2\{2i[\mathrm{Im}\Sigma^r,\omega-\epsilon(\vec{k}-(e/c)\vec{A})]+[\Sigma^a,\Sigma^r]\}\ ,\\
&[G^r,\Sigma^<]=-(G^r)^2\{[\omega-\epsilon(\vec{k}-(e/c)\vec{A}),\Sigma^<]-[\Sigma^r,\Sigma^<]\}\ ,\\
&[G^a,\Sigma^<]=-(G^a)^2\{[\omega-\epsilon(\vec{k}-(e/c)\vec{A}),\Sigma^<]-[\Sigma^a,\Sigma^<]\}\ .
\end{aligned} \tag{2.20}$$

We take the external fields constant in space and time which are switched on at $T=0$. For $T>0$ these fields are represented by the vector potential $\vec{A}(\vec{R},T)$

$$\vec{A}(\vec{R},T)=-\tfrac{1}{2}\vec{R}\times\vec{H}-cT\vec{E}\ . \tag{2.21}$$

Whenever the wave-vector $\vec{k}$ appears, it appears in combination with the vector potential. Therefore, we redefine the wave vector

$$\vec{q}\leftarrow\vec{k}+(e/2c)\vec{R}\times\vec{H}+eT\vec{E}\ . \tag{2.22}$$

This transformation also effects the $\vec{R}$ and T derivatives

$$\begin{aligned}
&\frac{\partial}{\partial T}\rightarrow\frac{\partial}{\partial T}+e\vec{E}\cdot\vec{\nabla}_q\ ,\\
&\vec{\nabla}_R\rightarrow\vec{\nabla}_R+(e/2c)\vec{H}\times\vec{\nabla}_q\ .
\end{aligned} \tag{2.23}$$

Combining (2.19), (2.20), and (2.23) we get, after some algebra,

$$\begin{aligned}
G^>\Sigma^<-\Sigma^>G^<=&iA(G^aG^r)\Sigma^<\left[\frac{\partial}{\partial T}+\vec{v}_q\cdot\vec{\nabla}_R\right]\mathrm{Im}\Sigma^r-i\tfrac{1}{2}A^2\left[\frac{\partial}{\partial T}+\vec{v}_q\cdot\vec{\nabla}_R\right]\Sigma^<\\
&-e\vec{E}\cdot\left[iA(G^aG^r)\Sigma^<\left[\vec{v}_q\frac{\partial}{\partial\omega}+\vec{\nabla}_q\right]\mathrm{Im}\Sigma^r+i\tfrac{1}{2}A^2\left[\vec{v}_q\frac{\partial}{\partial\omega}+\vec{\nabla}_q\right]\Sigma^<\right]\\
&+i\frac{e}{c}A(G^aG^r)\Sigma^<(\vec{H}\times\vec{v}_q)\cdot\vec{\nabla}_q\,\mathrm{Im}\Sigma^r+i\frac{e}{c}\frac{1}{2}A^2(\vec{H}\times\vec{v}_q)\cdot\vec{\nabla}_q\Sigma^<\\
&+iA(G^aG^r)\Sigma^r,\mathrm{Im}\Sigma^r]^{\sim}-iA\,\mathrm{Re}G^r[\mathrm{Im}\Sigma^r,\Sigma^<]^{\sim}+iA^2[\mathrm{Re}\Sigma^r,\Sigma^<]^{\sim},
\end{aligned} \tag{2.24}$$

$$[X,Y]^{\sim}=[X,Y]_{\omega,T}+eE[X,Y]_{\omega,\vec{q}}-[X,Y]_{\vec{q},\vec{R}}-[X,Y]_{\vec{q},(e/2c)H\times\vec{\nabla}_q}\ . \tag{2.25}$$

Equation (2.24) includes all terms consistent with our approximation of neglecting all derivations with respect to c.m. coordinates higher than first order. At this stage we linearize the equation further. We keep terms linear in the fields and self-energy contributions only up to linear in first-order derivatives. To collect the correct contributions we must insert the equilibrium value $\Sigma^<=2in\,\mathrm{Im}\Sigma^r$ in the second and fourth line of (2.24). Equation (2.24) becomes

$$\begin{aligned}
G^>\Sigma^<-\Sigma^>G^<=&iA(G^aG^r)\Sigma^<\left[\frac{\partial}{\partial T}+\vec{v}_q\cdot\vec{\nabla}_R\right]\mathrm{Im}\Sigma^r-i\tfrac{1}{2}A^2\left[\frac{\partial}{\partial T}+\vec{v}_q\cdot\vec{\nabla}_R\right]\Sigma^<\\
&+\left[\mathrm{Im}\Sigma^r\frac{\partial}{\partial T}\mathrm{Re}\Sigma^r+(\omega-\epsilon_q-\mathrm{Re}\Sigma^r)\frac{\partial}{\partial T}\mathrm{Im}\Sigma^r\right]\frac{\partial n}{\partial\omega}A^2\\
&+e\vec{E}\cdot[(\vec{v}_q+\vec{\nabla}_q\,\mathrm{Re}\Sigma^r)\,\mathrm{Im}\Sigma^r+\vec{\nabla}_q\,\mathrm{Im}\Sigma^r(\omega-\epsilon_q-\mathrm{Re}\Sigma^r)]\frac{\partial n}{\partial\omega}A^2\\
&+i\frac{e}{c}A(G^aG^r)\Sigma^<(\vec{H}\times\vec{v}_q)\cdot\vec{\nabla}_q\,\mathrm{Im}\Sigma^r+i\frac{e}{2c}A^2(\vec{H}\times\vec{v}_q)\cdot\vec{\nabla}_q\Sigma^<\ .
\end{aligned} \tag{2.26}$$

For steady-state conditions ($\partial/\partial T=\vec{\nabla}_R=0$) and no magnetic field ($H=0$) we discover the transport equation derived in our previous work [compare Eq. (5.39) in Ref. 1] by using the Kadanoff-Baym equations. When studying steady-state solutions, many terms in (2.26) vanish,

$$G^>\Sigma^<-\Sigma^>G^<=e\vec{E}\cdot[(\vec{v}_q+\vec{\nabla}_q \mathrm{Re}\Sigma^r)\mathrm{Im}\Sigma^r+\vec{\nabla}_q \mathrm{Im}\Sigma^r(\omega-\epsilon_q-\mathrm{Re}\Sigma^r)]\frac{\partial n}{\partial\omega}A^2$$
$$+i\frac{e}{c}A(G^aG^r)\Sigma^<(\vec{H}\times\vec{v}_q)\cdot\vec{\nabla}_q \mathrm{Im}\Sigma^r+i\frac{e}{2c}A^2(\vec{H}\times\vec{v}_q)\cdot\vec{\nabla}_q\Sigma^< . \quad (2.27)$$

Multiplying (2.10) by $(e/c)\vec{v}_q\cdot(\vec{H}\times\vec{\nabla}_q)$ and collecting terms in the order in which we are working, the second line of (2.27) can be changed to a more convenient form. After taking this last step we finally end up with our desired result: a many-body transport equation including the magnetic field,

$$G^>\Sigma^<-\Sigma^>G^<=e\vec{E}\cdot[(\vec{v}_q+\vec{\nabla}_q \mathrm{Re}\Sigma^r)\mathrm{Im}\Sigma^r+\vec{\nabla}_q \mathrm{Im}\Sigma^r(\omega-\epsilon_q-\mathrm{Re}\Sigma^r)]\frac{\partial n}{\partial\omega}A^2$$
$$+i\frac{e}{c}\vec{v}_q\cdot(\vec{H}\times\vec{\nabla}_qG^<)-2n\frac{e}{c}A^2\mathrm{Re}G^r\vec{v}_q\cdot(\vec{H}\times\vec{\nabla}_q \mathrm{Re}\Sigma^r)\mathrm{Im}\Sigma^r . \quad (2.28)$$

Equation (2.28) describes steady-state transport phenomena. It can be viewed as the asymptotic ($T\to\infty$) limit for a transport equation including all transients.

An equation very similar to (2.28) was derived by Langreth[23] who calculated the Hall coefficient of Hubbard's model. Neglecting the k dependence in the self-energies and replacing $\partial n/\partial\omega$ by its zero-temperature limit, Eq. (2.28) turns out to be the same as that derived by Langreth. He used the Kadanoff-Baym formalism [compare Eq. (3.6) in Ref. 23]. He found an approximate solution for high magnetic fields neglecting the scattering terms. We will solve (2.28) self-consistently with an ansatz

$$G^<=inA-i\frac{\partial n}{\partial\omega}\gamma . \quad (2.29)$$

The first term in (2.29) is the equilibrium expression and γ is a (vertex) function which is linear in the electric field. Once we know this function we can calculate the current density j

$$j=e^2\int\frac{d^3q}{(2\pi)^3}\int\frac{d\omega}{2\pi}\left[-\frac{\partial n(\omega)}{\partial\omega}\right]\vec{v}_q\gamma(\vec{q},\omega) \quad (2.30)$$

and henceforth the conductivity tensor $\overleftrightarrow{\sigma}$.

To continue our calculation we have to specify the scattering mechanisms in our system. We are considering electron-phonon and electron-impurity scattering. For a dilute alloy, the second-order Born approximation is a reasonable approximation for the latter. Phonon scattering is approximated, as usual, by a one-phonon process, which gives vertex corrections corresponding to the ladder series summation done by Holstein.[1,24] Using the rules for calculating the self-energies[3] $\Sigma^{\gtrless}$, we have for the electron-phonon part

$$\Sigma^{\gtrless}_{e\text{-ph}}(\vec{R},T,\vec{k},\omega)=-\sum_{\vec{q}}|g_{\vec{q}}|^2\int\frac{d\Omega}{2\pi}D^{\gtrless}(\vec{q},\Omega)G^{\gtrless}(\vec{R},T,\vec{k}-\vec{q},\omega-\Omega) , \quad (2.31)$$

where $D^{\gtrless}$ is the appropriate equilibrium phonon propagator and $g_{\vec{q}}$ the electron-phonon interaction matrix element. For the electron-impurity contribution we have

$$\Sigma^{\gtrless}_{\mathrm{im}}(\vec{R},T,\vec{k},\omega)=-n_i\sum_{\vec{q}}|v(\vec{q})|^2G^{\gtrless}(\vec{R},T,\vec{k}-\vec{q},\omega) . \quad (2.32)$$

Here, n_i is the density of impurities ($n_i\to 0$) and $v(\vec{q})$ is the electron-impurity interaction. Inserting (2.29) into (2.28), and by using the self-energy contributions according to (2.31) and (2.32), we obtain an equation for the function γ:

$$A^2e\vec{E}\cdot[(\vec{v}_q+\vec{\nabla}_q \mathrm{Re}\Sigma^r)\mathrm{Im}\Sigma^r+\vec{\nabla}_q \mathrm{Im}\Sigma^r(\omega-\epsilon_q-\mathrm{Re}\Sigma^r)]+\frac{e}{c}\vec{v}_q(\vec{H}\times\vec{\nabla}_q\gamma)$$
$$=2\,\mathrm{Im}\Sigma^r\gamma+A\sum_{\vec{k}}|g_{\vec{k}}|^2\{[n_B(\omega_{\vec{k}})+1-n_F(\omega-\omega_{\vec{k}})]\gamma(\vec{q}-\vec{k},\omega-\omega_{\vec{k}})$$
$$+[n_B(\omega_k)+n_F(\omega+\omega_{\vec{k}})]\gamma(\vec{q}-\vec{k},\omega+\omega_{\vec{k}})\}+An_i\sum_{\vec{k}}|v(\vec{k})|^2\gamma(\vec{q}-\vec{k},\omega) . \quad (2.33)$$

For the details of the calculation concerning the phonon part, we refer to our previous work (see Ref. 1). The subscripts B and F, respectively, refer to fermion and boson distribution functions. In a metal, the variations of the self-energies with wave vectors are usually small and their derivation with respect to $\vec{q}$ can be neglected. Calling $\Gamma=-\mathrm{Im}\Sigma^r$, we deduce a simplified version of (2.33)

$$e\vec{E}\cdot\vec{v}_q A^2\Gamma+\frac{e}{c}(\vec{H}\times\vec{v}_q)\cdot\vec{\nabla}_q\gamma=2\Gamma\gamma-A\sum_{\vec{k}}|g_{\vec{k}}|^2\{[\cdots]\gamma(\vec{q}-\vec{k},\omega-\omega_{\vec{k}})+[\cdots]\gamma(\vec{q}-\vec{k},\omega+\omega_{\vec{k}})\}$$
$$-An_i\sum_{\vec{k}}|v(\vec{k})|^2\gamma(\vec{q}-\vec{k},\omega)\ . \tag{2.34}$$

A solution of (2.34) in high magnetic fields is given in Sec. IV. In Appendix A we show that the Kadanoff-Baym method gives the very same Eq. (2.33) for the steady-state solution $G^<$ according to the ansatz (2.29).

So far we have implicitly assumed the free-electron picture and used it in deriving Eq. (2.14b). No further use has been made in the following steps leading to (2.34). Staying in the semiclassical regime with only moderately high fields so that we can neglect interband transitions, Eq. (2.14b) holds true for even a more general dispersion.[25] We simply incorporate an arbitrary band structure by labeling the energy by an index n. In what follows we omit this label because it is not important, but keep in mind that $\vec{v}_q\neq\vec{q}/m$.

IV. CONDUCTIVITY

Setting the magnetic field $\vec{H}$ in the z direction of our coordinate system, the system of reference is fixed with respect to this axis. The most general form of γ linear in the electric field is given by

$$\gamma(\vec{q},\omega)=(\vec{v}_q\cdot\vec{E})F_1(\vec{q},\omega)+\vec{v}_q\cdot(\vec{H}\times\vec{E})F_2(\vec{q},\omega)$$
$$+(\vec{v}_q\cdot\vec{H})(\vec{H}\cdot\vec{E})F_3(\vec{q},\omega)\ . \tag{3.1}$$

The functions F_i are not functions of $\vec{E}$ but can still have magnetic field contributions. Equation (3.1) can be deduced from the classical Boltzmann equation treatment in a magnetic field.[26] For an arbitrary electric field and the specified H direction it is convenient to introduce three new functions u_x, u_y, and u_z:

$$u_x(\vec{q},\omega)=v_xF_1(\vec{q},\omega)+v_yHF_2(\vec{q},\omega)\ ,$$
$$u_y(\vec{q},\omega)=v_yF_1(\vec{q},\omega)-v_xHF_2(\vec{q},\omega)\ , \tag{3.2}$$
$$u_z(\vec{q},\omega)=v_zF_1(\vec{q},\omega)+v_zH^2F_3(\vec{q},\omega)\ .$$

We used the abbreviation $v_i=\partial\epsilon/\partial q_i$. This set of functions turns γ into a remarkable simple form if we consider them as components of a vector $\vec{u}$,

$$\gamma=\vec{u}\cdot\vec{E}=u_xE_x+u_yE_y+u_zE_z\ . \tag{3.3}$$

Equation (2.34) separates into three decoupled equations, each determining one component of $\vec{u}$,

$$v_xA^2(\vec{q},\omega)\Gamma(\omega)+\frac{e}{c}H\left[v_x\frac{\partial}{\partial q_y}-v_y\frac{\partial}{\partial q_x}\right]u_x(\vec{q},\omega)=2\Gamma(\omega)u_x(\vec{q},\omega)-A(\vec{q},\omega)\sum_{\vec{k}}|g_{\vec{k}}|^2N_{\pm}u_x(\vec{q}-\vec{k},\omega)\ ,$$
$$v_yA^2(\vec{q},\omega)\Gamma(\omega)+\frac{e}{c}H\left[v_x\frac{\partial}{\partial q_y}-v_y\frac{\partial}{\partial q_x}\right]u_y(\vec{q},\omega)=2\Gamma(\omega)u_y(\vec{q},\omega)-A(\vec{q},\omega)\sum_{\vec{k}}|g_{\vec{k}}|^2N_{\pm}u_y(\vec{q}-\vec{k},\omega)\ , \tag{3.4}$$
$$v_zA^2(\vec{q},\omega)\Gamma(\omega)+\frac{e}{c}H\left[v_x\frac{\partial}{\partial q_y}-v_y\frac{\partial}{\partial q_x}\right]u_z(\vec{q},\omega)=2\Gamma(\omega)u_z(\vec{q},\omega)-A(\vec{q},\omega)\sum_{\vec{k}}|g_{\vec{k}}|^2N_{\pm}u_z(\vec{q}-\vec{k},\omega)\ .$$

We keep the phonon part in the scattering term only because its structure is more complicated than the impurity contribution. At the end of our calculation we put back the missing part. Every step done with the former one can be carried over to the latter. The operator $N_{\pm}$ takes care of the arrangements of the occupation numbers not listed in (3.4) [compare Eq. (2.33)]. Calculating the conductivity tensor $\overleftrightarrow{\sigma}^H$ with (2.30) and (3.3), we have for its components

$$\sigma_{ij}^H=e^2\frac{1}{(2\pi)^4}\int d\omega\left[-\frac{\partial n_F}{\partial\omega}\right]\int d\epsilon\int_{\epsilon>\epsilon_q}d^3q\frac{\partial u_j}{\partial q_i}\ ,\quad i,j=x,y,z\ . \tag{3.5}$$

In this work we are especially interested in the leading high magnetic field components contributing to the transverse conductivity [compare Eq. (1.7)]. Therefore, we have only to calculate u_x and u_y.

At this stage it is convenient to introduce cylinder coordinates, with its z direction parallel to the field $\vec{H}$ and

a plane perpendicular to $\vec{H}$. We illustrate this in Fig. 1, where an electron is moving along a trajectory of constant energy. Components with the subscripts z and $\perp$, respectively, lay along the magnetic field or in a plane perpendicular to $\vec{H}$.

We multiply the equation for u_y by i and add the result to u_x. The new function

$$g(\vec{q},\omega)=u_x(\vec{q},\omega)+iu_y(\vec{q},\omega) \tag{3.6}$$

obeys the equation

$$\left[v_{q_\perp}\frac{\partial g(\vec{q},\omega)}{\partial\phi}-v_\phi\frac{\partial g(\vec{q},\omega)}{\partial q_\perp}\right] = -\frac{c\Gamma(\omega)}{He}q_\perp\left[\left[v_{q_\perp}+i\frac{1}{q_\perp}v_\phi\right]e^{i\phi}A^2(\vec{q},\omega)-2g(\vec{q},\omega)+A(\vec{q},\omega)\frac{1}{\Gamma(\omega)}\sum_{\vec{k}}|g_{\vec{q}-\vec{k}}|^2N_{\pm}g(\vec{k},\omega)\right] \tag{3.7}$$

Now we keep the energy fixed $\epsilon=\epsilon_q$ and go from the parameters $q_\perp,\phi,q_z$ to a new set ϕ,q_z,ϵ. The function $q_\perp(\phi)$, defined by the intersection of a surface of constant energy ϵ and a plane perpendicular to the magnetic field which is located at q_z, is parametrized with

$$\frac{dq_\perp}{dt}=-v_\phi,\quad \frac{d\phi}{dt}=v_{q_\perp}\ . \tag{3.8}$$

The left-hand side of (3.7) is now a total differential with respect to t. Multiplying by dt we have

$$dg=-\frac{c\Gamma(\omega)}{He}\left[\left[q_\perp-i\frac{\partial q_\perp}{\partial\phi}\right]A^2(\epsilon,\omega)e^{i\phi}d\phi-2g(\phi,q_z,\epsilon,\omega)q_\perp dt + A(\epsilon,\omega)\frac{1}{\Gamma(\omega)}\sum_{\vec{k}}|g_{\vec{q}-\vec{k}}|^2N_{\pm}g(\Psi,k_z,\epsilon',\omega)q_\perp dt\right]\ . \tag{3.9}$$

We neglect angle-dependent contributions in the spectral function which come from the self-energies. Using (3.8) it is easy to show that the following relationship holds true:

$$q_\perp dt=\frac{q_\perp}{v_\perp}\left[1+\left[\frac{q_\perp'}{q_\perp}\right]^2\right]^{1/2}=\Phi(\phi,q_z,\epsilon)d\phi=\frac{dL}{v_\perp}\ . \tag{3.10}$$

Here dL is an arc of the trajectory under consideration and $v_\perp$ the velocity of the electron in the plane perpendicular to $\vec{H}$. The prime denotes a derivative with respect to ϕ. An angular integration along the trajectory of constant energy ϵ and fixed q_z gives

$$g(\phi,q_z,\epsilon)-g(\phi_0,q_z,\epsilon)=-\frac{c\Gamma(\omega)}{He}\left[A^2(\epsilon,\omega)\Delta(\phi,q_z,\epsilon)-2\int_{\phi_0}^{\phi}d\phi'\Phi(\phi',q_z,\epsilon)\times\left[g(\phi',q_z,\epsilon)-A(\epsilon,\omega)\frac{1}{2\Gamma(\omega)}\times\sum_{\vec{k}}|g_{q-k}|^2N_{\pm}g(\Psi,k_z,\epsilon',\omega)\right]\right], \tag{3.11}$$

$$\Delta(\phi,q_z,\epsilon)=-iq_\perp e^{i\phi}\Big|_{\phi_0}^{\phi}\ .$$

The continuum limit in the new set of variables is given by

$$\sum_{\vec{k}}f(\vec{k})\to\frac{1}{(2\pi)^3}\int d\epsilon\int dk_z\oint d\phi\,\Phi(\phi,k_z,\epsilon)f(\phi,k_z,\epsilon)\ . \tag{3.12}$$

To proceed it is convenient to write (3.11) in a shorthand notation. Omitting the arguments and setting all the complicated operations in an operator L_ϕ,

$$g = g_0 - \kappa A^2 \Delta - \kappa L_\phi g, \quad \kappa = \frac{c\Gamma}{He}, \tag{3.13}$$

$$(L_\phi g)(\phi, q_z, \epsilon, \omega) = -2 \int_{\phi_0}^{\phi} d\phi' \Phi(\phi', q_z, \epsilon) \times \left[g(\phi', q_z, \epsilon, \omega) - A(\epsilon, \omega) \frac{1}{2\Gamma(\omega)} \sum_{\vec{k}} |g_{\vec{q}-\vec{k}}|^2 N_{\pm} g(\Psi, k_z, \epsilon', \omega) \right]. \tag{3.14}$$

To look for high-field solutions, κ provides an appropriate expansion parameter. Therefore, the exact solution is represented by an expansion of the type

$$g = g^{(0)} + \kappa g^{(1)} + \kappa^2 g^{(2)} + \cdots . \tag{3.15}$$

Inserting (3.15) in (3.13) and collecting contributions of equal order we generate a sequence of equations

$$\begin{aligned} g^{(0)} &= g_0^{(0)} , \\ g^{(1)} &= g_0^{(1)} - A^2 \Delta - L_\phi g^{(0)} , \\ g^{(n)} &= g_0^{(n)} - L_\phi g^{(n-1)}, \quad n \geq 2 . \end{aligned} \tag{3.16}$$

We see at once that this system is not solvable in a unique way because g_0 itself is an integration constant and hence determined by boundary conditions. The kind of boundary conditions appropriate to our problem are closely related to the trajectories on which the electrons move. They move periodically on closed trajectories and in a complicated nonperiodical way on open ones. The influence of their topological structure on the high-field conductivity was first pointed out in the work by Lifshitz, Azbel, and Kaganov.[10]

For closed orbits a boundary condition of the form

$$g^{(n)}(\phi, q_z, \epsilon, \omega) = g^{(n)}(\phi + 2\pi, q_z, \epsilon, \omega) \tag{3.17}$$

is obvious. Equation (3.17) means the function g is a single valued function in a plane perpendicular to $\vec{H}$. If we indicate the integration over a closed path by $L_\phi \to L_{2\pi}$, Eq. (3.17) turns the system (3.16) into another one providing the integration constants $g_0^{(n)}$,

$$\begin{aligned} & L_{2\pi} g_0^{(0)} = 0 , \\ & L_{2\pi} g_0^{(1)} - L_{2\pi} A^2 \Delta - L_{2\pi} L_\phi g_0^{(0)} = 0 , \\ & L_{2\pi} g_0^{(n)} - L_{2\pi} L_\phi g^{(n-1)} = 0 , \quad n \geq 2 . \end{aligned} \tag{3.18}$$

Bearing in mind that g_0 does not depend on the angular variable ϕ and using (3.14), we can show that (3.18) is equivalent to

$$\begin{aligned} g_0^{(0)} &= B g_0^{(0)} , \\ g_0^{(n)} &= B g_0^{(n)} + c^{(n)} , \quad n \geq 1 \end{aligned} \tag{3.19a}$$

$$\begin{aligned} c^{(1)} &= Q^{-1}(L_{2\pi} A^2 \Delta + L_{2\pi} L_\phi g_0^{(0)}) , \\ c^{(n)} &= Q^{-1} L_{2\pi} L g^{(n-1)} , \quad n \geq 2 . \end{aligned} \tag{3.19b}$$

The actual form of B and Q can be deduced from (3.14), but is not important in what follows. Equations (3.16) and (3.19) determine the solution of (3.13) to be correct as far as closed orbits are concerned. We start the iteration at lowest order and work our way upwards. The very first step is simple. Because g is a single valued function, the first equation in (3.19a) yields $g^{(0)} = 0$. The next step is a little bit more cumbersome. Without lack of generality $g^{(1)}$ can be written like

$$g^{(1)}(\phi, q_z, \epsilon, \omega) = D^{(1)}(q_z, \epsilon, \omega) + i A^2(\epsilon, \omega) q_\perp(\phi) e^{i\phi} , \tag{3.20}$$

which is just a regrouping of angle-independent terms. To calculate $D^{(1)}$, we have to study the boundary conditions on $g^{(2)}$. This yields an integral equation of type (3.19a) with the inhomogeneity $\hat{c}^{(1)}$

$$\hat{c}^{(1)}(q_z, \epsilon, \omega) \sim -2i \oint d\phi \, \Phi(\phi, q_z, \epsilon) \left[A^2(\epsilon, \omega) q(\phi) e^{i\phi} - A(\epsilon, \omega) \frac{1}{2\Gamma(\omega)} \frac{1}{(2\pi)^3} \times \int dk_z \int d\epsilon' \oint d\Psi \, \Phi(\Psi, k_z, \epsilon') |g_{q-k}|^2 N_{\pm} A^2(\epsilon', \omega) k_\perp(\Psi) e^{i\Psi} \right]. \tag{3.21}$$

Equation (3.21) vanishes under certain conditions. The magnetic field must point in a direction of twofold or higher symmetry and the electron-phonon matrix element must be of the form $g_{\vec{q}} = g_{|\vec{q}|}$. The phonon dispersion is approximately correct in the Debye model $\omega_{\vec{k}} = \omega_{|\vec{k}|}$. We postpone the discussion of these restrictions for the moment and take them for granted. As a result, $D^{(1)}$ turns out to be zero for the same reasons $g_0^{(0)}$ was.

Further iteration is not possible in a simple way; therefore, we collect our results,

$$
\begin{aligned}
&g^{(0)}=0\ ,\\
&g^{(1)}=iA^2(\epsilon,\omega)q_\perp(\phi)e^{i\phi}\ ,\\
&g^{(2)}=g_0^{(2)}(q_z,\epsilon,\omega)+2iA^2(\epsilon,\omega)\int_0^\phi d\phi'\Phi(\phi',q_z,\epsilon)q_\perp(\phi')e^{i\phi'}\\
&\quad+iA(\epsilon,\omega)\frac{1}{\Gamma(\omega)}\int_0^\phi d\phi'\Phi(\phi',q_z,\epsilon)\sum_{\vec{k}}|g_{\vec{q}-\vec{k}}|^2N_\pm A^2(\epsilon',\omega)k_\perp(\Psi)e^{i\Psi}\ .
\end{aligned}
\tag{3.22}
$$

$g_0^{(2)}$ is still an unknown function, but does not depend on the angular variable ϕ. The energy-dependent part of the conductivity extracted from (1.6) and (3.5) is

$$
\sigma_{ij}^H(\omega)=\int d\epsilon\int_{\epsilon>\epsilon_q}d^3q\frac{\partial v_i(\vec{q},\omega)}{\partial q_i}\ ,\quad i,j=x,y,z\ .
\tag{3.23}
$$

Inserting the corresponding expressions u_x and u_y which are given by the real and imaginary parts of g, we finally get in lowest nonvanishing order

$$
\begin{aligned}
&\sigma_{xy}^H(\omega)=\frac{c\Gamma(\omega)}{He}\int d\epsilon A^2(\epsilon,\omega)[n_+(\epsilon)-n_-(\epsilon)]\ ,\\
&\sigma_{xy}^H(\omega)=-\sigma_{xy}^H(\omega)\ ,
\end{aligned}
\tag{3.24}
$$

$$
\begin{aligned}
\sigma_{xx}^H(\omega)=&\left[\frac{c\Gamma(\omega)}{He}\right]^2\left[2\int d\epsilon A^2(\epsilon,\omega)\int_{\epsilon>\epsilon_q}d^3q\,\Phi(\phi,q_z,\epsilon)\frac{q_y^2}{q_\perp^2}-\int d\epsilon\frac{A(\epsilon,\omega)}{\Gamma(\omega)}\int_{\epsilon>\epsilon_q}d^3q\,\Phi(\phi,q_z,\epsilon)\right.\\
&\left.\times\left[\sum_{\vec{k}}|g_{\vec{q}-\vec{k}}|^2\frac{q_yk_y}{q_\perp^2}N_\pm A^2(\epsilon',\omega)+n_i\sum_{\vec{k}}|v(q-k)|^2\frac{q_yk_y}{q_\perp^2}A^2(\epsilon',\omega)\right]\right]\ .
\end{aligned}
\tag{3.25}
$$

Changing x to y in (3.25) we get $\sigma_{yy}^H(\omega)$. $n_+(\epsilon)$ and $n_-(\epsilon)$ are, respectively, the volumes enclosed by an electronlike and holelike surface of constant energy.[10] The second equation, (3.24), is the Onsager relation in a magnetic field. If the linewidth Γ is not zero but sufficiently small, we can use the relation $A^2\approx A/\Gamma$.[27] We must evaluate the logarithmic derivations of $\sigma_{xy}^H(\omega)$ and $\sigma_{yy}^H(\omega)$. To this end we rewrite $\sigma_{xy}^H(\omega)$ and $\sigma_{yy}^H(\omega)$, and drop ω-independent factors

$$
\begin{aligned}
&\sigma_{xy}^H(\omega)\sim\int d\epsilon A(\epsilon,\omega)[n_+(\epsilon)-n_-(\epsilon)]\ ,\\
&\sigma_{yy}^H(\omega\sim\Gamma(\omega)\int d\epsilon A(\epsilon,\omega)L(\epsilon,\omega)\ ,
\end{aligned}
\tag{3.26a}
$$

$$
\begin{aligned}
L(\epsilon,\omega)=&\int_{\epsilon>\epsilon_q}d^3q\,\Phi(\phi,q_z,\epsilon)\left[\frac{q_y^2}{q_\perp^2}-\tfrac{1}{2}n_i\sum_{\vec{k}}|v(\vec{q}-\vec{k})|^2\frac{q_yk_y}{q_\perp^2}\frac{A(\epsilon',\omega)}{\Gamma(\omega)}\right.\\
&-\tfrac{1}{2}\sum_{\vec{k}}|g_{\vec{q}-\vec{k}}|^2\frac{q_yk_y}{q_\perp^2}\left[[n_B(\omega_{\vec{q}-\vec{k}})+1-n_F(\omega-\omega_{\vec{q}-\vec{k}})]\frac{A(\epsilon',\omega-\omega_{\vec{q}-\vec{k}})}{\Gamma(\omega-\omega_{\vec{q}-\vec{k}})}\right.\\
&\left.\left.+[n_B(\omega_{\vec{q}-\vec{k}})+n_F(\omega+\omega_{\vec{q}-\vec{k}})]\frac{A(\epsilon',\omega+\omega_{\vec{q}-\vec{k}})}{\Gamma(\omega-\omega_{\vec{q}-\vec{k}})}\right]\right]
\end{aligned}
\tag{3.26b}
$$

Here we restored the electron-impurity scattering term. The spectral function $A(\epsilon,\omega)$ is given by

$$
A(\epsilon,\omega)=\frac{2\Gamma(\omega)}{[\omega-\epsilon-\mathrm{Re}\Sigma^r(\omega)]^2+\Gamma^2(\omega)}\ .
\tag{3.27}
$$

The self-energy parts $\mathrm{Re}\Sigma^r(\omega)$ and $\Gamma(\omega)=-\mathrm{Im}\Sigma^r(\omega)$ are evaluated and averaged on the Fermi surface, so that only the ω dependence remains. In a metal, self-energy corrections are small compared to the Fermi energy, so that the main k dependence of the spectral function is attributed to $\epsilon=\epsilon_k$. The calculation of the ω derivation is straightforward. Neglecting contributions of the form $\partial\Gamma(\omega)/\partial\omega$ which are small[28] compared to the rest, we

get

$$\frac{d\sigma_{xy}^{H}(\omega)}{d\omega}\sim -2\left[1-\frac{\partial}{\partial\omega}\,\mathrm{Re}\Sigma^{r}(\omega)\right]\int d\epsilon\,\mathrm{Re}G^{r}(\epsilon,\omega)A(\epsilon,\omega)\{n_{+}(\omega)-n_{-}(\omega)\}\ , \tag{3.28}$$

$$\frac{d\sigma_{yy}^{H}(\omega)}{d\omega}\sim -2\left[1-\frac{\partial}{\partial\omega}\,\mathrm{Re}\Sigma^{r}(\omega)\right]\Gamma(\omega)\int d\epsilon\,\mathrm{Re}G^{r}(\epsilon,\omega)A(\epsilon,\omega)L(\epsilon,\omega)$$

$$+\Gamma(\omega)\int d\epsilon A(\epsilon,\omega)\frac{\partial}{\partial\omega}L(\epsilon,\omega)\ . \tag{3.29}$$

$\mathrm{Re}G^{r}(\epsilon,\omega)$ is the real part of the retarded Green's function $G^{r}(\epsilon,\omega)$:

$$G^{r}(\omega)=[\omega-\epsilon-\Sigma^{r}(\omega)]^{-1}\ . \tag{3.30}$$

In the small-$\Gamma(\omega)$ limit we can simplify (3.28) and (3.29) further,

$$\left.\frac{d\sigma_{xy}^{H}(\omega)}{d\omega}\right|_{\mathrm{re}}\sim 2\pi\left[1-\frac{\partial}{\partial\omega}\,\mathrm{Re}\Sigma^{r}(\omega)\right]\left[\frac{\partial}{\partial z}[n_{+}(z)-n_{-}(z)]\right]_{z=\omega-\mathrm{Re}\Sigma(\omega)}\ , \tag{3.31a}$$

$$\left.\frac{d\sigma_{yy}^{H}(\omega)}{d\omega}\right|_{\mathrm{re}}\sim 2\pi\left[1-\frac{\partial}{\partial\omega}\,\mathrm{Re}\Sigma^{r}(\omega)\right]\Gamma(\omega)\left[\frac{\partial}{\partial z}L(z,\omega)\right]_{z=\omega-\mathrm{Re}\Sigma(\omega)}$$

$$+2\pi\Gamma(\omega)\left[\frac{\partial}{\partial\omega}L(z,\omega)\right]_{z=\omega-\mathrm{Re}\Sigma(\omega)}\ . \tag{3.31b}$$

In the same order, we get for the conductivities $\sigma_{xy}^{H}(\omega)$ and $\sigma_{yy}^{H}(\omega)$ with the use of Eq. (3.26)

$$\sigma_{xy}^{H}(\omega)\,|_{\mathrm{re}}\sim 2\pi[n_{+}(z)-n_{-}(z)]_{z=\omega-\mathrm{Re}\Sigma(\omega)}\ , \tag{3.32a}$$

$$\sigma_{yy}^{H}(\omega)\,|_{\mathrm{re}}\sim 2\pi\Gamma(\omega)[L(z,\omega)]_{z=\omega-\mathrm{Re}\Sigma(\omega)}\ . \tag{3.32b}$$

For metal electrons, the small-$\Gamma(\omega)$ limit is not a serious restriction, because we must compare self-energy contributions of order 10 meV to the Fermi energy, which is of order 10 eV.

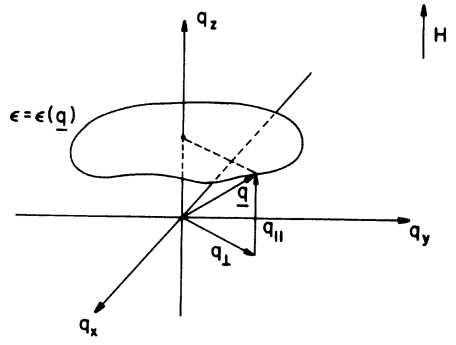

FIG. 1. The electron moves on a trajectory along the intersection of a surface of constant energy ϵ with a plane perpendicular to the magnetic field $\vec{H}$ located at $q_z=q_{\parallel}$. $q_{\perp}$ and $q_{\parallel}$ are, respectively, the components of $\vec{q}$ perpendicular and parallel to $\vec{H}$.

In comparison, we also evaluate the unrenormalized conductivities by approximating the spectral function in (3.26) by a δ function:

$$\left.\frac{d\sigma_{xy}^{H}(\omega)}{d\omega}\right|_{\mathrm{un}}\sim 2\pi\left[\frac{\partial}{\partial z}[n_{+}(z)-n_{-}(z)]\right]_{z=\omega}\ , \tag{3.33a}$$

$$\left.\frac{d\sigma_{yy}^{H}(\omega)}{d\omega}\right|_{\mathrm{un}}\sim 2\pi\Gamma(\omega)\left[\frac{\partial}{\partial z}L(z,\omega)\right]_{z=\omega}$$

$$+2\pi\Gamma(\omega)\left[\frac{\partial}{\partial\omega}L(z,\omega)\right]_{z=\omega}\ , \tag{3.33b}$$

$$\sigma_{xy}(\omega)\sim 2\pi[n_{+}(z)-n_{-}(z)]_{z=\omega}\ , \tag{3.34a}$$

$$\sigma_{yy}(\omega)\sim 2\pi\Gamma(\omega)[L(z,\omega)]_{z=\omega}\ . \tag{3.34b}$$

We are almost close to (1.9). The difference between our result and that proposed by Opsal *et al.*[4] is that the renormalized thermopower, according to (1.7), is not simply the unrenormalized one multiplied by $(1+\lambda)$ because the second term in (3.31b) has no enhancement factor. For the temperature region (2–5 K) of interest, impurity scattering is dominant. Kaveh and Wiser[29] reported a fraction of 10^3 for both scattering mechanisms in aluminum without a

magnetic field. Both mechanisms enter $L(\epsilon,\omega)$ the same way and we expect that their ratio will *not* change dramatically. Neglecting the phonon contribution we have

$$L(\epsilon,\omega)=\int_{\epsilon>\epsilon_q} d^3q\,\Phi(\phi,q_z,\epsilon) \times\left[\frac{q_y^2}{q_\perp^2}-\tfrac{1}{2}n_i\sum_{\vec{k}}|v(\vec{q}-\vec{k})|^2 \times\frac{q_y k_y}{q_\perp^2}\frac{A(\epsilon',\omega)}{\Gamma(\omega)}\right]. \tag{3.35}$$

It can be shown that (3.35) leads to the many-body generalization of a result obtained by Wagner,[12] who calculated the high-field conductivity with a classical Boltzmann equation approach.

Employing the same approximations as used in the derivation of Eq. (3.31), we can calculate $\partial L/\partial\omega$ in the second term of (3.31b) with $L(\epsilon,\omega)$ according to (3.35). As a result, we find that the renormalized version of that contribution is the unrenormalized one multiplied by the enhancement factor $(1+\lambda)$. We therefore have for the thermopower

$$S_d(H\to\infty)|_{\rm re}=\left[1-\frac{\partial\,{\rm Re}\Sigma^r}{\partial\omega}\right]_{\omega=\epsilon_F} S_d(H\to\infty)|_{\rm un}\,, \tag{3.36}$$

where Σ^r contains both electron-phonon and electron-impurity interactions. The electron-phonon self-energy gives a significant result in the ω derivative of its real part.[27] The impurity part is usually only weakly dependent on ω around $\omega-\epsilon_F$.[30] If this is true we can neglect it. Otherwise we do have a second contribution in the enhancement factor. We can confirm the result derived by Opsal *et al.*[4] with certain restrictions.

We would like to conclude this section with a remark on the Hall constant R in high magnetic fields[27]

$$R=\frac{E_y}{HJ_x}\,, \tag{3.37}$$

where E_y is the Hall field counterbalancing the influence of the Lorentz force on the electron flow in the x direction. The Hall constant R is related to the off-diagonal resistivity tensor element ρ_{yx}, where $\overleftrightarrow{\rho}$ is $\overleftrightarrow{\sigma}^{-1}$. Using the high-field limit of $\overleftrightarrow{\sigma}$ [Eqs. (3.24)] It is easy to calculate the corresponding resistivity tensor. The Hall coefficient is then

$$R=-\frac{1}{ce}\left[\left[\int\frac{d\epsilon}{2\pi}A(\epsilon,\omega)[n_+(\epsilon)-n_-(\epsilon)]\right]^{-1}\right]_{\omega=\epsilon_F} \tag{3.38}$$

This is exactly the well-known result for an uncompensated metal if we replace the spectral function $A(\epsilon,\omega)$ by $2\pi\delta(\epsilon-\omega)$, which is the usual quasiparticle approximation.

V. CONCLUSIONS

We presented a calculation of the low-temperature thermopower in a high magnetic field and essentially confirmed the suggestion made by Opsal *et al.*[4] Based on measurements on samples of aluminum they concluded that the low-temperature high-field magnetothermopower shall contain an electron-phonon mass enhancement factor. Our results show that we can separate two contributions [compare Eq. (3.31b)]; one that does contain enhancement and another that does not. This different behavior, however, disappears if we neglect the phonon scattering term. It is small compared to the impurity scattering at sufficient low temperatures. The conjecture of Opsal *et al.*[4] is correct in this case.

In this paper we continued our previous work[1] on many-body transport equations. Starting from the Keldysh formulation[3] of nonequilibrium quantum mechanics we derived a transport equation which now includes the magnetic field. We have shown that the Kadanoff-Baym method[2] gives the same equation for the steady-state solution.

Although we solved the transport equation for high magnetic fields, we have to restrict the solutions to a range of the magnetic field $\vec{H}$ and temperature T where the discrete structure of the Landau levels is not yet important. This is a usual restriction to transport theory.[22] Thus we are not able to explain, for instance, oscillations of the resistivity (Schubnikov–de Haas effect). To include these effects involves taking terms higher than first order in the magnetic field into account. The approximation of the retarded Green's function [compare Eq. (2.14b)] is then no longer justified. An extension of the presented formulation including quantum effects in the magnetic field is under current investigation.

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APPENDIX A: TRANSPORT EQUATION FOR STATIC ELECTRIC AND MAGNETIC FIELDS

In Sec. III we derived a new transport equation for particle motion in static electric and magnetic fields. This derivation employed the Keldysh formulation of the many-particle equations. Here we

will show that the same final equation is obtained starting from the Kadanoff-Baym formalism. The two derivations give the same result. This is expected, since there are very general proofs that the two formalisms are equivalent.[14] We include the derivation starting from the Kadanoff-Baym formulation simply because this method is familiar to most readers.

Earlier we derived the transport equation in a static electric field. There the electric field was introduced as a longitudinal vector potential. Now we introduce both fields as contributions to the vector potential

$$\vec{A}(\vec{r},t)=-et\vec{E}-\tfrac{1}{2}\vec{r}\times\vec{B}\ . \tag{A1}$$

With this form for the vector potential, we follow exactly the same steps as before.

The first step is to write equations of motion for the retarded G^r and other $G^<$ Green's functions in terms of their coordinates $x=(\vec{r},t)$,

$$\left[i\frac{\partial}{\partial t_1}-H(\vec{r}_1,\vec{p}_1,t_1)\right]G^<(x_1,x_2)=\int dx'[\Sigma_t(x_1,x')G^<(x',x_2)-\Sigma^<(x_1,x')G_{\bar{t}}(x',x_2)]\ , \tag{A2a}$$

$$\left[-i\frac{\partial}{\partial t_2}-H(\vec{r}_2,-\vec{p}_2,t_2)\right]G^<(x_1,x_2)=\int dx'[G_t(x_1,x')\Sigma^<(x',x_2)-G^<(x_1,x')\Sigma_{\bar{t}}(x',x_2)]\ , \tag{A2b}$$

$$\left[i\frac{\partial}{\partial t_1}-H(\vec{r}_1,\vec{p}_1,t_1)\right]G^r(x_1,x_2)=\delta(x_1-x_2)+\int dx'\Sigma^r(x_1,x')G^r(x',x_2)\ , \tag{A2c}$$

$$\left[-i\frac{\partial}{\partial t_2}-H(\vec{r}_2,-\vec{p}_2,t_2)\right]G^r(x_1,x_2)=\delta(x_1-x_2)+\int dx'G^r(x_1,x')\Sigma^r(x',x_2)\ . \tag{A2d}$$

The two Hamiltonian functions have the form

$$H(\vec{r}_1,\vec{p}_1,t_1)=\frac{1}{2m}\left[\vec{p}_1+et_1\vec{E}+\frac{e}{2c}\vec{r}_1\times\vec{B}\right]^2\ , \tag{A3a}$$

$$H(\vec{r}_2,-\vec{p}_2,t_2)=\frac{1}{2m}\left[\vec{p}_2-et_2\vec{E}-\frac{e}{2c}\vec{r}_2\times\vec{B}\right]^2\ . \tag{A3b}$$

Each pair of equations is first added, and then subtracted. Next we change to c.m. coordinates as defined in Sec. III. In these coordinates the new equations have on the left-hand sides

$$\left[2i\frac{\partial}{\partial t}-\frac{1}{m}\left[\vec{p}_r+eT\vec{E}+\frac{e}{2c}\vec{R}\times\vec{B}\right]^2-\frac{1}{4m}\left[\vec{P}_R+et\vec{E}+\frac{e}{2c}\vec{r}\times\vec{B}\right]^2\right]G$$

and

$$\left[i\frac{\partial}{\partial T}-\frac{1}{m}\left[\vec{p}_r+eT\vec{E}+\frac{e}{2c}\vec{R}\times\vec{B}\right]\cdot\left[\vec{P}_R+et\vec{E}+\frac{e}{2c}\vec{r}\times\vec{B}\right]\right]G\ .$$

The next step is to Fourier transform the variables $(\vec{r},t)$ to the set $(\vec{k},\omega)$. This transformation is easy to apply to the left-hand sides of the equations. In the scattering terms, on the right-hand sides of the equations, the transformation generates a series of terms in higher powers of space $(\vec{R})$ and time (T) derivatives. These latter series are described in Refs. 1, 2, and 14 so we only need to quote their results. We follow standard practice, and only retain first derivatives

$$\left[\omega-\frac{1}{2m}\left[\vec{k}+eT\vec{E}+\frac{e}{2c}\vec{R}\times\vec{B}\right]^2+\frac{1}{8m}\left[\vec{\nabla}_R+e\vec{E}\frac{\partial}{\partial\omega}+\frac{e}{2c}\vec{B}\times\vec{\nabla}_k\right]^2\right]$$

$$\times G^<(\vec{k},\omega,\vec{R},T)=(\mathrm{Re}\Sigma^r)G^<+\Sigma^<(\mathrm{Re}G^r)+\frac{i}{4}[\Sigma^>,G^<]-\frac{i}{4}[\Sigma^<,G^>]\ , \tag{A4a}$$

$$i\left[\frac{\partial}{\partial T}+\frac{1}{m}\left[\vec{k}+eT\vec{E}+\frac{e}{2c}\vec{R}\times\vec{B}\right]\cdot\left[\vec{\nabla}_R+e\vec{E}\frac{\partial}{\partial\omega}+\frac{e}{2c}\vec{B}\times\vec{\nabla}_k\right]\right]G^<(\vec{k},\omega,\vec{R},T)$$

$$=\Sigma^>G^<-\Sigma^<G^>+i[\mathrm{Re}\Sigma^r,G^<]+i[\Sigma^<,\mathrm{Re}G^r]\ , \tag{A4b}$$

$$\left[\omega-\frac{1}{2m}\left[\vec{k}+eT\vec{E}+\frac{e}{2c}\vec{R}\times\vec{B}\right]^2+\frac{1}{8m}\left[\vec{\nabla}_R+e\vec{E}\frac{\partial}{\partial\omega}+\frac{e}{2c}\vec{B}\times\vec{\nabla}_k\right]^2 - \Sigma^r(\vec{k},\omega,\vec{R},T)\right]G^r(\vec{k},\omega,\vec{R},T)=1\ , \quad \text{(A5a)}$$

$$i\left[\frac{\partial}{\partial T}+\frac{1}{m}\left[\vec{k}+eT\vec{E}+\frac{e}{2c}\vec{R}\times\vec{B}\right]\cdot\left[\vec{\nabla}_R+e\vec{E}\frac{\partial}{\partial\omega}+\frac{e}{2c}\vec{B}\times\vec{\nabla}_k\right]\right]G^r=i[\Sigma_r,G^r]\ . \quad \text{(A5b)}$$

Although these equations are correct, they are not in a useful form. Instead, we must change the k variable to $\vec{Q}=\vec{k}+eT\vec{E}+(e/2c)\vec{R}\times\vec{B}$. The motivation for this change is that the current is the average value of $\vec{Q}$, and not of $\vec{k}$. This variable change alters a number of terms in the equation [compare Eqs. (2.22), (2.23), and (2.25)]:

$$\vec{Q}\to\vec{k}+eT\vec{E}+\frac{e}{2c}\vec{R}\times\vec{B}\ ,$$

$$\frac{\partial}{\partial T}\to\frac{\partial}{\partial T}+e\vec{E}\cdot\vec{\nabla}_Q\ ,$$

$$\vec{\nabla}_R\to\vec{\nabla}_R+(e/2c)\vec{B}\times\vec{\nabla}_Q\ ,$$

$$[X,Y]\to[X,Y]-e\vec{E}\cdot\left[\frac{\partial X}{\partial\omega}\vec{\nabla}_Q Y-\frac{\partial Y}{\partial\omega}\vec{\nabla}_Q X\right]+\frac{e}{c}\vec{B}\cdot[(\vec{\nabla}_Q X)\times(\vec{\nabla}_Q Y)]\ .$$

After making these changes, we derive our final transport equation for static electric and magnetic field,

$$\left[\omega-\frac{Q^2}{2m}+\frac{1}{8m}\left[\vec{\nabla}_R+e\vec{E}\frac{\partial}{\partial\omega}+\frac{e}{c}\vec{B}\times\vec{\nabla}_Q\right]^2\right]G^<(\vec{Q},\omega,\vec{R},T)$$
$$=(\mathrm{Re}\Sigma^r)G^<+\Sigma^<(\mathrm{Re}G^r)+\frac{i}{4}[\Sigma^>,G^<]-\frac{i}{4}[\Sigma^<,G^>]$$
$$-i\frac{e\vec{E}}{4}\cdot\left[\frac{\partial}{\partial\omega}\Sigma^>\vec{\nabla}_Q G^<-\frac{\partial}{\partial\omega}\Sigma^<\vec{\nabla}_Q G^>-\frac{\partial G^<}{\partial\omega}\vec{\nabla}_Q\Sigma^>+\frac{\partial G^>}{\partial\omega}\vec{\nabla}_Q\Sigma^<\right]$$
$$+i\frac{e\vec{B}}{4L}\cdot(\vec{\nabla}_Q\Sigma^>\times\vec{\nabla}_Q G^<-\vec{\nabla}_Q\Sigma^<\times\vec{\nabla}_Q G^>)\ , \quad \text{(A6a)}$$

$$i\left[\frac{\partial}{\partial T}+\frac{\vec{Q}}{m}\cdot\vec{\nabla}_R+\left[1-\frac{\partial}{\partial\omega}\mathrm{Re}\Sigma^r\right]e\vec{E}\cdot\vec{\nabla}_Q+e\left[\frac{\vec{Q}}{m}+\vec{\nabla}_Q\mathrm{Re}\Sigma^r\right]\cdot\left[\vec{E}\frac{\partial}{\partial\omega}+\frac{1}{c}\vec{B}\times\vec{\nabla}_Q\right]\right]$$
$$\times G^<(\vec{Q},\omega,\vec{R},T)-ie\vec{E}\cdot\left[\frac{\partial\Sigma^<}{\partial\omega}\vec{\nabla}_Q\mathrm{Re}G^r-\left[\frac{\partial}{\partial\omega}\mathrm{Re}G^r\right]\vec{\nabla}_Q\Sigma^<\right]-i\frac{e}{c}\vec{B}\cdot[(\vec{\nabla}_Q\Sigma^<)\times(\vec{\nabla}_Q\mathrm{Re}G^r)]$$
$$=\Sigma^>G^<-\Sigma^<G^>+i[\mathrm{Re}\Sigma^r,G^<]+i[\Sigma^<,\mathrm{Re}G^r]\ , \quad \text{(A6b)}$$

$$\left[\omega-\frac{Q^2}{2m}+\frac{1}{8m}\left[\vec{\nabla}_R+e\vec{E}\frac{\partial}{\partial\omega}+\frac{e}{c}\vec{B}\times\vec{\nabla}_Q\right]^2-\Sigma^r\right]G^r(\vec{Q},\omega,\vec{R},T)=1\ , \quad \text{(A7a)}$$

$$i\left[\frac{\partial}{\partial T}+\frac{\vec{Q}}{m}\cdot\vec{\nabla}_R+\left[1-\frac{\partial}{\partial\omega}\Sigma^r\right]e\vec{E}\cdot\vec{\nabla}_Q+\left[\frac{\vec{Q}}{m}+\vec{\nabla}_Q\Sigma^r\right]\cdot\left[e\vec{E}\frac{\partial}{\partial\omega}+\frac{e}{c}\vec{B}\times\vec{\nabla}_Q\right]\right]G^r=i[\Sigma^r,G^r]\ . \quad \text{(A7b)}$$

One can show that these equations are gauge invariant: The same equations are obtained regardless of the form of the scalar or vector potential which is used to introduce the electric and magnetic fields. If we use in (A6a) or (A6b) the ansatz (2.29)

$$G^<=inA-i\frac{\partial n}{\partial\omega}\gamma \quad \text{(A8)}$$

we derive the transport equation (2.33).

APPENDIX B: GRADIENT EXPANSION OF G^r

The retarded Green's function solves Eq. (2.13)

$$G^r = G_0^r + \tfrac{1}{2}(G^r\phi G_0^r + G_0^r\phi G^r + G^r\Sigma^r G_0^r + G_0^r\Sigma^r G^r) . \tag{B1}$$

We make a gradient expansion of the right-hand side of (B1). The interesting feature comes from the one-point contribution ϕ. We drop the two-point part Σ^r and add its contribution later. Therefore, we study the simplified equation

$$G^r = G_0^r + \tfrac{1}{2}(G^r\phi G_0^r + G_0^r\phi G^r) . \tag{B2}$$

The two terms in the brackets are integrals represented by

$$G^r\phi G_0^r = \int d^3\bar{r}\int d\bar{t}\, G^r(\vec{r}_1,t_1,\bar{\vec{r}},\bar{t}) \times\phi(\bar{\vec{r}},\bar{t})G_0^r(\bar{\vec{r}}-\vec{r}_2,\bar{t}-t_2) , \tag{B3a}$$

$$G_0^r\phi G^r = \int d^3\bar{r}\int d\bar{t} G_0^r(\vec{r}_1-\bar{\vec{r}},t-\bar{t}) \times\phi(\bar{\vec{r}},\bar{t})G^r(\bar{\vec{r}}_1,\bar{t},\vec{r}_2,t_2) . \tag{B3b}$$

The operator $\phi(\bar{\vec{r}},\bar{t})$ is given by (2.12) and contains a gradient operator $\vec{\nabla}_{\vec{r}}$. In both Eqs. (B3a) and (B3b) we can extract it from the integral. In (B3a) the operator $\vec{\nabla}_{\bar{r}}$ acts on the unperturbed Green's function G_0^r which depends only on the difference $\bar{\vec{r}}-\vec{r}_2$. Thus we can write $\vec{\nabla}_{\bar{r}} \to -\vec{\nabla}_{r_2}$. In (B3b) we use the fact that the vector potential commutes with the operator $\vec{\nabla}_{\bar{r}}$ and shift it by a partial integration to act on G_0^r and again use $\vec{\nabla}_{\bar{r}} \to -\vec{\nabla}_{r_1}$. The minus sign created by partial integration compensates for the former one. We have

$$G^r\phi G_0^r = -i\frac{e}{mc}\vec{\nabla}_{r_2}\int d^3\bar{r}\int dt\, G_r(\vec{r}_1,t_1,\bar{\vec{r}},\bar{t})\vec{A}(\bar{\vec{r}},\bar{t})G_0^r(\bar{\vec{r}}-\bar{\vec{r}}_2,\bar{t}-t_2) + \frac{e^2}{2mc^2}\int d^3\bar{r}\int d\bar{t}\, G_r(\vec{r}_1,t_1,\bar{\vec{r}},\bar{t})A^2(\bar{\vec{r}},\bar{t})G_0^r(\bar{\vec{r}}-\bar{r},\bar{t}-t_2) , \tag{B4a}$$

$$G_0^r\phi G^r = i\frac{e}{mc}\vec{\nabla}_{r_1}\int d^3\bar{r}\int d\bar{t}\, G_0^r(\vec{r}_1-\bar{\vec{r}},t_1-\bar{t})\vec{A}(\bar{\vec{r}},\bar{t})G^r(\bar{\vec{r}},\bar{t},\vec{r}_2,t_2) + \frac{e^2}{2mc^2}\int d^3\bar{r}\int d\bar{t}\, G_0^r(\vec{r}_1-\bar{\vec{r}},t_1-\bar{t})A^2(\bar{\vec{r}},\bar{t})G^r(\bar{\vec{r}},\bar{t},\vec{r}_2,t) . \tag{B4b}$$

In (B4) we go over to c.m. coordinates (2.7). In particular, we must replace

$$\vec{\nabla}_{r_1} \to \tfrac{1}{2}\vec{\nabla}_R + \vec{\nabla}_r , \qquad \vec{\nabla}_{r_2} \to \tfrac{1}{2}\vec{\nabla}_R - \vec{\nabla}_r . \tag{B5}$$

The gradient expansion of the integral is now straightforward. After taking the Fourier transform with respect to the relative coordinates we have

$$G^r(\vec{R},T,\vec{k},\omega) = G_0^r(\vec{k},\omega) - \frac{e}{mc}G_0^r(\vec{k},\omega)\vec{k}\cdot\vec{A}(\vec{R},T)G^r(\vec{R},T,\vec{k},\omega) + \frac{e^2}{2mc^2}G_0^r(\vec{k},\omega)A^2(\vec{R},T)G^r(\vec{R},T,\vec{k},\omega) + \frac{e}{4mc}\vec{\nabla}_R\{G_0^r(\vec{k},\omega)[G^r(\vec{R},T,\vec{k},\omega),\vec{A}(\vec{R},T)] - G^r(\vec{R},T,\vec{k},\omega)[G_0^r(\vec{k},\omega),\vec{A}(\vec{R},T)] - \vec{A}(\vec{R},T)[G_0^r(\vec{k},\omega),G^r(\vec{R},T,\vec{k},\omega)]\} . \tag{B6}$$

Adding the contribution from the two-point function Σ^r, where first-order terms cancel each other (similar to the $G_0^r A^2 G^r$ term), we have (2.14).

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